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RESEARCH ARTICLE

Analytical computation of the correlation of spreading sequences for nonlinear OQPSK-type modulations

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ABSTRACT

In this paper we study the behavior and correlation properties of some spreading sequences when applied to a system using nonlinear offset quadrature phase-shift keying-type modulations. These signals, which include as special cases continuous-phase modulation schemes, can be designed to have very low envelope fluctuations or even a constant envelope, making them compatible with a very low-cost and power-efficient grossly nonlinear amplification. We use an analytical method to derive the correlation of those signals. The resulting expressions are then applied to maximum-length sequences, Kasami, and Tomlinson, Cercas, Hughes sequences. It is shown that Tomlinson, Cercas, Hughes sequences have better correlation properties than other spreading sequences. Copyright © 2012 John Wiley & Sons, Ltd.

KEY WORDS
PN sequences; nonlinear modulations; correlation behavior

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1. INTRODUCTION

Pseudorandom sequences are widely employed in communication systems. Pseudorandom spreading sequences are used to differentiate between users in code division multiple access although their use is not restricted to spread spectrum systems. In fact, pseudorandom sequences are employed for synchronization [1] and channel estimation purposes [2], and position and range measurements [3]. When employed in code division multiple access communication systems, spreading sequences associated with different users should have a small correlation. Although the cross-correlation between orthogonal codes (e.g., Hadamard codes [4]), is zero, the corresponding sequences are no longer orthogonal after being submitted to non-ideal channels (e.g., the system is not synchronous and/or we have time-dispersive channels). The sequences used for synchronization, channel estimation, and positioning purposes should also have good autocorrelation properties, close to a Dirac function. Many families of pseudorandom sequences have been intensively studied to identify good sets that exhibit good correlation properties, that is, autocorrelation and/or cross-correlation. However, the majority of these studies usually assume ideal conditions that can be easily modeled and studied, such as linear modulations and ideal linear transmitters (i.e., with linear amplifiers), which do not exist in the real world.

On the other hand, it is well-known that efficient power amplification is important for communication systems in general and wireless communications in particular. Typical quasi-linear amplifiers are only linear for signals with small amplitude, which means that we either have high back-off (and, consequently, small amplification efficiency) or we need to work in the nonlinear region with significant signal distortion. Although this is especially serious for multicarrier schemes [5, 6], it is also important for single-carrier modulations [7, 8]. Whenever we need very high power amplifiers with low complexity we have to employ grossly nonlinear power amplifiers. This class of amplifiers has lower implementation complexity, higher amplification efficiency, and output power than linear amplifiers. However, grossly nonlinear power amplification should only be used with signals with constant or quasi-constant envelope, to avoid nonlinear distortion effects. Because of its importance in present and future systems, it is important to derive a model that can easily characterize the signals at the output of these nonlinear, but efficient, amplifier devices.

Continuous phase modulation [9, 10] schemes are constant-envelope modulations that include as special cases minimum shift keying (MSK) [11] and Gaussian MSK [12], among others. These modulations can be denoted as OQPSK-type (offset quadrature phase-shift keying) modulations because they can be decomposed as a sum of OQPSK components [13, 14]. OQPSK-type modulations also include other 'non-continuous phase modulation' schemes with almost constant envelope and good tradeoffs between spectral and power efficiencies [15–17]. OQPSK-type schemes are particularly important in the context of nonlinear amplification because an OQPSK-type signal retains its OQPSK-type structure when submitted to bandpass memoryless nonlinear devices, the usual model for power amplifiers [18], which simplifies the performance evaluation and receiver design [19, 20].

