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RESEARCH ARTICLE

# Reduced feedback load using user selection algorithms for the multiuser multi-input single-output systems

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## ABSTRACT

For wireless multiuser multi-input single-output communication systems with adaptive scheduling and precoding, the users have to feed back their channel state information. However, the amount of feedback data increases with the number of users and the number of transmit antennas. In this paper, we propose a criterion based on the orthogonality between the users' channels to reduce this uplink load. Without cooperation between the users, we only allow users that are semi-orthogonal to feed back their quantized channel information to the base station. We propose a lower bound on the sum rate for the proposed criterion. We show that the performance is almost independent of the number of quantized channel quality information and only channel direction information is important at the transmitter. Then, we consider a noisy feedback channel and propose an algorithm to reduce the noise effect on the analogue feedback. Copyright © 2012 John Wiley & Sons, Ltd.

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## 1. INTRODUCTION

In a multiuser downlink system, it is possible to increase the total throughput by using multiple antennas at the transmission. In a multi-antenna transmission scheme, high data rate is achieved when the base station (BS) transmits to more than one user simultaneously and exploits spatial diversity offered by multiple transmit antennas by means of space division multiple access. The optimal space division multiple access strategy using dirty paper coding (DPC) can reach the maximum throughput of a multiple-input multiple-output downlink channel with a single BS and $K$ user terminals, by exploiting perfect instantaneous channel state information at the transmitter (CSIT) and at the receiver. The complete characterisation of the capacity region by DPC was shown in [1]. Nevertheless, this optimal coding scheme based on coding for known interference is difficult to implement especially when the number of users is large; some solutions using linear beamforming have been proposed to decrease the complexity.

Such a reduced complexity technique is proposed by [2] and combines DPC with QR channel decomposition. This scheme is called zero-forcing (ZF) DPC. This solution is only optimal at high signal-to-noise ratio (SNR) when the number of transmit antennas $N_t$ is higher than the number of users $K$ equipped with a single receive antenna. In order to decrease the complexity further, some solutions using linear beamforming have been proposed, such as ZF precoding where the precoding filter is the channel pseudo-inverse [3, 4].

When $N_t \leqslant K$, the architecture of a multiuser multi-antenna system requires not only a sophisticated precoding but also an efficient user selection algorithm. When the number of users becomes large, the complexity of the user selection algorithm increases. In [5], the authors proposed low-complexity algorithm where the users are scheduled from a subset of users with high channel magnitudes and for which their channel directions are semi-orthogonal. It has been shown in [6] that the transmitted signals achieving the capacity are mutually orthogonal with respect to time among the transmitter antennas and the constituent orthonormal unit vectors are isotropically distributed and statistically independent of the signal magnitudes. Recently in [7], the authors have presented a user selection algorithm based on chordal distance to reduce the complexity at the BS. In fact, only the users whose channel directions maximise the mutual chordal distance are selected for the scheduling. Nevertheless, in order to perform these algorithms, all user's CSIT and at the receiver are required at the BS side. Such a quantity of information is too important and overloads the uplink.

Several techniques aiming to reduce this feedback information have been studied and developed. The compression of the feedback information by techniques such as vector quantization also called limited feedback has been studied [8, 9]. These techniques quantify the channel direction with a common vector codebook to the transmitter and the receivers [10–12] and in the more general multiuser multiple-input multiple-output–orthogonal frequency-division multiple access case [13]. These techniques need the knowledge of all the users' channels at the transmitter. Jindal [11] and Kountouris *et al.* [14] have used the signal-to-interference-plus-noise ratio (SINR) as a scalar metric that incorporates information on the channel gain and the quantized channel direction, as well as on the channel quantization error. In order to reduce the load of the feedback link, only relevant information for the transmitter is allowed. In [15], the authors reduce the number of users that send back their channel gains by applying a threshold to all the users' channel norm. In [16], the Cluster-eigenbeamforming scheme is proposed where similar spatially users are grouped and for each group is assigned only one beamformer tailored on the spatial covariances of the clustered users. In [17], the authors have improved this technique applied to multi-antenna case by proposing two thresholds, one on the channel gain and the other on the channel direction quantization error.

Recently, a two-stage feedback scheme was proposed [18], where in the first step coarse estimates of all user channels are fed back to the BS and then in the second step only $N_t$ users are selected to feed back more accurate channel quantization. From the previous work [18], an interesting study of Ravindran and Jindal [19] has been raised, where the tradeoff between obtaining coarse channel feedback from large number of users and providing multiuser diversity gain versus obtaining high-quality channel feedback from a low number of users was underscored. Al-Harthi [20] has proposed a two-stage scheduling process based on partial probing of the users, in fact to reduce the feedback, and at the second stage the probing process stops and only the remaining users are requested to feed back their channel quality.

Because the users having a poor channel, such as low norm or/and interfering with other good users, are unlikely to schedule, these users do not need to feed back their channel to avoid redundant information at the BS. Thus, the complexity of the scheduling algorithm and the rate of the feedback channel are reduced by performing an intelligent selection at the user side. In this paper, we propose the algorithms to reduce the feedback load for multiuser multi-input single-output systems using the selection of a group of semi-orthogonal users at the user side. The derivation of a lower bound on the ergodic sum rate is provided for the proposed user selection technique. This lower bound can be used to find the thresholds according to the number of users and the SNR. The channel state information (CSI) of selected users at the receiver side can be fed back through the uplink channels using analogue or quantized feedback channel. Firstly, we evaluate the impact of the quantization of the channel quality indicator (CQI) and the channel direction information (CDI) on the system performance. Secondly, we consider a noisy uplink channel for both analogue and quantized feedback information. Finally, we propose an algorithm to reduce the noise effect on the analogue feedback.

This paper is organised as follows. The system model and the precoding strategy are given in Section 2. In Section 3, we propose the reduced feedback load algorithms at the user side. The limited feedback link will be provided in Section 4 including the proposed quantization methods for the CDI and CQI, the sum rate lower bound and noisy analogue link. In Section 5, we present the simulation and comparison results for quantization and analogue feedback in noisy and noise-free uplink. The conclusion will be drawn in Section 6.

## 2. SYSTEM MODEL AND PRECODING STRATEGY

We consider a system with a BS having $N_t$ transmit antennas and a cluster of $K$ users equipped with a single receive antenna as shown in Figure 1. We consider a block fading channel model and that the received signals are corrupted by independent additive white Gaussian noise. We assume that the channel is perfectly known at the receiver and the BS receives the CSIT using the uplink channels.

