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# Spatial Frequency Scheduling for Uplink SC-FDMA based Linearly Precoded LTE Multiuser MIMO Systems

Zihuai Lin<sup>†</sup>, Pei Xiao<sup>††</sup>, Troels B. Sørensen<sup>†††</sup>, Branka Vucetic<sup>†</sup>

## Abstract

This paper investigates the performance of the 3GPP Long Term Evolution (LTE) uplink Single Carrier (SC) Frequency Division Multiple Access (FDMA) based linearly precoded multiuser Multiple Input Multiple Output (MIMO) systems with frequency domain packet scheduling. A mathematical expression of the received Signal to Interference plus Noise Ratio (SINR) for the studied systems is derived and a utility function based spatial frequency packet scheduling algorithms is investigated. The schedulers are shown to be able to exploit the available multiuser diversity in time, frequency and spatial domains.

## I. INTRODUCTION

The Single Carrier (SC) Frequency Division Multiple Access (FDMA) technique for uplink transmission has attracted much attention due to its low Peak to Average Power Ratio (PAPR) property in comparison to Orthogonal FDMA (OFDMA) technique [1, 2]. In 3GPP Long Term Evolution (LTE) (also known as Evolved-UMTS Terrestrial Radio Access (E-UTRA)), SC-FDMA and OFDMA have been selected for uplink and downlink transmissions, respectively [3]. The SC-FDMA signal can be obtained by using Discrete Fourier Transform (DFT) spread OFDMA, where the DFT is applied to transform the time domain input data symbols to the frequency domain before feeding them into an OFDMA modulator.

For broadband wireless transmissions, e.g., LTE OFDMA downlink and SC-FDMA uplink [4, 5], several consecutive subcarriers are usually grouped together in order to simplify the scheduling task. A basic scheduling unit is called a Resource Block (RB). The scheduler in a E-UTRAN NodeB (eNB) may assign single or multiple RBs to a User Equipment (UE).

Two MIMO schemes for SC-FDMA uplink transmission are being investigated under 3GPP LTE, namely, multi-user MIMO and single user MIMO [3]. For single user MIMO, the eNB only schedules a single user into one RB; whereas for multi-user MIMO, multiple UEs are allowed to transmit simultaneously on each RB. Both open loop and closed loop MIMO have been proposed. However, the latter provides both diversity and array gain, and hence superior

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performance. Due to its simplicity and robust performance, the use of linear precoding has been widely studied as a closed loop scheme in the open literature [6].

For broadband radio transmission, the channel may experience frequency selective fading. For SC-FDMA uplink transmission, data transmission of different users may experience different channel gain even in the same subcarrier. By assigning subcarriers to their favorable users with large channel gains via the scheduler at the base station, the frequency selective diversity can be provided. Thus, the overall system throughput can be improved. Previous works on channel dependent frequency domain only (without spatial domain) scheduling can be found in the literature, e.g., [7, 8] for downlink OFDMA systems and [4, 9] for uplink SC-FDMA systems. In [7], a multiuser subcarrier and power allocation scheme for OFDM systems was considered. A multiuser adaptive subcarrier and bit allocation algorithm was derived, it was shown that with their proposed algorithm the overall required transmit power can be reduced about 5 – 10 dB compared with the conventional OFDM without adaptive modulation. In [8], a transmit power adaptation algorithm was developed for multiuser OFDM systems in downlink transmission, the algorithm was used to maximize the total data rate. In [10], a fair and efficient channel dependent scheduling algorithm for High Speed Downlink Packet Access (HSPDA) systems was investigated. That algorithm aims to enhance the average throughput for each user by giving more priority for those users with low average throughput.

In [11], a frequency domain channel dependent scheduling for pilot channel employing adaptive transmission bandwidth in uplink SC-FDMA system was proposed. In their proposed scheme, the users with good Channel Quality Indicator (CQI) measurements were assigned with a wide pilot transmission bandwidth, whereas the users with poor CQI measurements were assigned with a narrow pilot transmission bandwidth. Compared with the system using a fixed pilot transmission bandwidth, the proposed adaptive pilot transmission scheme can significantly improve the cell throughput. All of the above mentioned works deal with open loop systems with single input single output channel under the assumption that the channel state information is not available at the transmitter.

In this paper, we investigate the system performance of a SC-FDMA based linearly precoded LTE uplink multi-user MIMO system, with spatial and frequency domain packet scheduling. The objective of this paper is to develop low complexity scheduling algorithms to provide higher throughput for the uplink transmission system. We will derive an novel mathematical expression of the received Signal Interference plus Noise Ratio (SINR) for the system under

investigation. With the proposed scheduling scheme, users are appropriately grouped to form a virtual (or distributed) multiuser MIMO configuration with linear precoding. Such kind of uplink MIMO is termed as uplink “Cooperative MIMO (Co-MIMO)” in [12, p.27]. When Co-MIMO is used in downlink by sharing data streams between the eNBs for precoding, it is sometimes called as network MIMO [12]. As stated in [12], “Co-MIMO is also possible in the uplink but is fundamentally more difficult to implement as no physical connection exists between the UE to share the data streams.” In this paper, we investigate the scenarios where the uplink Co-MIMO is possible. Practical situations where such assumption could apply: (1) users are close, such as they are within the range of WLAN, Bluetooth, etc. 2) for eNB to Relay communications where the relays play the role of users; Relays could be assumed to be deployed as a kind of meshed sub-network and therefore able to cooperate in transmission over the uplink interference channel. In both cases, one could foresee the need in connection with hot-spots - specific areas where capacity needs to be relieved by multiplexing transmissions in the uplink. It will be shown how the system throughput increases significantly compared with scheduling based on random users grouping. The main contributions of this paper are the derivation of the received SINR for both open loop and closed loop SC-FDMA based uplink MIMO systems, the development of a low complexity spatial frequency channel dependent scheduling algorithm and the proposal of a per RB based resource fair scheduling algorithm.

We start with a description of the system model in Section II, based upon which we derive an analytical expression of the received SINR. Multiuser scheduling algorithms are discussed in Section III, and simulation results are presented in Section IV. Finally, conclusions are drawn in Section V.

## II. SYSTEM MODEL

We consider a cellular multiple access system with  $n_R$  receiver antennas at eNB and single transmit antenna at the  $i$ th user terminal,  $i = 1, 2, \dots, K_T$  where  $K_T$  is the total number of users in the system. We consider the multi-user MIMO system in which  $K$  ( $K < K_T$ ) users are served at each time slot, and we assume  $K = n_R$ . The system model for the SC-FDMA based MIMO transmitter and receiver are shown by Fig. 1 and 2, respectively. On the transmitter side, each user data block containing  $N$  symbols is firstly transformed by an  $N$  point DFT to a frequency domain representation. At the  $n$ th subcarrier,  $n \in \{1, \dots, N\}$ , the frequency domain representation of the users' data are then passed through a precoding matrix  $\mathbf{B}_n$  of size  $K \times K$  in Fig. 1. The precoding matrix will be further explained later. The outputs are

then mapped to  $M$  ( $M > N$ ) orthogonal subcarriers followed by a  $M$  point Inverse Fast Fourier Transform (IFFT) to convert them to a time domain complex signal sequence. A Cyclic Prefix (CP) is inserted into the signal sequence before it is passed to the Radio Frequency (RF) module. On the receiver side, the opposite operating procedure is performed after the noisy signals are received at the receiver antennas. A MIMO Frequency Domain Equalizer (FDE) is applied to the frequency domain signals after subcarrier demapping as shown in Fig. 2. For simplicity, we employ a linear Minimum Mean Squared Error (MMSE) equalizer, which provides a good tradeoff between the noise enhancement and the multiple stream interference mitigation [13].

