

# Sincere-Strategy Preference-Based Approval Voting Fully Resists Constructive Control and Broadly Resists Destructive Control\*

Gábor Erdélyi<sup>†</sup> and Markus Nowak and Jörg Rothe<sup>‡</sup>  
 Institut für Informatik  
 Heinrich-Heine-Universität Düsseldorf  
 40225 Düsseldorf  
 Germany

June 12, 2008

## Abstract

We study sincere-strategy preference-based approval voting (SP-AV), a system proposed by Brams and Sanver [1] and here adjusted so as to *coerce* admissibility of the votes (rather than excluding inadmissible votes *a priori*), with respect to procedural control. In such control scenarios, an external agent seeks to change the outcome of an election via actions such as adding/deleting/partitioning either candidates or voters. SP-AV combines the voters' preference rankings with their approvals of candidates, where in elections with at least two candidates the voters' approval strategies are adjusted—if needed—to approve of their most-preferred candidate and to disapprove of their least-preferred candidate. This rule coerces admissibility of the votes even in the presence of control actions, and hybridizes, in effect, approval with plurality voting.

We prove that this system is computationally resistant (i.e., the corresponding control problems are NP-hard) to 19 out of 22 types of constructive and destructive control. Thus, SP-AV has more resistances to control than is currently known for any other natural voting system with a polynomial-time winner problem. In particular, SP-AV is (after Copeland voting, see Faliszewski et al. [2, 3]) the second natural voting system with an easy winner-determination procedure that is known to have full resistance to constructive control, and unlike Copeland voting it in addition displays broad resistance to destructive control.

---

\*Supported in part by the DFG under grants RO 1202/12-1 (within the European Science Foundation's EUROCORES program LogICCC: "Computational Foundations of Social Choice") and RO 1202/11-1 and by the Alexander von Humboldt Foundation's TransCoop program. Preliminary versions of this paper have been presented at the 33rd International Symposium on Mathematical Foundations of Computer Science (MFCS-08) [4] and at the 2nd International Workshop on Computational Social Choice (COMSOC-08). Work done in part while the first author was visiting Universität Trier and while the third author was visiting the University of Rochester.

<sup>†</sup>URL: [ccc.cs.uni-duesseldorf.de/~erdelyi](http://ccc.cs.uni-duesseldorf.de/~erdelyi).

<sup>‡</sup>URL: [ccc.cs.uni-duesseldorf.de/~rothe](http://ccc.cs.uni-duesseldorf.de/~rothe).

# 1 Introduction

Voting provides a particularly useful method for preference aggregation and collective decision-making. While voting systems were originally used in political science, economics, and operations research, they are now also of central importance in various areas of computer science, such as artificial intelligence (in particular, within multiagent systems). In automated, large-scale computer settings, voting systems have been applied, e.g., for planning [5] and similarity search [6], and have also been used in the design of recommender systems [7] and ranking algorithms [8] (where they help to lessen the spam in meta-search web-page rankings). For such applications, it is crucial to explore the computational properties of voting systems and, in particular, to study the complexity of problems related to voting (see, e.g., the survey by Faliszewski et al. [9]).

The study of voting systems from a complexity-theoretic perspective was initiated by Bartholdi, Tovey, and Trick’s series of seminal papers about the complexity of winner determination [10], manipulation [11], and procedural control [12] in elections. This paper contributes to the study of electoral control, where an external agent—traditionally called *the chair*—seeks to influence the outcome of an election via procedural changes to the election’s structure, namely via adding/deleting/partitioning either candidates or voters (see Section 2.2 for the formal definitions of our control problems). We consider both *constructive* control (introduced by Bartholdi, Tovey, and Trick [12]), where the chair’s goal is to make a given candidate the unique winner, and *destructive* control (introduced by Hemaspaandra, Hemaspaandra, and Rothe [13]), where the chair’s goal is to prevent a given candidate from being a unique winner.

We investigate the same twenty types of constructive and destructive control that were studied for approval voting [13] and two additional control types introduced by Faliszewski et al. [14] (see also [2]), and we do so for a variant of a voting system that was proposed by Brams and Sanver [1] as a combination of preference-based and approval voting. Approval voting was introduced by Brams and Fishburn [15] as follows: Every voter either approves or disapproves of each candidate, and every candidate with the largest number of approvals is a winner. One of the simplest preference-based voting systems is plurality: All voters report their preference rankings of the candidates, and the winners are the candidates that are ranked first-place by the largest number of voters. The purpose of this paper is to show that Brams and Sanver’s combined system (adapted here so as to keep its useful features even in the presence of control actions) combines the strengths, in terms of computational resistance to control, of plurality and approval voting.