By exploiting the available CSIT, $N_t$ users are selected by the BS for the transmission. Let $\mathcal{S} \subset \{1, \cdots, K\}$, $|\mathcal{S}| \leqslant N_t$ be the subset of the selected users. Let $\mathbf{h}_k$ and $\mathbf{w}_k \in \mathbb{C}^{N_t \times 1}$ be respectively the channel and its precoding vector for the $k$th user. The BS applies precoding for the $N_t$ selected users' data symbols $\mathbf{s}(\mathcal{S})$, and the transmitted vector $\mathbf{x}(\mathcal{S})$ is given by

$$\mathbf{x}(\mathcal{S}) = \mathbf{W}(\mathcal{S})\mathbf{P}(\mathcal{S})\mathbf{s}(\mathcal{S}) \tag{1}$$

where $\mathbf{W}(\mathcal{S})$ is the concatenation of the beamforming vectors for the set of the $N_t$ scheduled users and $\mathbf{P}(\mathcal{S}) = \text{diag}\left(\sqrt{\frac{P}{N_t}} \quad \cdots \quad \sqrt{\frac{P}{N_t}}\right)$ for uniform power loading.

The zero-forcing beamforming (ZFBF) precoding strategy is chosen to avoid interference among user streams and the precoding vector can be found by inverting the composite channel matrix of the selected users $\mathbf{H}(\mathcal{S})$ by

$$\mathbf{W}(\mathcal{S}) = \alpha \mathbf{H}(\mathcal{S})^H \left(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H\right)^{-1} \tag{2}$$

where $\alpha = \frac{1}{\sqrt{tr((\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H)^{-1})}}$ in order to keep the short-term power constraint.


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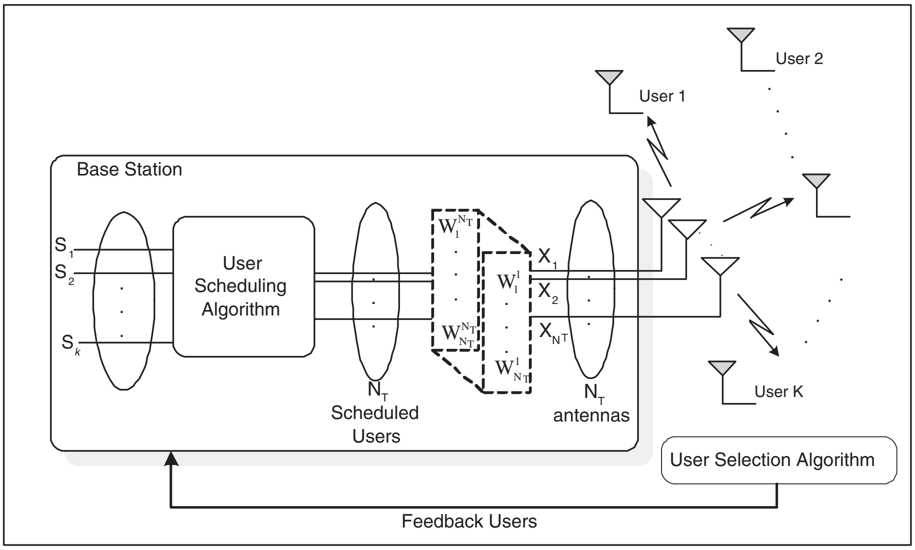


**Figure 1.** Beamforming transmission strategy in downlink multi-input single-output multiusers system.

The sum rate achieved by the ZFBF scheme for the selected user group $\mathcal{S}$ is

$$R_{\mathrm{ZFBF}}(\mathcal{S}) = \sum_{k \in \mathcal{S}} \log(1 + \mathrm{SINR}_k) \tag{3}$$

where the received SINR at each user is

$$\mathrm{SINR}_k = \frac{\frac{P}{N_t}\left|\mathbf{h}_k^H \mathbf{w}_k\right|^2}{\sigma^2 + \frac{P}{N_t}\sum_{j \neq k}^{N_t}\left|\mathbf{h}_k^H \mathbf{w}_j\right|^2} \tag{4}$$

The achievable sum rate of ZFBF is found by considering every possible choice of user groups $\mathcal{S}$:

$$R_{\mathrm{ZFBF}} = \max_{\mathcal{S} \subset \{1,\cdots,K\}:|\mathcal{S}|=N_t} R_{\mathrm{ZFBF}}(\mathcal{S}) \tag{5}$$

The ZFBF has been chosen to precode the users' data at the BS side. Without any user selection method at the receiver side, the scheduler has to perform a heuristic search or one of the referred algorithms in [5, 7] to select the users for the scheduling, assuming all the users CSIs are available at the BSs. In the following, we introduce the proposed algorithms to reduce this quantity of information.

## 3. REDUCED FEEDBACK LOAD USING USER SELECTION

In the precoding scheme, the number of active users is limited by the number of transmit antennas at the BS. The exhaustive search, which consists in evaluating $\begin{pmatrix} K \\ N_t \end{pmatrix}$ combinations, quickly becomes prohibitive. However, the users having a poor channel (low norm or/and interfering with other good users) should not take part in the user selection algorithm nor feed back their channel information.

By using a self-discrimination criterion at receiver side, it will be possible to reduce the feedback load and the complexity of the user selection algorithm at BS. In order to describe the less complex user selection and reduced feedback load, we propose the selection of three different sets of users.

Let $\mathcal{R}$ be the set of all the users in the cell. From $\mathcal{R}$, we define three sets of users corresponding to three different self-discrimination criteria:

$\mathcal{T}_1$ : The set of users satisfying the norm-only criterion [15].

$$\mathcal{T}_1 = \left\{k \in \mathcal{R} : \|\mathbf{h}_k\|^2 > \gamma_{th}\right\} \tag{6}$$

$\mathcal{T}_2$ : The set of semi-orthogonal users. We define the set of $N_t$-dimensional complex random orthogonal vectors $\boldsymbol{\phi}_i$ where $i = 1, \ldots, N_t$. The $\boldsymbol{\phi}_i$ vectors are generated according to an isotropic distribution [6, 21] and are equally likely pointing in any direction in the complex space. Based on the method of Heiberger, for the construction of $\boldsymbol{\phi}_i$, a $N_t \times N_t$ matrix $\mathbf{X}$ is first generated with entries $x_{ij} \sim \mathrm{Normal}(0, 1)$ then a QR factorisation is computed. The method provides a matrix $\mathbf{Q}$ and $\mathbf{R}$ where the matrix $\mathbf{Q}$ composed of $N_t$ orthogonal column vectors represents the $\boldsymbol{\phi}_i$ matrix.

In this work, each user generates the same $N_t$ random orthonormal vectors $\boldsymbol{\phi}_i$, which are also known by the BS.