In the following, we let  $\mathbf{D}_{\mathbf{F}_M} = \mathbf{I}_K \otimes \mathbf{F}_M$  and denote by  $\mathbf{F}_M$  the  $M \times M$  Fourier matrix with the element  $[\mathbf{F}_M]_{m,k} = \exp(-j\frac{2\pi}{M}(m-1)(k-1))$  where  $k, m \in \{1, \dots, M\}$  are the sample number and the frequency tone number, respectively. Here  $\otimes$  is the Kronecker product,  $\mathbf{I}_K$  is an identity matrix of dimension  $K$ . We denote by  $\mathbf{D}_{\mathbf{F}_M}^{-1} = \mathbf{I}_K \otimes \mathbf{F}_M^{-1}$  the  $KM \times KM$  dimension inverse Fourier matrix, where  $\mathbf{F}_M^{-1}$  is the  $M \times M$  inverse Fourier matrix with each element  $[\mathbf{F}_M^{-1}]_{m,k} = \frac{1}{M} \exp(j\frac{2\pi}{M}(m-1)(k-1))$ . The  $N \times N$  matrices  $\mathbf{D}_{\mathbf{F}_N}$  and  $\mathbf{D}_{\mathbf{F}_N}^{-1}$  are defined in the similar way as  $\mathbf{D}_{\mathbf{F}_M}$  and  $\mathbf{D}_{\mathbf{F}_M}^{-1}$ . Furthermore, we let  $F_n$  represent the subcarrier mapping matrix of size  $M \times N$  and  $F_n^{-1}$  the subcarrier demapping matrix of size  $N \times M$ .

The received signal after RF and removing CP can be expressed as  $\mathbf{r} = \mathbf{H}\mathbf{D}_{\mathbf{F}_M}^{-1}(\mathbf{I}_K \otimes F_n)\mathbf{D}_{\mathbf{F}_N}\tilde{\mathbf{x}} + \mathbf{w}$ , where  $\tilde{\mathbf{x}} = [\tilde{\mathbf{x}}^{(1)}, \dots, \mathbf{x}^{(K)}]^T \in \mathbb{C}^{KN \times 1}$  is the data sequence of all  $K$  users, and  $\tilde{\mathbf{x}}^{(i)} \in \mathbb{C}^{1 \times N}$ ,  $i \in \{1, \dots, K\}$  is the data block for the  $i$ th user.  $\mathbf{w} \in \mathbb{C}^{Mn_R \times 1}$  is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix  $N_0\mathbf{I} \in \mathbb{R}^{Mn_R \times Mn_R}$ , i.e.,  $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ .  $\mathbf{H} \in \mathbb{C}^{n_R \times K}$  is the block channel matrix defined as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \cdots & \mathbf{H}_{1,K} \\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2} & \cdots & \mathbf{H}_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{n_R,1} & \mathbf{H}_{n_R,2} & \cdots & \mathbf{H}_{n_R,K} \end{bmatrix},$$

where  $\mathbf{H}_{i,j} \in \mathbb{C}^{M \times M}$  is the diagonal channel matrix between the  $j$ th transmit antenna and the  $i$ th receive antenna, the  $n$ th diagonal element of  $\mathbf{H}_{i,j}$  corresponds to the  $n$ th symbol of the transmitted data block at the  $j$ th transmit antenna.

With the frequency domain equalizer, the signal at the detector becomes

$$\tilde{\mathbf{z}} = \mathbf{D}_{\mathbf{F}_N}^{-1}\mathbf{A}(\mathbf{I}_K \otimes F_n^{-1})\mathbf{D}_{\mathbf{F}_M}\mathbf{r} \quad (1)$$

where  $\mathbf{A}$  is a  $K \times K$  block diagonal equalization matrix with the  $i$ th matrix on the diagonal equal to  $\mathbf{A}^{(i)}$  which is a

$N \times N$  frequency domain equalization matrix for the  $i$ th user,  $i \in \{1, \dots, K\}$ .

In frequency domain, the transmitted symbols from the  $i$ th user at the  $n$ th subcarrier before linear precoding is  $x_{n,i} = \sqrt{p_{n,i}}s_{n,i}$  where  $p_{n,i}$  and  $s_{n,i}$ ,  $i \in \{1, 2, \dots, K\}$ , are the transmitted power and the data symbol for the  $i$ th user at the  $n$ th subcarrier, respectively. If gathering all the  $K$  users' data and arrange them into a vector, from the receiver point of view, all the  $K$  users's data can be considered as from a virtual single user with  $K$  antennas. The transmitted signal from the virtual user at the  $n$ th subcarrier before linear precoding is  $\mathbf{x}_n = \mathbf{P}_n \mathbf{s}_n$ , where  $\mathbf{P}_n = \text{diag}(\sqrt{p_{n,1}}, \sqrt{p_{n,2}}, \dots, \sqrt{p_{n,K}})$  is a  $K \times K$  diagonal matrix.  $\mathbf{s}_n \in \mathbb{C}^{K \times 1}$  represents the transmitted data symbol vector from the virtual user with  $E[\mathbf{s}_n \mathbf{s}_n^H] = \mathbf{I}$ , where  $\mathbf{I}$  is an  $K \times K$  identity matrix. Each antenna of the virtual user corresponds to one user.

The received signal in frequency domain at the  $n$ th subcarrier can be expressed as  $\mathbf{y}_n = \mathbf{H}_n \mathbf{B}_n \mathbf{P}_n \mathbf{s}_n + \mathbf{w}_n$ , where  $\mathbf{B}_n \in \mathbb{C}^{K \times K}$  is the transmit precoding matrix of the virtual user at the  $n$ th subcarrier. As shown later,  $\mathbf{B}_n$  can be obtained from the channel matrix  $\mathbf{H}_n$  and the channel noise covariance matrix.  $\mathbf{H}_n$  is the  $n_R \times K$  complex channel matrix between the virtual user and the eNB (note that we assume  $n_R = K$ ), each element of which represents the complex channel propagation gain and  $\mathbf{y}_n = [y_n^1, \dots, y_n^K]^T$  (see Fig. 2).  $\mathbf{H}_n$  is obtained from the matrix  $\mathbf{\Lambda}$ , which is defined as  $\mathbf{\Lambda} = (\mathbf{I}_K \otimes F_n^{-1}) \mathbf{D}_{\mathbf{F}_M} \mathbf{H} \mathbf{D}_{\mathbf{F}_M}^{-1} (\mathbf{I}_K \otimes F_n)$ . The  $K \times K$  block matrix  $\mathbf{\Lambda}$  consists of  $N \times N$  diagonal submatrices  $\mathbf{\Lambda}_{i,j}$ ,  $i, j \in \{1, 2, \dots, K\}$ . The  $(i, j)$ th element of  $\mathbf{H}_n$  is the  $n$ th diagonal element of  $\mathbf{\Lambda}_{i,j}$ .  $\mathbf{w}_n \in \mathbb{C}^{n_R \times 1}$  is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix  $N_0 \mathbf{I} \in \mathbb{R}^{n_R \times n_R}$ , i.e.,  $\mathbf{w}_n \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I})$ .