Some voting systems are *immune* to certain types of control in the sense that it is never possible for the chair to reach his or her goal via the corresponding control action. Immunity to any type of control unconditionally shields the voting system against this particular control type. However, like most voting systems approval voting is *susceptible* (i.e., not immune) to many types of control, and plurality voting is susceptible to all types of control.<sup>1</sup> However, and this was Bartholdi, Tovey, and Trick’s brilliant insight [12], even for systems susceptible to control, the chair’s task of controlling a given election may be too hard computationally (namely, NP-hard) for him or her to succeed. The

---

<sup>1</sup>A related line of research has shown that, in principle, all natural voting systems can be manipulated by strategic voters. Most notable among such results is the classical work of Gibbard [16] and Satterthwaite [17]. The study of strategy-proofness is still an extremely active and interesting area in social choice theory (see, e.g., Duggan and Schwartz [18]) and in artificial intelligence (see, e.g., Everaere et al. [19]).

Number of	Condorcet	Approval	Llull	Copeland	Plurality	SP-AV
resistances	3	4	14	15	16	19
immunities	4	9	0	0	0	0
vulnerabilities	7	9	8	7	6	3
References	[12, 13]	[12, 13]	[2, 14, 3]	[2, 14, 3]	[12, 13, 2, 14]	Theorem 3.1

Table 1: Number of resistances, immunities, and vulnerabilities to our 22 control types. (Regarding the “Condorcet” column, see Footnote 3.)

voting system is then said to be *resistant* to this control type. If a voting system is susceptible to some type of control, but the chair’s task can be solved in polynomial time, the system is said to be *vulnerable* to this control type.

The quest for a natural voting system with an easy winner-determination procedure that is universally resistant to control has lasted for more than 15 years now. Among the voting systems that have been studied with respect to control are plurality, Condorcet, approval, cumulative, Llull, and (variants of) Copeland voting [12, 13, 20, 21, 14, 3, 22, 2]. Among these systems, plurality and Copeland voting (denoted Copeland<sup>0.5</sup> in [2, 3]) display the broadest resistance to control, yet even they are not universally control-resistant. The only system currently known to be fully resistant—to the 20 types of constructive and destructive control studied in [13, 20]—is a highly artificial system constructed via hybridization [20]. (We mention that this system was not designed for direct, real-world use as a “natural” system but rather was intended to rule out the existence of a certain impossibility theorem [20].)

While approval voting nicely distinguishes between each voter’s acceptable and unacceptable candidates, it ignores the preference rankings the voters may have about their approved (or disapproved) candidates. This shortcoming motivated Brams and Sanver [1] to introduce a voting system that combines approval and preference-based voting, and they defined the related notions of sincere and admissible approval strategies, which are quite natural requirements. We adapt their sincere-strategy preference-based approval voting system in a natural way such that, for elections with at least two candidates, admissibility of approval strategies (see Definition 2.1) can be ensured even in the presence of control actions such as deleting candidates and partitioning candidates or voters.<sup>2</sup> The purpose of this paper is to study if, and to what extent, this system inherits the control resistances of plurality (which is perhaps the simplest preference-based system) and approval voting. Denoting this system by SP-AV, we show that SP-AV does combine all the resistances of plurality and approval voting.

More specifically, we prove that sincere-strategy preference-based approval voting is resistant to 19 and vulnerable to only three of the 22 types of control considered here. For comparison, Table 1 shows the number of resistances, immunities, and vulnerabilities to our 22 control types that are

---

<sup>2</sup>Note that in control by partition of voters (see Section 2.2) the run-off may have a reduced number of candidates.

known for each of Condorcet,<sup>3</sup> approval, Llull, plurality,<sup>4</sup> and Copeland voting (see [12, 13, 2, 14, 3]), and for SP-AV (see Theorem 3.1 and Table 2 in Section 3).

This paper is organized as follows. In Section 2, we define and discuss sincere-strategy preference-based approval voting, the types of control studied in this paper, and the notions of immunity, susceptibility, vulnerability, and resistance. In Section 3, we prove our results on SP-AV. Finally, in Section 4 we give our conclusions and state some open problems.