Using chordal distance that measures distances between subspaces of Euclidean $m$-dimensional space, the users measure the orthogonality between their normalised channel vector $\bar{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ and orthonormal vectors $\boldsymbol{\phi}_i$ as follows:

$$d^2(\bar{\mathbf{h}}_k, \boldsymbol{\phi}_i) = 1 - |\bar{\mathbf{h}}_k^H \boldsymbol{\phi}_i|^2 = \sin^2(\Theta)$$

where $\Theta$ is the angle between the lines generated by the column spaces of $\bar{\mathbf{h}}_k^H$ and $\boldsymbol{\phi}_i$.

Let $\mathcal{O}^{N_t}$ be the unit sphere lying in $\mathcal{C}^{N_t}$ and centered at the origin. For any $0 < \epsilon_{th} < 1$, we can define a spherical cap on $\mathcal{O}^{N_t}$ with center $\mathbf{o}$ and square radius $\epsilon_{th}$ as the open set $\mathcal{B}_\epsilon(\mathbf{o}) = \{\mathbf{f} \in \mathcal{O}^{N_t} : d^2(\mathbf{f}, \mathbf{o}) \leqslant \epsilon_{th}\}$. Then, we have

$$\mathcal{T}_2 = \left\{ k \in \mathcal{R} : \bar{\mathbf{h}}_k \in \bigcup_{i=1}^{N_t} \mathcal{B}_\epsilon(\boldsymbol{\phi}_i) \right\} \tag{7}$$

$\mathcal{T}_3$ : The set of users satisfying the *two* previous criteria:

$$\mathcal{T}_3 = \left\{ k \in \mathcal{R} : \bar{\mathbf{h}}_k \in \bigcup_{i=1}^{N_t} \mathcal{B}_\epsilon(\boldsymbol{\phi}_i) \text{ and } \|\mathbf{h}_k\|^2 \geqslant \gamma_{th} \right\} \tag{8}$$

### 3.1. Threshold predetermination values

In this section, the threshold values for the three sets are determined in order to allow the number of average users $K_{\text{avg}}$ to feed back their CDI and CQI.

The channel norm and channel direction are independent and we have

$$\begin{aligned} K_{\text{avg}} &= K\mathcal{P}\{k \in \mathcal{T}_3\} \\ &= K\mathcal{P}\{k \in \mathcal{T}_1\} \times \mathcal{P}\{k \in \mathcal{T}_2\} \\ &= K\mathcal{P}\left\{\|\mathbf{h}_k\|^2 > \gamma_{th}\right\} \\ &\quad \times \mathcal{P}\left\{ k \in \mathcal{R} : \bar{\mathbf{h}}_k \in \bigcup_{i=1}^{N_t} \mathcal{B}_\epsilon(\boldsymbol{\phi}_i) \right\} \end{aligned} \tag{9}$$

According to [8]

$$\mathcal{P}\left\{ k \in \mathcal{R} : \bar{\mathbf{h}}_k \in \bigcup_{i=1}^{N_t} \mathcal{B}_\epsilon(\boldsymbol{\phi} i) \right\} \leqslant N_t \epsilon_{th}^{N_t - 1}$$

Using the union bound theorem

$$K_{avg} \leqslant K N_t \mathcal{P}\left\{\|\mathbf{h}_k\|^2 \geqslant \gamma_{th}\right\} \epsilon_{th}^{N_t-1} \tag{10}$$

The set $\mathcal{T}_1$ is determined by the incomplete gamma distribution $\gamma(N_t, 1)$, which can be bounded by [17, 21]

$$\left[1 - e^{-\beta\gamma}\right]^{N_t} \leqslant \int_0^{\gamma_{th}} f_\gamma(\gamma) \mathrm{d}\gamma \leqslant \left[1 - e^{-\gamma}\right]^{N_t}$$

where $\beta = (N_t!)^{-\frac{1}{N_t}}$ and $f_\gamma(\gamma)$ is the probability density function $\chi^2_{2N_t}$.

In Figure 2, we present the curves of the pair $(\gamma_{th}, \epsilon_{th})$ allowing predetermined probability $\mathcal{P}\{k \in \mathcal{T}_3\}$ (10–50 per cent). For each probability, it will be necessary to choose a pair by focusing the criterion on either the norm or orthogonality.

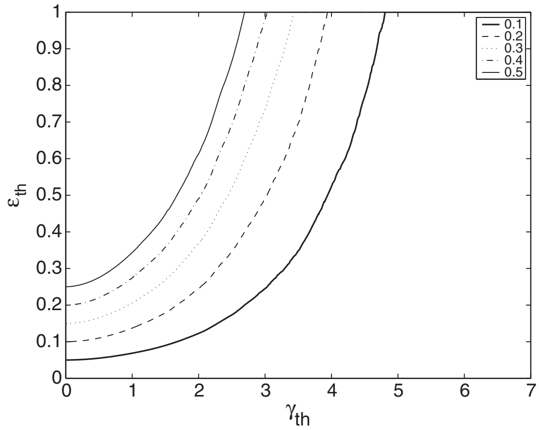


**Figure 2.** $\epsilon_{th}$ and $\gamma_{th}$ for $\mathcal{P}\{k \in \mathcal{T}_3\} = 0.1$ to 0.5.

The threshold values for the three sets are chosen analytically or by simulation in order to guarantee the number of average users $K_{\text{avg}}$ to feed back their CDI and CQI to the BS.

### 3.2. The user selection algorithm

First, different thresholds are defined and transmitted by the BS to the active users in the cell. The calculation of the threshold values is explained in Section 3.1. The BS can adjust the threshold values according to the traffic load or the available resources.

Second, only the users satisfying the criteria fixed by the BS feed back their CDI or CQI or both. By performing this self-discrimination, the number of the users requesting to be scheduled is reduced.

Third, among the set of selected users in the cell, it will be necessary to carry out the choice of the $N_t$ users, which will maximise the total throughput by exhaustive or heuristic search. In this work, we use the algorithm proposed by [22].

When $K$ users fed back their CDI and CQI to the BS, the $N_t$ users to be scheduled are selected among $\binom{K}{N_t}$ user pairs. However, with the help of the user selection algorithms at the user side, instead of $K$ users, the scheduling is performed among $\binom{K_{\text{avg}}}{N_t}$.

## 4. THE QUANTIZED FEEDBACK LINK

### 4.1. CDI and CQI quantization

To address the lack of perfect CSIT, a classical solution is to quantize CDI and CQI before transmission over the finite rate feedback link. The quantized channel vector $\mathbf{w}$ is taken from a set of $2^B$ vectors where $B$ is the number



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of feedback bits. In contrast to the normalised independent and identically distributed channel isotropically distributed in $\mathcal{O}^{N_t}$, an important aspect of a limited feedback codebook tailored to a spherical cap region is the quantization of the localised region or local packing. A local Grassmannian packing with parameters $N_t$,$N$, $\mathbf{o}$, $\epsilon_{th}$ is a set of $N$ vectors, $\mathbf{c}_i$, $i = 1, \ldots, N$, where $N$ is the codebook size, constrained to a spherical cap $\mathcal{B}_\epsilon(\mathbf{o})$ in $\mathcal{O}^{N_t}$ with the largest minimum chordal distance:

$$\min_{1 \leqslant i < j \leqslant N} d^2(\mathbf{c}_i, \mathbf{c}_j)$$

is maximised where each $\mathbf{c}_i \in \mathcal{C}$ is constrained to a spherical cap $\mathcal{B}_\epsilon(\mathbf{o})$.