With linear frequency domain equalizers, which are designed for each subcarrier, the signal after equalization becomes<sup>1</sup>

$$\mathbf{z}_n = \mathbf{A}_n [\mathbf{H}_n \mathbf{B}_n \mathbf{P}_n \mathbf{s}_n + \mathbf{w}_n] \quad (2)$$

The equalization matrix  $\mathbf{A}_n \in \mathbb{C}^{K \times n_R}$  can be derived under the MMSE criterion (such that  $E[|\mathbf{s}_n - \mathbf{z}_n|^2]$  is minimized) as

$$\mathbf{A}_n = \mathbf{P}_n \mathbf{B}_n^H \mathbf{H}_n^H [\mathbf{H}_n \mathbf{B}_n \mathbf{R}_{X,n} \mathbf{B}_n^H \mathbf{H}_n^H + \mathbf{R}_{W,n}]^{-1} \quad (3)$$

where  $\mathbf{R}_{X,n} = E[\mathbf{x}_n \mathbf{x}_n^H] = \mathbf{P}_n \mathbf{P}_n^H$  and  $\mathbf{R}_{W,n} = E[\mathbf{w}_n \mathbf{w}_n^H] = N_0 \mathbf{I}$ . Note that the matrix  $\mathbf{A}_n^H$  can also be interpreted as a receiver beamforming matrix with each column representing a receiver beamvector corresponding to each transmitted

<sup>1</sup>Note that the size of  $\tilde{\mathbf{z}}$  of eq. (1) is  $KN \times 1$  and  $\tilde{\mathbf{z}}$  is in the time domain, whereas the size of  $\mathbf{z}_n$  is  $K \times 1$  and  $\mathbf{z}_n$  is in the frequency domain.

symbol. Substituting (3) into (2) and with some simple matrix manipulations, we get

$$\begin{aligned}\mathbf{z}_n &= [(\mathbf{R}_{W,n} + \mathbf{H}_n \mathbf{B}_n \mathbf{P}_n \mathbf{P}_n^H \mathbf{B}_n^H \mathbf{H}_n^H)^{-1} \mathbf{H}_n \mathbf{B}_n \mathbf{P}_n]^H [\mathbf{H}_n \mathbf{B}_n \mathbf{P}_n \mathbf{s}_n + \mathbf{w}_n] \\ &= \left( \mathbf{I} + \mathbf{P}_n \mathbf{B}_n^H (\mathbf{R}_{W,n}^{-1/2} \mathbf{H}_n)^H (\mathbf{R}_{W,n}^{-1/2} \mathbf{H}_n) \mathbf{B}_n \mathbf{P}_n \right)^{-1} \mathbf{P}_n \mathbf{B}_n^H (\mathbf{R}_{W,n}^{-1/2} \mathbf{H}_n)^H \mathbf{R}_{W,n}^{-1/2} (\mathbf{H}_n \mathbf{B}_n \mathbf{P}_n \mathbf{s}_n + \mathbf{w}_n). \quad (4)\end{aligned}$$

The second equality of (4) follows from the fact that  $(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}\mathbf{B}\mathbf{D}^{-1} = \mathbf{A}^{-1}\mathbf{B}(\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1}$ . Denoting  $\Phi_n = \mathbf{R}_{W,n}^{-1/2} \mathbf{H}_n$ , and using the Singular Value Decomposition (SVD),  $\Phi_n$  can be expressed as

$$\Phi_n = \mathbf{U}_n \Lambda_n \mathbf{V}_n^H, \quad (5)$$

where  $\mathbf{U}_n$  and  $\mathbf{V}_n$  are unitary matrices, and the columns of  $\mathbf{U}_n$  and  $\mathbf{V}_n$  are the eigenvectors of  $\Phi_n \Phi_n^H$  and  $\Phi_n^H \Phi_n$ , respectively. The singular values  $\lambda_{n,i}$ , ( $i \in \{1, 2, \dots, K\}$ ), of  $\Phi_n$  are the diagonal entries of  $\Lambda_n$  and are arranged in the descending order. To diagonalize the colored channel matrix  $\Phi_n$ , we can choose  $\mathbf{B}_n = \mathbf{V}_n$ , Equ. (4) then becomes

$$\mathbf{z}_n = (\mathbf{I} + \mathbf{P}_n \Lambda_n^H \Lambda_n \mathbf{P}_n)^{-1} \mathbf{P}_n \Lambda_n^H \Lambda_n \mathbf{P}_n \mathbf{s}_n + (\mathbf{I} + \mathbf{P}_n \Lambda_n^H \Lambda_n \mathbf{P}_n)^{-1} \mathbf{P}_n \Lambda_n^H \mathbf{U}_n \mathbf{R}_{W,n}^{-1/2} \mathbf{w}_n. \quad (6)$$

Consequently, the  $i$ th symbol  $\mathbf{z}_n(i)$  can be expressed as

$$\mathbf{z}_n(i) = \frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}} \mathbf{s}_n(i) + \frac{\sqrt{p_{n,i} \lambda_{n,i} / N_0}}{1 + p_{n,i} \lambda_{n,i}} \mathbf{w}'_n(i), \quad (7)$$

where  $\mathbf{w}'_n(i)$  is the  $i$ th element of the vector  $\mathbf{U}_n^H \mathbf{w}_n$ , and  $p_{n,i}$  is the power of the  $i$ th symbol.

The time-domain signal vector at the receiver at time interval  $k$  is

$$\begin{aligned}\tilde{\mathbf{z}}_k &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} nk} \mathbf{z}_n \\ &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} nk} \mathbf{A}_n [\mathbf{H}_n \mathbf{B}_n \sqrt{\mathbf{P}_n} \mathbf{s}_n + \mathbf{W}_n] \quad (8)\end{aligned}$$

where  $N$  is the number of occupied subcarriers,  $\tilde{\mathbf{z}}_k \in \mathbb{C}^{K \times 1}$ . The  $i$ th symbol of  $\tilde{\mathbf{z}}_k$  can be represented by

$$\tilde{\mathbf{z}}_k(i) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} nk} \left( \frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}} \mathbf{s}_n(i) + \frac{\sqrt{p_{n,i} \lambda_{n,i} / N_0}}{1 + p_{n,i} \lambda_{n,i}} \mathbf{w}'_n(i) \right). \quad (9)$$

Since  $\mathbf{s}_n(q) = \sum_{m=0}^{N-1} e^{-j \frac{2\pi}{N} mn} \tilde{\mathbf{s}}_m(q)$ , where  $\tilde{\mathbf{s}}_m(q)$  is the  $q$ th user's  $m$ th data symbol in the time domain. Alternatively,

$$\begin{aligned}
\tilde{\mathbf{z}}_k(i) &= \frac{1}{N} \tilde{\mathbf{s}}_k(i) \sum_{n=0}^{N-1} \frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}} + \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=0, m \neq k}^{N-1} \frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}} \tilde{\mathbf{s}}_m(i) e^{j \frac{2\pi}{N} (k-m)n} \\
&+ \frac{1}{N} \sum_{n=0}^{N-1} e^{j \frac{2\pi}{N} nk} \frac{\sqrt{p_{n,i} \lambda_{n,i} / N_0}}{1 + p_{n,i} \lambda_{n,i}} \mathbf{w}'_n(i).
\end{aligned} \tag{10}$$

The first term of the right hand side of (19) represents the desired signal, the second term is the intersymbol interference from the same substream, and the third term is due to the noise. The power of the desired signal is then  $P_s = |\frac{1}{N} \sum_{n=0}^{N-1} \frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}}|^2$ . The total power of the received signal can be obtained from (18) as  $P_{total} = \frac{1}{N} \sum_{n=0}^{N-1} |\frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}}|^2$ . The power of the noise is  $P_{noise} = \frac{1}{N} \sum_{n=0}^{N-1} \frac{p_{n,i} \lambda_{n,i}}{(1 + p_{n,i} \lambda_{n,i})^2}$ .