In this paper we consider the use of pseudorandom spreading sequences with nonlinear OQPSK-type signals. The main purpose of this study is to present analytical expressions to evaluate the correlation properties of pseudorandom sequences when submitted to a modulated system including nonlinearities. Its performance is studied and compared for three families of pseudorandom sequences, specifically maximum-length sequences (MLS), Kasami sequences, and Tomlinson, Cercas, Hughes (TCH) sequences, derived from the corresponding code sets. The analytical expressions deduced can be easily used with other types of pseudorandom sequences and the obtained results provide more information than the correlation properties of the sequences alone, or when they are submitted to a linear system. These results may contribute to the implementation of more reliable systems.

This paper is structured as follows. After this introductory section, Section 2 presents OQPSK schemes, distinguishing its parallel and serial representations. In Section 3 the effects of nonlinearity applied to OQPSK-type modulations is studied. Section 4 presents the analytical derivation of the correlation properties for the type of signals used in this paper. Section 5 briefly introduces the pseudorandom sequences considered in this paper. Section 6 presents the main results obtained with the derived expressions and Section 7 concludes the paper.

2. PARALLEL AND SERIAL OQPSK SCHEMES

In a QPSK scheme, the signal to be transmitted can be denoted as $x_{BP}(t) = Re\{x_p(t)\exp(j2\pi f_c t)\}$, where f_c is the carrier frequency and $x_p(t)$ is the complex envelope, given by

$$x_p(t) = \sum_n a_n r_p(t - 2nT) \tag{1}$$

where $a_n = a_n^I + ja_n^Q$ with $a_n^I = \pm 1$ and $a_n^Q = \pm 1$ are the 'in-phase' and 'quadrature' bits of a length-N data block to be transmitted, T is the bit duration and $r_p(t)$ is the adopted pulse shape. Clearly, we can assume the QPSK signal as a sum of pulse-amplitude modulation signals with complex symbols separated by $2T$, because we are transmitting 2 bits per symbol.

For an OQPSK scheme, we have the same signal to be transmitted, $x_{BP}(t) = Re\{x_p(t)\exp(j2\pi f_c t)\}$, but in this case the OQPSK signal can be regarded as a sum of two pulse-amplitude modulation signals with symbols separated by $2T$ and an offset T between them. This corresponds to the parallel representation of an OQPSK signal given by

$$x_p(t) = \sum_n a_n^I r_p(t - 2nT) + \sum_n a_n^Q r_p(t - 2nT - T) \tag{2}$$

The average bit rate is $1/T$, because we transmit 2 bits per symbol.

An alternative form to describe the OQPSK signal is

$$x_p(t) = \sum_n a_n^p r_p(t - nT) \tag{3}$$

In this case, a_n^p has both in-phase and quadrature components, which appear alternately, that is,

$$a_n^p = \begin{cases} a_{n/2}^I = \pm 1 & , n \text{ even} \\ ja_{(n+1)/2}^Q = \pm j & , n \text{ odd} \end{cases} \tag{4}$$

In both representations of the OQPSK signal (i.e., (1) and (3)), we are assuming that the complex envelope is referred to f_c. By shifting the reference carrier from f_c to $f_1 = f_c + 1/4T$, the signal can be represented in a 'serial' (BPSK-type) format. In fact, we have $x_{BP}(t) = Re\{x_s(t)\exp(j2\pi f_1)\}$, where

$$x_s(t) = x_p(t)\, e^{-j\pi \frac{t}{2T}} \tag{5}$$

which means that

$$\begin{aligned} x_s(t) &= x_p(t)\, e^{-j\pi \frac{t}{2T}} = \\ &= \left[\left(\sum_n a_n^p \delta(t - nT)\right) r_p(t)\right] e^{-j\pi \frac{t}{2T}} = \\ &= \left[\left(\sum_n a_n^p \delta(t - nT)\right) e^{-j\pi \frac{t}{2T}}\right]\left[r_p(t) e^{-j\pi \frac{t}{2T}}\right] = \\ &= \sum_n a_n^p e^{-j\pi \frac{t}{2T}}\ r_p(t)\, e^{-j\pi \frac{t}{2T}} \end{aligned} \tag{6}$$