As in the independent and identically distributed case, we use the Lloyd algorithm to design the proposed local packing. The codebook $\mathcal{C}$ is fixed for each pair $(\gamma_{th}, \epsilon_{th})$ and known by the transmitter and the receivers.

For the $\mathcal{T}_2$ and $\mathcal{T}_3$ criteria, the codebook has to be adapted according to the orthogonal vectors $\boldsymbol{\phi}_i$. From the local packing, it is possible to compute the local packing associated to the center $\boldsymbol{\phi}_i$ using the rotation matrix $\mathbf{U}_{\mathrm{rot}_i} = \boldsymbol{\phi}_i \mathbf{o}^{-1}$ [23]. Each user of the set checks which region that it belongs to

$$i = \underset{1 \leqslant m \leqslant N_t}{\arg\min}\ d^2\left(\bar{\mathbf{h}}_k, \boldsymbol{\phi}_m\right) \tag{11}$$

The user will feed back $\log_2(N_t)$ bits corresponding to the index $i$ of the vector $\boldsymbol{\phi}_i$ and $\log_2(N)$ bits corresponding to the codebook index $n$. For a codebook size $N$, $B = \log_2(N \times N_t)$ bits will be necessary to quantify the CDI. The users through the user selection process quantize their CDI according to the following rule:

$$\begin{cases} n = \underset{1 \leqslant j \leqslant N}{\arg\min}\, d^2\left(\bar{\mathbf{h}}_k, \mathbf{U}_{\mathrm{rot}_i} \mathbf{c}_j\right) \\ \hat{\mathbf{h}}_k = \mathbf{U}_{\mathrm{rot}_i} \mathbf{c}_n \end{cases} \tag{12}$$

where index $i$ is found by Equation (11) and $\hat{\mathbf{h}}_k$ is the quantized channel direction.

Let $\theta_k$ be the angle between $\hat{\mathbf{h}}_k$ and $\bar{\mathbf{h}}_k$. From [8] and [10]. The CDF of the CDI quantization error $z = \sin^2\theta_k$ can be approximated by

$$F_Z(z) \simeq \begin{cases} \dfrac{2^B}{N_t \epsilon_{th}^{N_t - 1}} z^{N_t - 1} & 0 \leqslant z \leqslant \left(\dfrac{N_t}{2^B}\right)^{\frac{1}{N_t - 1}} \epsilon_{th} = \alpha \epsilon_{th} \\ 1 & z \geqslant \alpha \epsilon_{th} \end{cases} \tag{13}$$

The selected users also feed back their CQI that could be the channel norm or the SINR [12]. In this paper, we choose the channel norm because we are interested in the evaluation of the norm threshold effects. A scalar quantizer is used to quantize the channel norm $\|\mathbf{h}_k\|^2$.

The channel norm $\|\mathbf{h}_k\|^2$ has a $\sim\gamma(N_t, 1)$ distribution with the probability distribution function (PDF):

$$f_{\|h_k\|^2}(x) = \frac{e^{-x} x^{N_t - 1}}{(N_t - 1)!} \tag{14}$$

The PDF of channel norm $\|\mathbf{h}_k\|^2$ restricted to $\|\mathbf{h}_k\|^2 \geqslant \gamma_{th}$ is given as

$$f_{\|h_k\|^2 \geqslant \gamma_{th}}(x) = \frac{e^{-x} x^{N_t - 1}}{(N_t - 1)!} \frac{1}{\sum\limits_{i=0}^{N_t - 1} \frac{e^{-\gamma_{th}} (\gamma_{th})^i}{(i)!}} \tag{15}$$

### 4.2. Performance analysis

The SINR for user $k \in \mathcal{T}_3$ can be expressed as

$$\mathrm{SINR}_k = \frac{\frac{P}{N_t} \|\mathbf{h}_k\|^2 \left|\bar{\mathbf{h}}_k^H \mathbf{w}_k\right|^2}{\sigma^2 + \frac{P}{N_t} \|\mathbf{h}_k\|^2 \sum\limits_{\substack{1 \leqslant j \leqslant N_t \\ j \neq k}} \left|\bar{\mathbf{h}}_k^H \mathbf{w}_j\right|^2} \tag{16}$$

Using the relation $\bar{\mathbf{h}}_k = \hat{\mathbf{h}}_k \cos\theta_k + \check{\mathbf{h}}_k \sin\theta_k$ and because $\left|\hat{\mathbf{h}}_k^H \mathbf{w}_j\right|^2 = 0$, the expected $\mathrm{SINR}_k$ for the user $k$ over all the channel realisations is given by [11]

$$E(\mathrm{SINR}_k) = \frac{\frac{P}{N_t} \|\mathbf{h}_k\|^2 \left|\bar{\mathbf{h}}_k^H \mathbf{w}_k\right|^2}{\sigma^2 + \frac{P}{N_t} \|\mathbf{h}_k\|^2 \sin^2\theta_k \sum\limits_{\substack{1 \leqslant j \leqslant N_t \\ j \neq k}} \left|\check{\mathbf{h}}_k^H \mathbf{w}_j\right|^2} \tag{17}$$

A lower bound of the expected $\mathrm{SINR}_k$ for the user $k$ is given by

$$E(\mathrm{SINR}_k) \geqslant \frac{\frac{P}{N_t} \|\mathbf{h}_k\|^2 E\left(\left|\bar{\mathbf{h}}_k^H \mathbf{w}_k\right|^2\right)}{\sigma^2 + \frac{P}{N_t} \|\mathbf{h}_k\|^2 \sin^2\theta_k} \tag{18}$$

#### 4.2.1. Expected SINR approximation.

The worst case is considered to express $\left|\bar{\mathbf{h}}_k^H \mathbf{w}_k\right|$, which means the channel directions of the selected users are less orthogonal as possible. Then the angle between the channel direction and its corresponding precoding vector is $\angle\left(\bar{\mathbf{h}}_k^H, \mathbf{w}_k\right) \leqslant (2\Psi_{th} - \theta_k)$ where $\Psi_{th} = \sin^{-1}\left(\sqrt{\epsilon_{th}}\right)$. The lower bound of the expected $\mathrm{SINR}_k$ is expressed by

$$E(\mathrm{SINR}_k) \geqslant \frac{\frac{P}{N_t} \|\mathbf{h}_k\|^2 \cos^2(2\Psi_{th} - \theta_k)}{\sigma^2 + \frac{P}{N_t} \|\mathbf{h}_k\|^2 \sin^2\theta_k} \tag{19}$$