In the time domain, the received SINR for the  $i$ th symbol at time interval  $k$  is then  $\gamma_{k,i} = \frac{P_s}{P_{total} - P_s + P_{noise}}$ . With some simple mathematical manipulations, we can then obtain

$$\gamma_{k,i} = \left[ \frac{1}{\frac{1}{N} \sum_{n=0}^{N-1} \frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}}} - 1 \right]^{-1}. \tag{11}$$

Since  $\gamma_{k,i}$  is not a function of  $k$  any more, we just denote  $\gamma_{k,i}$  as  $\gamma_i$ .

It can be seen that the instantaneous SINR for the  $i$ th user  $\gamma_i$  is completely determined by  $\mathbf{H}_n$ , the noise variance  $N_0$  (implied by the  $\Phi_n$  matrix), the number of occupied subcarriers  $N$  and the transmitted power matrix  $\mathbf{P}_n$ . Note that in [14], SINR expression for an open-loop SC-FDMA system with frequency domain equalizer is derived for a single antenna case. However, the SINR expressed by (11) applies to a linearly precoded MIMO system with multiple antennas, it is therefore more general.

The maximum achievable spectrum efficiency in bits/second/Hz based on Shannon's capacity is

$$r_i = \log_2(1 + \gamma_i) = \log_2 \left( 1 + \left[ \frac{1}{\frac{1}{N} \sum_{n=0}^{N-1} \frac{p_{n,i} \lambda_{n,i}}{1 + p_{n,i} \lambda_{n,i}}} - 1 \right]^{-1} \right). \tag{12}$$

For broadband wireless communication systems, e.g., 3GPP LTE uplink, the total bandwidth  $B$  is usually divided into a number of  $M$  subcarriers. Among  $M$  subcarriers,  $N$  subcarriers ( $N < M$ ) are allocated for data transmission.  $L$  contiguous subcarriers form a scheduling RB. Let  $I_{sub,i}$  and  $|I_{sub,i}|$  be the index set of subcarriers assigned to user  $i$  and the length of the set  $I_{sub,i}$ , respectively. Denote by  $P_t^i$  the total transmitted power of user  $i$ . Assuming that the power is equally allocated over  $I_{sub,i}$ , then  $p_{n,i} = P_t^i / |I_{sub,i}|$ . The maximum achievable rate in bits per second for the  $i$ th user can then be written as



$$C_i = \frac{B|I_{sub,i}|}{M} \log_2 \left( 1 + \left[ \frac{1}{\frac{1}{|I_{sub,i}|} \sum_{n \in I_{sub,i}} \frac{P_t^i \lambda_{n,i}}{|I_{sub,i}| + P_t^i \lambda_{n,i}}} - 1 \right]^{-1} \right). \quad (13)$$

Above we derived the received SINR and the maximum achievable rate for linearly precoded uplink SC-FDMA based MIMO systems. In the Appendix, the received SINR for open loop uplink SC-FDMA MIMO systems is derived. The open loop uplink SC-FDMA MIMO systems is served as a reference for the studied linearly precoded system.

### III. SPATIAL FREQUENCY MULTIUSER SCHEDULING

For localized FDMA uplink multiuser MIMO transmission<sup>2</sup>, each SC-FDMA uplink transmission sub-frame can be partitioned into several RBs for the convenience of multiple user channel aware packet scheduling [3, 4]. Let  $I_{RB,i}$  be the index set of RBs assigned to user  $i$  within one sub-frame and  $|I_{RB,i}|$  be the length, the number of total RBs in one sub-frame is  $|I_{RB}|$ . Then  $|I_{RB,i}|L = |I_{sub,i}|$ . Multiple contiguous RBs can be assigned to one user within one sub-frame.

Denote by  $\phi_j$  the  $j$ th set of  $K$  users which are selected from the total  $K_T$  users in the system and let  $\Phi$  be the whole set of  $K$  users chosen from total  $K_T$  users,  $\phi_j \in \Phi, \forall j \in \{1, 2, \dots, |\Phi|\}$ , where  $|\Phi|$  is the size of  $\Phi$ , and

$$|\Phi| = \binom{K_T}{K}.$$

Let us define  $U_j(\phi)$  as the utility function for the  $j$ th RB.

The objective is to maximize the utility function by selecting the users group with appropriate channel condition and optimizing the set of RBs assigned to each user within one subframe. The optimization problem can be described as

$$\begin{aligned} & \max_{\forall \phi \in \Phi; \phi: I_{RB,i}, P_t^i, \forall i \in \phi} \sum_{j=1}^{|I_{RB}|} U_j(\phi), \\ & s.t.1 : \bigcup_{\forall i \in \phi} |I_{RB,i}|L = N, \\ & s.t.2 : I_{sub,i}^{k+1} - I_{sub,i}^k = 1, \forall k \in \{1, 2, \dots, |I_{sub,i}| - 1\}, \end{aligned} \quad (14)$$

where  $I_{sub,i}^k$  is the  $k$ th element in the set  $I_{sub,i}$ . The subconstraint 1 ensures that all the available RBs are assigned to the users in  $\phi$ . The subconstraint 2 corresponds to the localized FDMA transmission, i.e., the user data is transmitted by a group of consecutive subcarriers. Note that the above optimization problem is not equivalent to maximize the utility

<sup>2</sup>In the localized FDMA transmission scheme, each user's data is transmitted by consecutive subcarriers, while for the distributed FDMA transmission scheme, the user's data is transmitted by distributed subcarriers [3].

function for each RB subject to the user's power constraint. This is because if we consider each RB independently, the channel frequency selective fading property cannot be exploited. The frequency selective fading channel property may make some users experience excellent channel condition for two or more consecutive RBs. Since the power of the  $i$ th user is constrained to be  $P_t^i$ , and the power for each subcarrier is obtained by  $p_{n,i} = P_t^i / |I_{sub,i}|$ , in the case of multiple RBs assigned for the  $i$ th user, the power for each subcarrier  $p_{n,i}$  of that user is reduced.