Assuming that $\theta_k$ is very small compared with $2\Psi_{th}$, the lower bound on the expected rate can then be approximated by

$$\begin{aligned} E\{R\} &\geqslant E\left\{\sum_{k \in \mathcal{S}} \log_2\left(1 + \frac{\eta \|\mathbf{h}_k\|^2 \cos^2(2\Psi_{th} - \theta_k)}{1 + \eta \|\mathbf{h}_k\|^2 \sin^2\theta_k}\right)\right\} \\ &\simeq E\left\{\sum_{k \in \mathcal{S}} \log_2\left(1 + \frac{\eta \|\mathbf{h}_k\|^2 \cos^2(2\Psi_{th})}{1 + \eta \|\mathbf{h}_k\|^2 \sin^2\theta_k}\right)\right\} \end{aligned} \tag{20}$$

where $\eta = \frac{P}{N_t \sigma^2}$.

*4.2.1.1. Distribution of the expected SINR under $\gamma_{th}$ and $\epsilon_{th}$.* Let $\mathrm{SINR}_{\Psi_{th}} = \frac{\eta\|\mathbf{h}_k\|^2\cos^2(2\Psi_{th})}{1+\eta\|\mathbf{h}_k\|^2\sin^2\theta_k}$. The expected sum-rate can be lower bounded as follows:

$$E\{R\} \geqslant \sum_{k\in\mathcal{S}} \int_0^\infty f_{\mathrm{SINR}_{\Psi_{th}}}(x)\log_2(1+x)\mathrm{d}x \tag{21}$$

The probability distributed function of the expected SINR is derived for $N_t = 2$ as

$$\begin{array}{ll} \text{case} \quad x_1 \leqslant x_2 \leqslant x_3 & \text{case} \quad x_1 \leqslant x_3 \leqslant x_2 \\ \begin{cases} f_{2\mathrm{SINR}_{\Psi_{th}}}(x), & x_1 \leqslant x \leqslant x_2 \\ f_{3\mathrm{SINR}_{\Psi_{th}}}(x), & x_2 \leqslant x \leqslant x_3 \\ f_{4\mathrm{SINR}_{\Psi_{th}}}(x), & x \geqslant x_3 \end{cases} & \begin{cases} f_{5\mathrm{SINR}_{\Psi_{th}}}(x), & x_1 \leqslant x \leqslant x_3 \\ f_{6\mathrm{SINR}_{\Psi_{th}}}(x), & x_3 \leqslant x \leqslant x_2 \\ f_{7\mathrm{SINR}_{\Psi_{th}}}(x), & x \geqslant x_2 \end{cases} \end{array} \tag{22}$$

where

$$\begin{aligned} f_{2\mathrm{SINR}_{\Psi_{th}}}(x) &= f_{5\mathrm{SINR}_{\Psi_{th}}}(x) = C_1(\eta a)^2\frac{(\gamma_{th}+1)}{x^2}e^{-\gamma_{th}} - C_1(\eta a)^2\left[\frac{1}{\eta x(a - x\alpha\varepsilon_{th})} + \frac{1}{x^2}\right]e^{-\frac{x}{\eta(a-x\alpha\varepsilon_{th})}} \\ f_{3\mathrm{SINR}_{\Psi_{th}}}(x) &= C_1\eta a\frac{e^{-\frac{x}{\eta a}}}{x} + C_1(\eta a)^2\frac{e^{-\frac{x}{\eta a}}}{x^2} - C_1(\eta a)^2\frac{e^{-\frac{x}{\eta(a-x\alpha\varepsilon_{th})}}}{\eta x\,(a - x\alpha\varepsilon_{th})} - C_1(\eta a)^2\frac{e^{-\frac{x}{\eta(a-x\alpha\varepsilon_{th})}}}{x^2} \\ f_{4\mathrm{SINR}_{\Psi_{th}}}(x) &= f_{7\mathrm{SINR}_{\Psi_{th}}}(x) = C_1(\eta a)^2\left(\frac{1}{\eta a x} + \frac{1}{x^2}\right)e^{-\frac{x}{\eta a}} \\ f_{6\mathrm{SINR}_{\Psi_{th}}}(x) &= C_1(\eta a)^2(\gamma_{th}+1)e^{-\gamma_{th}}\left(\frac{1}{x^2}\right) \end{aligned} \tag{23}$$

with

$$\begin{cases} a = \cos^2(2\Psi_{th}) \\ \eta = \dfrac{P}{N_t\sigma^2} \\ C_1 = \dfrac{2^B}{\eta^2 a2\varepsilon_{th}e^{-\gamma_{th}}(1+\gamma_{th})} \end{cases} \quad \text{and} \quad \begin{cases} x_1 = \dfrac{\eta a\gamma_{th}}{1+\eta\alpha\varepsilon_{th}\gamma_{th}} \\ x_2 = \eta a\gamma_{th} \\ x_3 = \dfrac{a}{\alpha\varepsilon_{th}} \end{cases} \tag{24}$$

The derivation is given in the Appendix.

From Equation (23), we can give a lower bound for $E\{R\}$ as follows:

$$E\{R\} \geqslant N_t \int_0^\infty f_{SINR_{\Psi_{th}}}(x)\log_2(1+x)\mathrm{d}x \tag{25}$$

assuming that the transmitter chooses exactly $N_t$ users randomly from the set of selected users.

## 5. ANALOGUE FEEDBACK LINK

In analogue feedback [24–26], the channel coefficients are explicitly transmitted on the uplink using unquantized quadrature amplitude modulation (QAM). Because the dimension of the channel vectors is $N_t$, $N_t$ channels users will be necessary to send the channel coefficients. The signal is corrupted by additive white Gaussian noise.

Each selected user returns its channel information to the transmitter through the noisy uplink channel. The received vector $\mathbf{g}_k$ of dimension $N_t \times 1$ from the $k$th user at the BS is

$$\mathbf{g}_k = \beta\mathbf{h}_k + \mathbf{b}_k \tag{26}$$

where $\beta = \sqrt{\bar{P}}$ and $\bar{P}$ is the average transmit power, $\mathbf{b}_k \in \mathcal{C}^{N_t\times 1}$ is the independent and identically distributed zero mean Gaussian noise added to the uplink channel.

When using the $\mathcal{T}_1$ criterion, the received signal at the BS is directly exploited to calculate the sum rate, select the users and perform the precoding. However, when using the $\mathcal{T}_3$ criterion, it is possible to improve the performance of the system by exploiting the semi-orthogonal constraint to suppress part of the uplink noise. The estimated channel at the BS $\hat{\mathbf{h}}_k$ should lie in the spherical cap described by the square radius $\epsilon_{th}$. In this paper, we propose two algorithms to exploit this propriety.


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### 5.1. The proposed noise reduction algorithms

Option 1:

$$
\begin{aligned}
&\text{if} \min_{i=1,\ldots,N_t}\left(1-|\bar{\mathbf{g}}_k^H \boldsymbol{\phi}_i|^2\right) \leqslant \epsilon_{th} \qquad \hat{\hat{\mathbf{h}}}_k \longleftarrow \bar{\mathbf{g}}_k = \frac{\mathbf{g}_k}{\|\mathbf{g}_k\|}\\
&\text{else} \quad \text{find} \quad \hat{\hat{\mathbf{h}}}_k \quad \text{such as}\\
&\qquad \min \|\bar{\mathbf{g}}_k - \hat{\hat{\mathbf{h}}}_k\|^2\\
&\qquad \text{subject to} \quad 1-|\hat{\hat{\mathbf{h}}}_k^H \boldsymbol{\phi}_j|^2 = \epsilon_{th}\\
&\qquad \text{where} \quad j = \arg\min_{i=1,\ldots,N_t}\left(1-|\bar{\mathbf{g}}_k^H \boldsymbol{\phi}_i|^2\right)
\end{aligned} \tag{27}
$$

$\hat{\hat{\mathbf{h}}}_k$ is the closest vector from $\bar{\mathbf{g}}_k$ and situated at the surface of the spherical cap $\mathcal{B}_\epsilon(\boldsymbol{\phi}_j)$.

Because $\hat{\hat{\mathbf{h}}}_k$ and $\bar{\mathbf{g}}_k$ are unit norm, it is equivalent to minimise $\left(-\hat{\hat{\mathbf{h}}}_k^H \bar{\mathbf{g}}_k\right)$. This problem is then a second order cone problem and can be solved efficiently using a software for optimisation over symmetric cones.

A simpler solution (option 2) consists in directly associating the closest orthogonal vector $\boldsymbol{\phi}_j$ to $\hat{\hat{\mathbf{h}}}_k$:

$$\hat{\hat{\mathbf{h}}}_k = \boldsymbol{\phi}_j \tag{28}$$

## 6. SIMULATION RESULTS

In this section, we perform the simulation results for different user selection criteria to reduce feedback load considering $N_t = 2$ antennas at the BS. The sum-rate results are obtained depending on the quantization error and on the chosen threshold values $(\gamma_{th}, \epsilon_{th})$, which are adjusted according to Equation (10). Only these users feed back their $Q$ bits for the quantization CQI and $B$ bits corresponding to the codebook index of their quantized CDI. We define $F = B + Q$ the number of feedback bits per user.

The threshold $\gamma_{th}$ for criterion $\mathcal{T}_1$ and the threshold pair $(\gamma_{th}, \epsilon_{th})$ for criterion $\mathcal{T}_3$ are theoretically calculated to have $K_{\text{avg}} = 4$. The threshold values are chosen as $\gamma_{th} = [0 \;\; 1.7 \;\; 3 \;\; 4 \;\; 5 \;\; 5.8]$ for criterion $\mathcal{T}_1$, $\epsilon_{th} = [1 \;\; 0.25 \;\; 0.11 \;\; 0.05 \;\; 0.02 \;\; 0.01]$ for criterion $\mathcal{T}_2$ and the pair $(\gamma_{th}, \epsilon_{th}) = [(0, 1)(1, 0.35)(2, 0.23)(2.5, 0.18)(3, 0.1)(3.8, 0.09)]$ for criterion $\mathcal{T}_3$ corresponding the number of active users in the cell to $K = [8 \;\; 16 \;\; 20 \;\; 40 \;\; 100 \;\; 200]$.

The sum-rate performance versus the $(\gamma_{th}, \epsilon_{th})$ thresholds for $K = 100$ users at SNR$= 10$ dB is presented in Figure 3. We can show that the lower bound can be used to determinate the optimal pair $(\gamma_{th}, \epsilon_{th})$ values efficiently instead of performing Monte Carlo simulations. The lower bound is accurate for low to moderate $\gamma_{th}$ and $\epsilon_{th}$. For high values of $\epsilon_{th}$, the lower bound is weak because the selected users are not orthogonal and interference between selected users increases. For small values of $\epsilon_{th}$, the quantization error is very small thanks to the localised codebook.

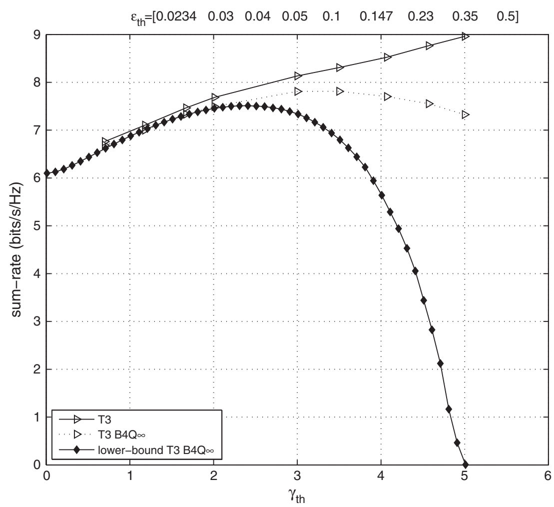


**Figure 3.** Sum rate of $\mathcal{T}_3$ criterion versus $(\gamma_{th}, \epsilon_{th})$ for $K = 100$, $K_{\text{avg}} = 4$ and $N_t = 2$.

In Figure 4, we compare the sum rate performances of full feedback ZFBF, multiuser random beamforming (RBF), $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ schemes. As it is shown, $\mathcal{T}_1$ outperforms $\mathcal{T}_3$ in the case of perfect feedback, but the two criteria give slightly the same performance for $F = 6$. For the case of less feedback bits such as $F = 4$ is used, not only $\mathcal{T}_3$ sum-rate performance becomes better than $\mathcal{T}_1$ performance, but also the $\mathcal{T}_1$ degradation is larger relative to $\mathcal{T}_3$ loss. This is due to the smaller vector quantization error for $\mathcal{T}_3$ criterion because all the codebook vectors are lying in the spherical cap described by the square radius $(\epsilon_{th})$ instead of all the hypersphere. We also show the sum-rate performances of various CQI quantization $Q = 0$ (no channel norm information) and $Q = \infty$ (without norm quantization) for SNR $= 15$ dB. Thanks to the channel norm constraint, the sum-rate performance is almost not