For the multiuser MIMO scheduling scheme, the set of the RBs should be optimized for each user within each transmitted sub-frame. The optimization problem can be summarized as: among  $K_T$  users, we choose  $K$  of them and allocate these  $K$  users to the available RBs to maximize the utility function (14). The optimal solution to the optimization problem involves a high computational complexity. Therefore, low complexity suboptimal algorithms are needed for practical implementation. In what follows, we propose a greedy algorithm to solve the above optimization problem. The algorithm is performed in two steps: the first step is to schedule users for each RB, i.e., find users group or paired users for each RB to optimize the utility function. The second step is to assign available RBs for the paired users.

For the first step, we need to find the best users group for each RB. At this stage, we can maximize the utility function for each RB. We can define  $U(\phi) = \sum_{i \in \phi} C_i$ . Maximization of this utility function is equivalent to optimization of the total system capacity. This may result in an unfair situation, i.e., only the users with good channel conditions get resources.

To tackle this problem, we propose a resource fair allocation algorithm for each RB based utility function maximization. The key idea of the proposed fair resource allocation algorithm is to limit the users with more RBs used in a past certain period  $T_{win}$ , and give priority to those users with less transmissions in the period  $T_{win}$ . The algorithm works as follow: Let  $\alpha_i$  be the moving average of used RBs by the  $i$ th user in the past  $T_{win}$  at interval  $k$  and  $\alpha_i^k = (1 - \frac{1}{T_{win}})\alpha_i^{(k-1)} + \frac{1}{T_{win}}\delta$ , where  $\delta = 1$  if the user  $i$  gets scheduled, otherwise  $\delta = 0$ . We define the utility function at the  $k$ th interval as  $U_k(\phi) = \sum_{i \in \phi} f(\alpha_i^k, C_i)$ , where  $f(\alpha_i^k, C_i)$  is a function of  $\alpha_i^k$  and  $C_i$ . The per RB based scheduling problem then becomes

$$\phi^* = \arg \max_{\forall \phi \in \Phi} \sum_{i \in \phi} f(\alpha_i^k, C_i). \quad (15)$$

For comparison purpose, the Proportional Fair (PF) scheduling algorithm [5, 15] is also investigated in this work. For a set of  $K$  users who share the same wireless link, a PF scheduler allocate the rate  $R_i$  for the  $i$ th user  $i \in \{1, 2, \dots, K\}$  such that for any other rate allocation  $\hat{R}_i, i \in \{1, 2, \dots, K\}$ , there exists  $\sum_{i=1}^K \frac{\hat{R}_i - R_i}{R_i} \leq 0$ . In other words, some users may perform better in terms of the relative rate with the rate allocation  $\hat{R}_i, i \in \{1, 2, \dots, K\}$ , but overall gain cannot be achieved. Such scheduler is called a proportional fair scheduler as it provides fairness among users in the system. It can be shown that under the constraint of the overall capacity of the shared link, the PF scheduling algorithm maximizes  $\sum_{i=1}^K \log_2(R_i)$ . It is shown in [16] that a scheduler is PF if the instantaneous rate  $\{C_i\}$  maximize  $\sum_{i=1}^K \left[ \frac{C_i}{\bar{R}_{k,i}} \right]$ , where  $\bar{R}_{k,i}$  is the moving average of the maximum achievable rate of user  $i$  at the  $k$ th time slot<sup>3</sup> over a sliding window of  $T_{win}$  time slots.

After the best user groups for each RB are determined, we are ready for the second step. Let  $I_{RB,i}^q$  denote the set of RBs allocated to user group  $i$  with an additional adjacent RB  $q$  added into the set  $I_{RB,i}$  within one sub-frame. Let  $\Lambda_{k,i}^q$  be the rate increment at time interval  $k$  when the  $i$ th user group is allocated  $I_{RB,i}^q$  instead of  $I_{RB,i}$ . Then the greedy RB allocation based on rate increment can be described as follows. First, we pick the  $Q$  best user groups for each RB in relation to the rate increment, and add them into the available user group set  $S_{ugp}$ .

Step 1. Add all available  $Q$  RBs into a set  $S_{RB} = \{1, 2, \dots, Q\}$ .

Step 2. For each RB, find the best user group in terms of the highest rate increment. That is, for each user group  $i^*$  and RB  $q^*$ , find the pair

$$[i^*, q_i^*] = \arg \max_{q \in S_{RB}, i \in S_{ugp}} \Lambda_{k,i}^q \quad (16)$$

Step 3. For each user group  $j^*$ , find an adjacent RB  $q_o^*$  of  $q_j^*$  from the set  $\{S_{RB} - q_j^*\}$  which has the maximum rate increment  $\Lambda_{k,j^*}^{q_o^*}$ .

Step 4. Choose the user group and the additional adjacent RB pair  $[h^*, q_t^*]$ , which has the maximum rate increment  $\Lambda_{k,h^*}^{q_t^*}$  among all the available user groups and RBs.

Step 5. Delete the RB  $q_t^*$  from the available RB set  $S_{RB}$  and its corresponding user group  $t^*$  in the available user group set  $S_{ugp}$ . Repeat Step 3, 4 and 5 until all RBs get assigned.

<sup>3</sup>One time slot corresponds one Transmit Time Interval (TTI) which is defined as the time duration for one sub-frame transmission, e.g., 0.5 ms [3].

#### IV. NUMERICAL RESULTS

For the results presented in this section, we consider the 3GPP LTE baseline antenna configuration with two receive antennas at the eNB and one transmit antenna at the UE [17]. Two UEs are grouped together and synchronized to form a virtual MIMO channel between eNB and UEs. we consider the multipath fading channel. A typical urban channel model with six paths is assumed, each path suffers from independent Rayleigh fading. The Ergodic fading channel capacity [18] can be computed by averaging the instantaneous channel capacity over a large number of sub-frames. The system bandwidth is set to 900 kHz with a subcarrier spacing of 15 kHz. Hence there are 60 occupied subcarriers for full band transmission. We further assume these 60 subcarriers are arranged in 5 consecutive RBs per sub-frame, so that each RB contains 12 subcarriers. At each Monte-Carlo run, 100 sub-frames are used for data transmission and the power of each user is randomly generated to simulate the fact that users maybe in different locations. The simulation results are averaged over 50 Monte-Carlo run.

Fig. 3 shows the simulation results of the so called unified effective SINR distribution for the 3GPP LTE uplink MIMO system. The unified effective SINR is defined as the equivalent single SINR which offers the same instantaneous (Shannon) capacity as a MIMO system with multiple streams [6, 19]. The results for both open loop and closed loop MIMO schemes, e.g., spatial division multiplexing and linear precoding, are shown. The number of users which is available for scheduling in the system is 20. The transmitted Signal to Noise Ratio (SNR), which is defined as the total transmitted power of the paired users divided by the variance of the complex Gaussian noise, is equal to 20 dB. Random user Pairing Scheduling (RPS) algorithm described in [20] is also investigated for a baseline comparison. For random pairing scheduling, the first user is selected in a round robin fashion, while the second user is randomly selected from the rest of the users in the system.

From Fig. 3, it can be seen that the system with linear precoding (curves labeled with w.LPC in Fig. 3) has a better SINR distribution than the one without precoding (curves labeled with w.o.LPC). This observation is valid for all the investigated scheduling algorithms. For the system with precoding, at the 10th percentile of the post scheduled SINR, using maximum rate sum scheduling algorithm can achieve about 4 dB gain compared with the one using the resource fair scheduling algorithm. The latter is about 1 dB and about 6 dB better than the system with PF scheduling algorithm and the RPS scheduling algorithm, respectively.