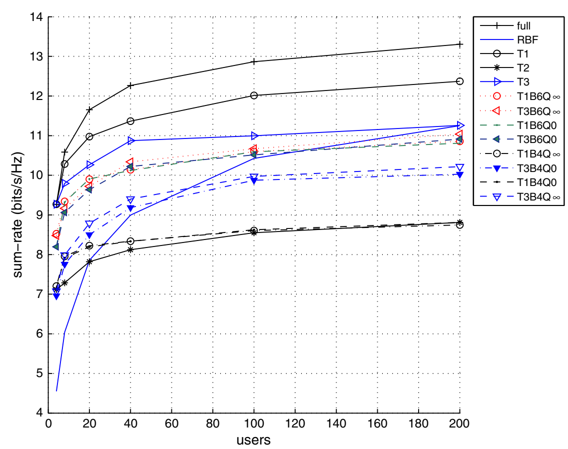


**Figure 4.** Sum rate versus the number of users for criteria $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ under $N_t = 2$, SNR $= 15$ dB and various B and Q. RBF, random beamforming.

affected when there is no feedback information about the channel norm of the selected users. On the opposite, the number of CDI bits has a high impact on the performance. We only plot the $\mathcal{T}_2$ criterion for the non-quantized version because it gives very poor performances.

In Figure 5, we present the sum rate versus average SNR for the system with $K = 100$ and $F = 4$, 8 bits It is known that RBF performs in a system with large number of users with requirement of all users' channel feedback. However, in the case of $F = 8$ bits, $\mathcal{T}_1$ and $\mathcal{T}_3$ outperform the RBF scheme with reduced feedback. $\mathcal{T}_3$ criterion outperforms $\mathcal{T}_1$ criterion at high SNR where the system becomes more sensitive to the interference and the users' orthogonality is more important. For the case of $F = 4$ bits, the performance of $\mathcal{T}_3$ is almost the same as $\mathcal{T}_1$ at low SNR and the same as RBF at high SNR. In this case, $\mathcal{T}_1$ degradation becomes important compared with the $F = 8$ bits case. We also present the lower bound for $B = 4$, which performs very closely to the performance of $\mathcal{T}_3$.

In Figure 6, we evaluate the performance of criterion $\mathcal{T}_3$ using the proposed noise reduction algorithm considering the two options at different $(E_s/N_0)_{UL}$. If the SNR is high (10 dB), the improvement is limited; however, at low SNR, the performance of the $\mathcal{T}_3$ criterion can be increased. In that case, the $\mathcal{T}_3$ criterion outperforms the $\mathcal{T}_1$ criterion when the number of active users in the cell is high.

In Figure 7, we compare the performance of the analogue and quantized feedback links for $(E_s/N_0)_{UL} = 0$ dB assuming SNR $= 10$ dB in the downlink. For these schemes, two symbols are needed to transmit the analogue channel or the codeword index. The modulation is 16 QAM when the number of bits is $B = 8$, whereas the modulation is quadrature phase shift keying when $B = 4$. In order to fairly compare the different criteria when the feedback link is noisy, we use the same average transmit energy per symbol for both analogue and quantized feedback link. According to the simulation results, when the uplink noise variance increases, the performance of the $\mathcal{T}_3$ criterion with analogue feedback gives the same result as the $\mathcal{T}_1$ criterion. $\mathcal{T}_3$ criterion in analogue feedback gives almost the same results at $(E_s/N_0)_{UL} = 0$ dB than the quantized one with 4 bits. Moreover, with this scheme, it is preferable to use only 4 quantized bits in the feedback link because 16 QAM is less robust to the noise than quadrature phase shift keying modulation. It is noticed that $\mathcal{T}_1$ is more sensitive to the transmission errors.

We have shown that the feedback link rate is significantly reduced without sacrificing the capacity performance of the multiple antenna-based wireless networks. Moreover, the next-generation wireless networks will have more intelligent properties such as context awareness with both physical context and user contexts by having many heterogeneous devices. In order to avoid inconsistency detection for physical context information [27], it is possible to adapt the proposed reduced rate feedback

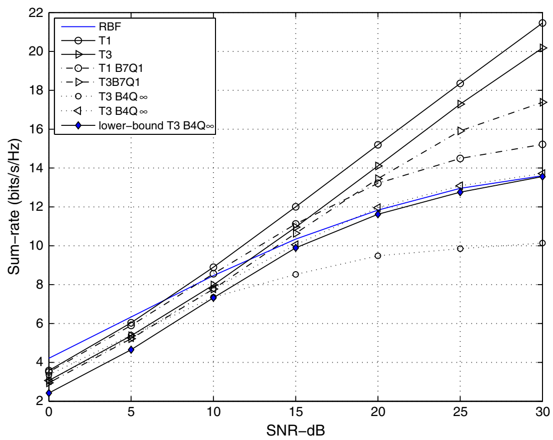


**Figure 5.** Sum rate versus signal-to-noise ratio (SNR) for random beamforming (RBF), criteria $\mathcal{T}_1$ and $\mathcal{T}_3$ under $N_t$=2, $K = 100$ and 4, 8 bits.

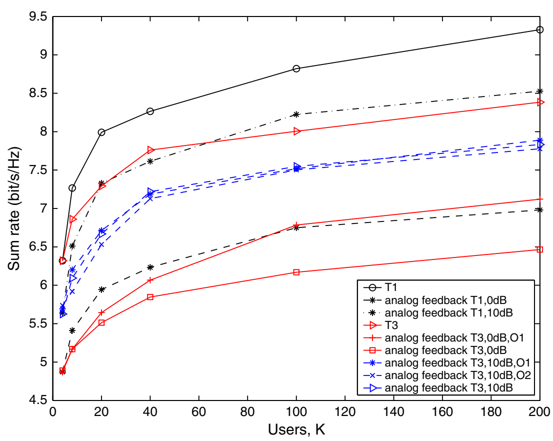


**Figure 6.** Sum rate versus number of users for $\mathcal{T}_3$ with the proposed algorithm for $N_t = 2$ in noisy feedback channel with different signal-to-noise ratio values.

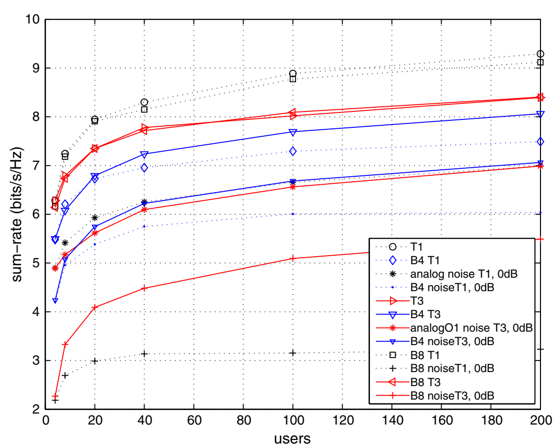


**Figure 7.** Sum rate versus number of users for criteria $\mathcal{T}_1$ and $\mathcal{T}_3$ for $N_t = 2$ in noisy feedback channel with $(E_s/N_0)_{UL} = 0$ dB.