Compared with the open loop spatial division multiplexing MIMO scheme, about 7 dB precoding gain can be achieved by using RPS, and approximately 4 dB gain for both the proposed resource fair scheduling algorithm and the PF algorithm. For the maximum rate sum scheduling algorithm, the precoding gain is about 3 dB.

Fig. 4 shows the simulation results for the maximum achievable rate in bits/second versus the number of available users for the open loop and the closed loop uplink MIMO with various scheduling algorithms, respectively. The transmitted SNR is 20 dB. It can be seen that as the number of users increases, the multiuser diversity gain can be achieved for all the investigated systems except the one with the RPS algorithm. This observation is valid for both open loop and closed loop uplink MIMO. The reason is that those non-random pairing schedulers have more freedom to choose the Ms with good channel condition and multiuser diversity can thus be exploited. Compared with the open loop MIMO, the multiuser diversity gain for closed loop MIMO is smaller. This can be explained by the fact that the diversity gain is already exploited by precoding, additional diversity gain obtained by multiuser MIMO cannot contribute too much in this case.

In terms of the fairness, we can look at the outage probabilities of these different algorithms. Here, the outage probability is defined as the the probability that the user data rate is less than a certain value. It is obtained by computing the ratio of the number of users whose data rates are lower than a certain value divided by the total number of users in the system. Throughout the simulations, the predetermined value is set to 0.01. The simulation results are shown in Fig. 5 for both the open loop and the closed loop uplink MIMO. For all the simulations, zero outage probabilities of PF scheduling for both the open loop and the closed loop uplink MIMO has been observed, which means that the PF scheduling provides fairness for both schemes. The proposed resource fair scheduling algorithm has slightly worse outage probability performance compared with PF scheduling. For both the open loop and closed loop systems, the max sum rate scheduling algorithm has the worst performance in terms of the outage probability. This is because the maximum rate sum algorithm always chooses the users with the good channel conditions, the users with poor channel quality have less opportunities to be scheduled.

Interestingly, compared with the open loop uplink MIMO, the closed loop uplink MIMO has better outage probability performance for the maximum rate sum scheduling algorithm. This is because for the maximum rate sum algorithm, precoding can increase the channel capacity between the users' group and the eNB, it can make the user selection more

diverse, therefore, the outage probability can be improved.

## V. CONCLUSION

In this work, we derived an analytical expression of the received SINR for SC-FDMA based uplink MIMO system with linear precoding, and proposed a channel-aware sub-optimal spatial frequency scheduling algorithm. The SINR distribution and the maximum achievable rate per RB for the system with different multiuser scheduling algorithms were investigated. Compared with the open loop uplink MIMO system with SC-FDMA, linear precoding can improve the system performance in terms of the SINR and the maximum achievable sum rate. Compared with RPS, all the other investigated scheduling algorithms yield better SINR distribution and can achieve multiuser diversity gain. The results presented in this paper are obtained under the assumption that perfect Channel State Information (CSI) at the UEs is available. Further works considering limited CSI, packet length, queuing, etc. will be the future research topics for the authors.

## APPENDIX

### *Derivation of the received SINR for open loop SC-FDMA MIMO Systems*

In this section, we will derive the received SINR for open loop uplink SC-FDMA based MIMO systems. The open loop system model is similar to the linearly precoded system model described by Figs. 1 and 2, the difference is that the precoding matrix is removed in the open loop system. With frequency domain equalizers, which are designed for each subcarrier, the signal after the equalizer is then  $\varphi_n = \Psi_n[\mathbf{H}_n \mathbf{x}_n + \mathbf{w}_n]$ , where  $\mathbf{H}_n$  is the  $n_R \times K$  complex channel matrix with each element of  $\mathbf{H}_n$  representing the complex channel propagation gain.  $\mathbf{H}_n$  is obtained from the matrix  $\mathbf{\Lambda}$ , which is defined as  $\mathbf{\Lambda} = (\mathbf{I}_K \otimes F_n^{-1}) \mathbf{D}_{\mathbf{F}_M} \mathbf{H} \mathbf{D}_{\mathbf{F}_M}^{-1} (\mathbf{I}_K \otimes F_n)$ .  $\mathbf{\Lambda}$  is a  $K \times K$  block matrix with the  $(i, j)$ th submatrix  $\mathbf{\Lambda}_{i,j}$ ,  $i, j \in \{1, 2, \dots, K\}$ .  $\mathbf{\Lambda}_{i,j}$  is a  $N \times N$  diagonal matrix. The  $(i, j)$ th element of  $\mathbf{H}_n$  is the  $n$ th diagonal value of  $\mathbf{\Lambda}_{i,j}$ .  $\mathbf{x}_n$  is the transmitted data symbols from the  $K$  users at the  $n$ th subcarrier and where  $\mathbf{w}_n \in \mathbb{C}^{n_R \times 1}$  is a circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix  $N_0 \mathbf{I} \in \mathbb{R}^{n_R \times n_R}$ .  $\mathbf{x}_n$  can be further expressed as  $\mathbf{x}_n = \mathbf{P}_n \cdot \mathbf{s}_n$ , where  $\mathbf{P}_n = \text{diag}\{\sqrt{p_{n,1}}, \sqrt{p_{n,2}}, \dots, \sqrt{p_{n,K}}\}$  is a  $K \times K$  diagonal matrix and  $p_{n,i}$  ( $i \in \{1, 2, \dots, K\}$ ) is the transmitted power for the  $i$ th user at the  $n$ th subcarrier.  $\mathbf{s}_n \in \mathbb{C}^{K \times 1}$  represents the transmitted data symbol vector from different users with  $E[\mathbf{s}_n \mathbf{s}_n^H] = \mathbf{I}_K$ .

The frequency domain equalization matrix  $\Psi_n \in \mathbb{C}^{K \times K}$  can be obtained by using the Minimum Mean Squared Error

(MMSE) criterion such that  $E[|\mathbf{s}_n - \boldsymbol{\varphi}_n|^2]$  is the minimum. By taking the partial derivative of the mean square error with respect to  $\boldsymbol{\Psi}_n^H$ . The Linear MMSE (LMMSE) equalizer matrix is then  $\boldsymbol{\Psi}_n = \mathbf{R}_{x,n} \mathbf{H}_n^H [\mathbf{H}_n \mathbf{R}_{x,n} \mathbf{H}_n^H + \mathbf{R}_{w,n}]^{-1}$ , where  $\mathbf{R}_{x,n} = E[\mathbf{x}_n \mathbf{x}_n^H] = \mathbf{P}_n \mathbf{P}_n^H$  and  $\mathbf{R}_{w,n} = E[\mathbf{w}_n \mathbf{w}_n^H] = N_0 \mathbf{I}_K$ . The signal vector detected at the receiver in the time domain at time interval  $k$  can be expressed as

$$\tilde{\boldsymbol{\varphi}}_k = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nk} \boldsymbol{\Psi}_n [\mathbf{H}_n \mathbf{P}_n \mathbf{s}_n + \mathbf{w}_n] \quad (17)$$

where  $\tilde{\boldsymbol{\varphi}}_k \in \mathbb{C}^{K \times 1}$  and  $N$  is the number of occupied subcarriers.