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architectures, which provide less noisy information related to physical context of the users.

## 7. CONCLUSION

In this paper, we have evaluated the performance of user selection criteria at the user side based on the norm and semi-orthogonality of the channels considering both analogue and quantized CDI and CQI feedback link. Therefore, we have designed a codebook with local packing depending on the number of users in the cell. We have shown that because both criteria perform user selection using the norm of their channels, the performances are almost independent of the number of quantized CQI, and only CDI is important at the transmitter to maximise the sum-rate performance. We have derived a lower bound on the ergodic sum rate to determine the thresholds for the proposed $\mathcal{T}_3$ user selection algorithm. Afterwards, we have considered noisy feedback channels. We have shown that for analogue feedback it is possible to improve the performance of the criterion $\mathcal{T}_3$ by exploiting the semi-orthogonal constraint. For noisy quantized CDI feedback link, we have shown that the criterion $\mathcal{T}_1$ for the same number of symbols per codeword is much more sensitive to uplink noise than criterion $\mathcal{T}_3$. This sensitivity increases when the number of codewords decreases. Finally, the performance of quantized feedback with few bits can be very close to analogue feedback.

## APPENDIX

The cumulative distribution function (CDF) of the $\mathrm{SINR}_{\Psi_{th}}$.

Let us define $X_1 = \|\mathbf{h}_k\|^2$, $X_2 = \sin^2\theta_k$ and the noise variance as $\sigma^2 = 1$.

$$\begin{cases} Y_2 = \eta X_1 X_2 + 1 \\ Y_1 = \eta a X_1 \end{cases} \quad \text{where } a = \cos^2(2\Psi_{th}) \tag{29}$$

The joint distribution of $(Y_1, Y_2)$ is related to that of $(\eta\|\mathbf{h}_k\|^2\cos^2(2\Psi_{th}), \eta\|\mathbf{h}_k\|^2\sin^2\theta_k)$, so the CDF of $\mathrm{SINR}_{\Psi_{th}}$ is

$$CDF(x) = P\left(\frac{Y_1}{Y_2} < x\right) = \iint\limits_{Y_1 Y_2} f_{Y_1,Y_2}(y_1, y_2)\,\mathrm{d}y_2\mathrm{d}y_1 \tag{30}$$

Let $X$ be the domain of definition of the random variables $X_1$ and $X_2$, $X = \{(x_1, x_2) : \gamma_{th} \leqslant x_1$ and $0 \leqslant x_2 \leqslant \alpha\epsilon_{th}$; or $x_2 > \alpha\epsilon_{th}\}$. For $Y_1$ and $Y_2$ range as

$$\begin{cases} X_1 = \dfrac{1}{\eta a} Y_1 \\ X_2 = a\dfrac{(Y_2 - 1)}{Y_1} \end{cases} \text{where} \begin{cases} y_1 \geqslant \eta a \gamma_{th} \\ y_1 \frac{\alpha\varepsilon_{th}}{a} + 1 \geqslant y_2 \geqslant 1 \end{cases} \text{or} \begin{cases} y_1 \geqslant \eta a \gamma_{th} \\ y_2 \geqslant 1 + \alpha\eta\varepsilon_{th}\gamma_{th} \end{cases} \tag{31}$$

$$f_{Y_1,Y_2}(y_1, y_2) = \frac{1}{\eta y_1} \times f_{X_1}\left(\frac{1}{\eta a} y_1\right) f_{X_2}\left(\frac{a(y_2 - 1)}{y_1}\right)$$

Because the random variables $X_1$ and $X_2$ are independent, we have

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} \dfrac{2^B}{\eta^{N_t} a N_t \sum\limits_{i=0}^{N_t-1} \frac{e^{-\gamma_{th}}(\gamma_{th})^i}{(i)!}\varepsilon_{th}^{N_t-1}} e^{-\frac{y_1}{\eta a}} \frac{(y_2-1)^{Nt-2}}{(Nt-2)!}, & y_1 \geqslant \eta a \gamma_{th} \quad \text{and } 0 \leqslant \frac{a(y_2-1)}{y_1} \leqslant \alpha\varepsilon_{th}, \\ 0 & \text{else.} \end{cases} \tag{32}$$

Case $x_1 \leqslant x_2 \leqslant x_3$

$$CDF(x) = \begin{cases} \displaystyle\int_{\eta a \gamma_{th}}^{\frac{ax}{a - x\alpha\varepsilon_{th}}} \int_{\frac{y_1}{x}}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2)\mathrm{d}y_2\mathrm{d}y_1, & x_1 \leqslant x \leqslant x_2 \\ \displaystyle\int_{\eta a \gamma_{th}}^{x} \int_{1}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2)\mathrm{d}y_2\mathrm{d}y_1 + \int_{x}^{\frac{ax}{a - x\alpha\varepsilon_{th}}} \int_{\frac{y_1}{x}}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2)\mathrm{d}y_2\mathrm{d}y_1, & x_2 \leqslant x \leqslant x_3 \\ \displaystyle\int_{\eta a \gamma_{th}}^{x} \int_{1}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2)\mathrm{d}y_2\mathrm{d}y_1 + \int_{x}^{+\infty} \int_{\frac{y_1}{x}}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2)\mathrm{d}y_2\mathrm{d}y_1, & x \geqslant x_3 \end{cases} \tag{33}$$

Case $x_1 \leqslant x_3 \leqslant x_2$

$$CDF(x) = \begin{cases} \int_{\eta a \gamma_{th}}^{\frac{ax}{a - x\alpha\varepsilon_{th}}} \int_{\frac{y_1}{x}}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2) \mathrm{d}y_2 \mathrm{d}y_1, & x_1 \leqslant x \leqslant x_3 \\ \int_{\eta a \gamma_{th}}^{+\infty} \int_{\frac{y_1}{x}}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2) \mathrm{d}y_2 \mathrm{d}y_1, & x_3 \leqslant x \leqslant x_2 \\ \int_{\eta a \gamma_{th}}^{x} \int_{1}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2) \mathrm{d}y_2 \mathrm{d}y_1 + \int_{x}^{+\infty} \int_{\frac{y_1}{x}}^{\frac{\alpha\varepsilon_{th}}{a} y_1 + 1} f_{Y_1 Y_2}(y_1, y_2) \mathrm{d}y_2 \mathrm{d}y_1, & x \geqslant x_2 \end{cases} \tag{34}$$

Finally, the derivation of (33) and (34) gives the PDFs of Equations (23).

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