The signal vector detected at the receiver in the time domain at time interval  $k$  can be expressed as (17). Let  $\boldsymbol{\Omega}_n = \boldsymbol{\Psi}_n \mathbf{H}_n$ , then the  $i$ th symbol of  $\tilde{\mathbf{y}}_k$  can be represented by

$$\tilde{\varphi}_k(i) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nk} \left\{ \sum_{q=1}^K \sqrt{p_{n,q}} [\boldsymbol{\Omega}_n]_{i,q} \mathbf{s}_n(q) + \sum_{q=1}^K [\boldsymbol{\Psi}_n]_{i,q} \mathbf{w}_n(q) \right\}, \quad (18)$$

where the  $(n, k)$  element of a matrix  $\mathbf{A}$  is represented by  $[\mathbf{A}]_{n,k}$ .

Since  $\mathbf{s}_n(q) = \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}mn} \tilde{\mathbf{s}}_m(q)$ , where  $\tilde{\mathbf{s}}_m(q)$  is the  $m$ th data symbol in the time domain of user  $q$ . Then,

$$\begin{aligned} \tilde{\varphi}_k(i) &= \frac{1}{N} \tilde{\mathbf{s}}_k(i) \sum_{n=0}^{N-1} \sqrt{p_{n,i}} [\boldsymbol{\Omega}_n]_{i,i} + \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=0, m \neq k}^{N-1} \sqrt{p_{n,i}} [\boldsymbol{\Omega}_n]_{i,i} \tilde{\mathbf{s}}_m(i) e^{j\frac{2\pi}{N}(k-m)n} + \frac{1}{N} \sum_{q=1, q \neq i}^K \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \\ &\quad e^{j\frac{2\pi}{N}(k-m)n} \sqrt{p_{n,q}} [\boldsymbol{\Omega}_n]_{i,q} \tilde{\mathbf{s}}_m(q) + \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nk} \sum_{q=1}^K [\boldsymbol{\Psi}_n]_{i,q} \mathbf{w}_n(q). \end{aligned} \quad (19)$$

The first term of the right hand side of (19) represents the received desired signal, the second term is the intersymbol interferences from the same substream, the third term is the interference from the other substreams, and the fourth one is the noise.

The power of the received desired signal is then  $P_s = |\frac{1}{N} \sum_{n=0}^{N-1} \sqrt{p_{n,i}} [\boldsymbol{\Omega}_n]_{i,i}|^2$ .

The total power of the received signal can be obtained from (18),

$$P_{total} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{q=1}^K p_{n,q} |[\boldsymbol{\Omega}_n]_{i,q}|^2 \quad (20)$$

The power of the noise is

$$P_{noise} = \frac{N_0}{N} \sum_{n=0}^{N-1} \sum_{q=1}^K |[\boldsymbol{\Psi}_n]_{i,q}|^2 = \frac{N_0}{N} \sum_{n=0}^{N-1} [\boldsymbol{\Psi}_n \boldsymbol{\Psi}_n^H]_{i,i}. \quad (21)$$

The received SINR for the  $i$ th symbol at time interval  $k$  is then

$$\gamma_{k,i} = \frac{P_s}{P_{total} - P_s + P_{noise}} = \left[ \frac{\sum_{n=0}^{N-1} \left\{ \sum_{q=1}^K p_{n,q} |\mathbf{\Omega}_n|_{i,q}|^2 + N_0 [\mathbf{\Psi}_n \mathbf{\Psi}_n^H]_{i,i} \right\}}{\frac{1}{N} \sum_{n=0}^{N-1} \sqrt{p_{n,i}} |\mathbf{\Omega}_n|_{i,i}|^2} - 1 \right]^{-1}. \quad (22)$$

It can be seen  $\gamma_{k,i}$  is not a function of  $k$ , thus we can denote it by  $\gamma_i$ . Since both  $\mathbf{\Omega}_n$  and  $\mathbf{\Psi}_n$  are a function of the  $n$ th subcarrier channel matrix  $\mathbf{H}_n$ ,  $\gamma_i$  is completely determined by  $\mathbf{H}_n$ , the noise variance  $N_0$ , the number of occupied subcarriers  $N$  and the transmitted power matrix  $\mathbf{P}_n$ .  $\gamma_i$  represents the instantaneous SINR for the  $i$ th user.

#### REFERENCES

- [1] H. G. Myung, J. Lim, and D. J. Goodman, "Single Carrier FDMA for Uplink Wireless Transmission," *IEEE Vehicular Technology Magazine*, vol. 1, no. 3, pp. 30–38, Sept. 2006.
- [2] K. Fazel and S. Kaiser, *Multi-Carrier and Spread Spectrum Systems*, John Wiley & Sons Ltd., 2003.
- [3] 3GPP TR 25.814 V7.0.0, "Physical Layer Aspects for Evolved UTRA," Tech. Rep., June 2006.
- [4] R1060048, "Channel dependent packet scheduling for single carrier FDMA in E-UTRA uplink," Tech. Rep., 3GPP TSG-RAN1 WG1 LTE ad hoc, Helsinki, Finland, Jan. 2006.
- [5] J. L. Lim, H. G. Myung, K. Oh, and D. J. Goodman, "Proportional fair scheduling of uplink single carrier FDMA systems," in *PIMRC*, 2006.
- [6] Z. Lin, T. B. Sørensen, and P. E. Mogensen, "Downlink SINR distribution for linearly Precoded MIMO Systems," *IEEE Communications Letters*, vol. 11, no. 11, Nov. 2007.
- [7] C. Wong, R. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser ofdm with addaptive subcarrier, bit and power allocation," *IEEE Selected Areas in Comm.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.
- [8] J. Jang and K.B. Lee, "Transmit power adaptation for multiuser ofdm systems," *IEEE Selected Areas in Comm.*, vol. 21, no. 2, pp. 171–178, Feb. 2003.
- [9] J. Lim, H. H. Myung, K. Oh, and D. J. Goodman, "Channel-Dependent Scheduling of Uplink Single Carrier FDMA Systems," in *IEEE VTC*, September 2006, pp. 1–5.
- [10] B. Al-Manthari, N. Nasser, and H. Hassanein, "Fair and Efficient Channel Dependent Scheduling Algorithm for HSPDA System," in *Proc. of 2nd international symposium on wireless communication systems*, Sept. 2005, pp. 238–241.
- [11] Y. Ofuji, K. Higuchi, and M. Sawahashi, "Frequency domain channel-dependent scheduling employing an adaptive transmission bandwidth for pilot channel in uplink single-carrier-fdma radio access," in *Proc. of Vehicular Technology Conference, VTC 2006-Spring. IEEE 63rd.* IEEE, May 2006, vol. 1, pp. 7–10.
- [12] Agilent Technologies, "Agilent 3gpp long term evolution: System overview, product development, and test challenges," <http://cp.literature.agilent.com/litweb/pdf/5989-8139EN.pdf>.
- [13] A.J. Paulraj, R. Nabar & D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 1 edition, September 2003.
- [14] R1051335, "Simulation Methodology for EUTRA UL: IFDMA and DFT-Spread-OFDMA," Tech. Rep., 3GPP TSG-RAN1 WG1 Number 43, Seoul, Korea, Nov. 2005.
- [15] T. Park, O.-S. Shin, and K. B. Lee, "Proportional Fair Scheduling for Wireless Communication with Multiple Transmit and Receive Antennas," in *IEEE VTC*, Florida, USA, October 2003, vol. 3, pp. 1573 – 1577.
- [16] V. K. N. Lau, "Proportional fair space-time scheudling for wireless communications," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1353–1360, Aug. 2005.
- [17] 3GPP TR 25.913 V7.3.0, "Requirements for Evolved UTRA (E-UTRA) and Evolved UTRAN (E-UTRAN)," Tech. Rep., Mar. 2006.
- [18] E. Biglieri and G. Taricco, *Foundations and Trends in Communications and Information Theory*, vol. 1, now Publishers Inc, 2004.
- [19] N. Wei, T. B. Sørensen, T. E. Kolding, and P. E. Mogensen, "Analysis and Evaluation of Link Adaptation with MIMO adaptation," in *IEEE VTC*, Sept. 2006, pp. 1–5.
- [20] R1051422, "UL Virtul MIMO System Level Performance Evaluation for E-UTRA," Tech. Rep., 3GPP TSG-RAN1 WG1 Number 43, Seoul, Korea, Nov. 2005.



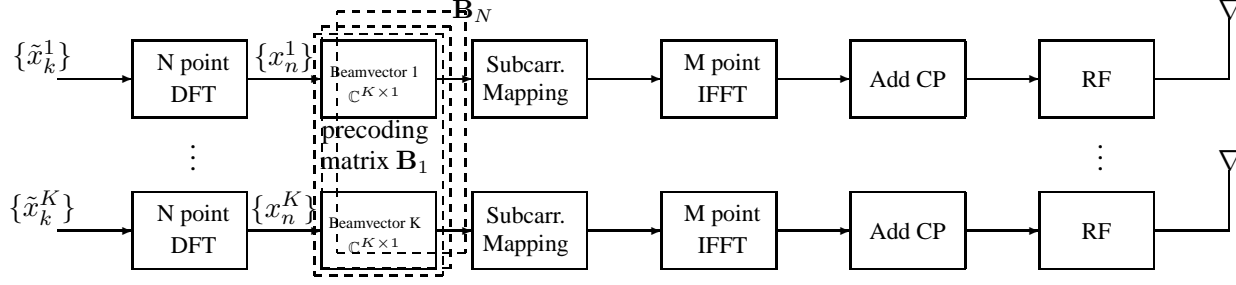


Fig. 1. SC-FDMA based Precoded MIMO Transmitter.

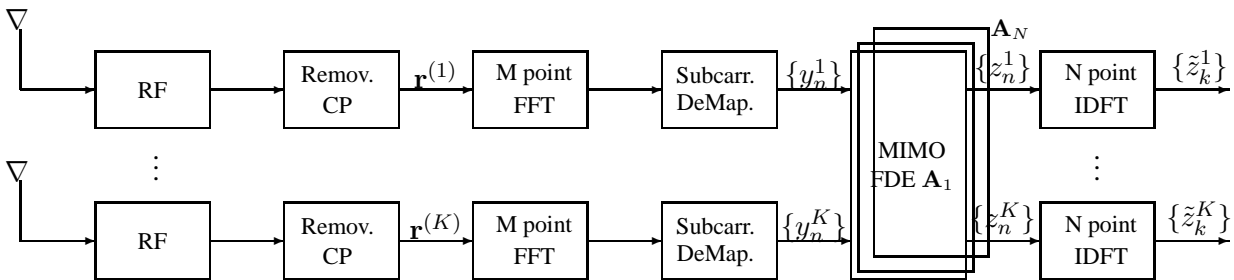


Fig. 2. SC-FDMA based Precoded MIMO Receiver.

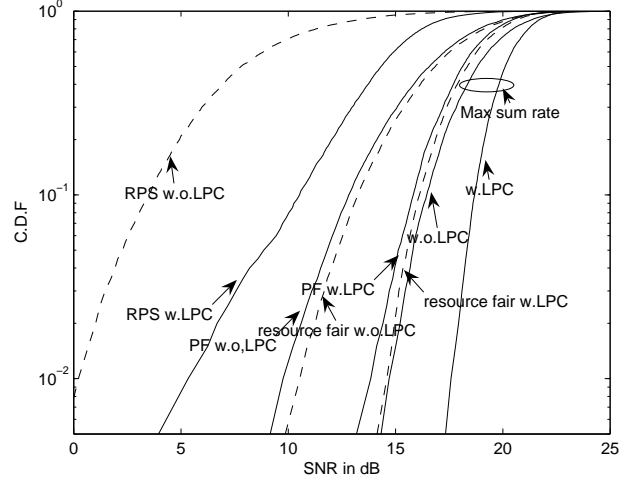


Fig. 3. SINR distribution for different scheduling algorithms with and without precoding, the number of users is 20 and the transmitted SNR is 20 dB.

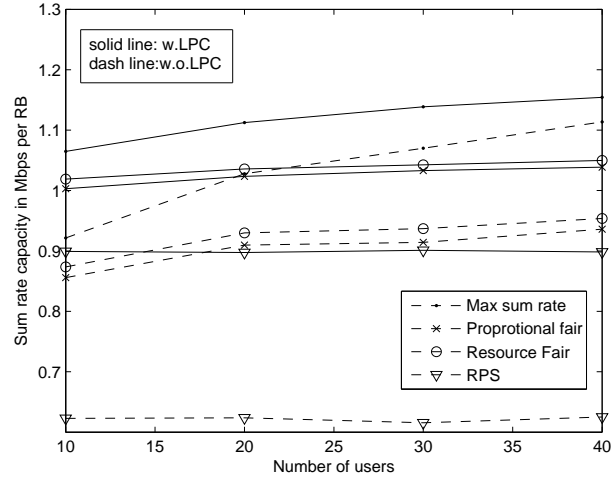


Fig. 4. Rate sum capacity for both open loop and closed loop uplink MIMO with various scheduling algorithms versus the number of users, the transmitted SNR is 20 dB.

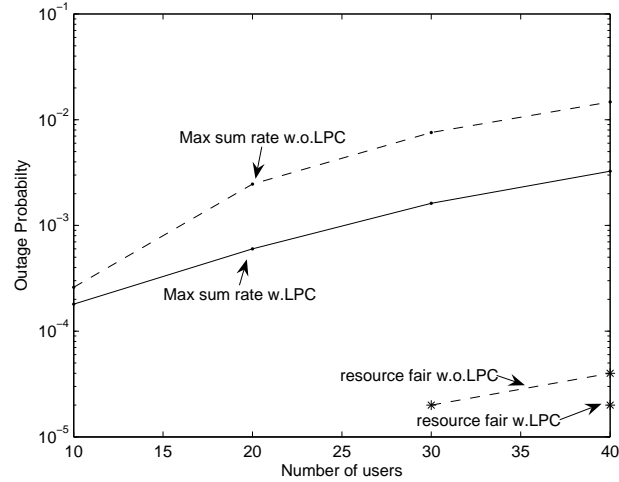


Fig. 5. Outage probability for both open loop and closed loop uplink MIMO with various scheduling algorithms versus the number of users, the transmitted SNR is 20 dB.