## 2 Preliminaries

### 2.1 Preference-Based Approval Voting

An election  $E = (C, V)$  is specified by a finite set  $C$  of candidates and a finite collection  $V$  of voters who express their preferences over the candidates in  $C$ , where distinct voters may, of course, have the same preferences. How the voter preferences are represented depends on the voting system used. In approval voting (AV, for short), every voter draws a line between his or her acceptable and unacceptable candidates (by specifying a 0-1 approval vector, where 0 represents disapproval and 1 represents approval), yet does not rank them. In contrast, many other important voting systems (e.g., Condorcet voting, Copeland voting, all scoring protocols, including plurality, Borda count, veto, etc.) are based on voter preferences that are specified as tie-free linear orderings of the candidates. As is most common in the literature, votes will here be represented nonsuccinctly: one ballot per voter. Note that some papers (e.g., [23, 2, 14, 3]) also consider succinct input representations for elections where multiplicities of votes are given in binary.

Brams and Sanver [1] introduced a voting system that combines approval and preference-based voting. To distinguish this system from other systems that these authors introduced with the same purpose of combining approval and preference-based voting [24], we call the variant considered here (including the assumption of sincerity as explained below and including Rule 1 below, which will coerce admissibility) *sincere-strategy preference-based approval voting* (SP-AV, for short).

**Definition 2.1** (Brams and Sanver [1]). *Let  $(C, V)$  be an election, where the voters both indicate approvals/disapprovals of the candidates and provide a tie-free linear ordering of all candidates. For each voter  $v \in V$ , an AV strategy of  $v$  is a subset  $S_v \subseteq C$  such that  $v$  approves of all candidates in  $S_v$  and disapproves of all candidates in  $C - S_v$ . The list of AV strategies for all voters in  $V$  is*

---

<sup>3</sup> Note that Table 1 lists only 14 instead of 22 types of control for Condorcet. The reason is that, as in [13], we consider two types of control by partition of candidates (namely, with and without run-off) and one type of control by partition of voters, and for each partition case we use the rules TE (“ties eliminate”) and TP (“ties promote”) for handling ties that may occur in the corresponding subelections (see Section 2.2). However, since Condorcet winners are always unique when they exist, the distinction between TE and TP is not made for the partition cases within Condorcet voting. Note further that the two additional control types in Section 2.2.1 (namely, constructive and destructive control by adding a limited number of candidates [2, 14]) have not been considered for Condorcet voting [12, 13].

<sup>4</sup>Regarding the references given in Table 1 for plurality, Faliszewski et al. [2, 14] note that plurality is resistant to constructive and destructive control by adding a limited number of candidates (see Section 2.2 for the definition of this problem). Hemaspaandra et al. [13] obtained all other results for destructive control within plurality, and for the constructive partitioning control cases in models TE and TP. The remaining results for plurality are due to Bartholdi et al. [12].

called an AV strategy profile for  $(C, V)$ . (We sometimes also speak of  $V$ 's AV strategy profile for  $C$ .) For each  $c \in C$ , let  $\text{score}_{(C, V)}(c) = \|\{v \in V \mid c \in S_v\}\|$  denote the number of  $c$ 's approvals. Every candidate  $c$  with the largest  $\text{score}_{(C, V)}(c)$  is a winner of election  $(C, V)$ .

An AV strategy  $S_v$  of a voter  $v \in V$  is said to be admissible if  $S_v$  contains  $v$ 's most-preferred candidate and does not contain  $v$ 's least-preferred candidate.<sup>5</sup>  $S_v$  is said to be sincere if for each  $c \in C$ , if  $v$  approves of  $c$  then  $v$  also approves of each candidate ranked higher than  $c$  (i.e., there are no gaps allowed in sincere approval strategies). An AV strategy profile for  $(C, V)$  is admissible (respectively, sincere) if the AV strategies of all voters in  $V$  are admissible (respectively, sincere).

Admissibility and sincerity are quite natural requirements. In particular, requiring the voters to be sincere ensures that their preference rankings and their approvals/disapprovals are not contradictory. Note that sincere strategies for at least two candidates are always admissible if voters are neither allowed to approve of everybody nor to disapprove of everybody (i.e., if we require voters  $v$  to have only AV strategies  $S_v$  with  $\emptyset \neq S_v \neq C$ ), an assumption adopted by Brams and Sanver [1].<sup>6</sup> Henceforth, we will assume that only sincere AV strategy profiles are considered, which—assuming that the trivial cases  $S_v = \emptyset$  and  $S_v = C$  are excluded—necessarily are admissible whenever there are at least two candidates.<sup>7</sup> A vote with an insincere strategy will be considered void.

The following notation was used by Brams and Sanver for a different election system [24], but is useful for SP-AV as well: Preferences are represented by a left-to-right ranking (separated by a space) of the candidates (e.g.,  $a b c$ ), with the leftmost candidate being the most-preferred one, and approval strategies are denoted by inserting a straight line into such a ranking, where all candidates left of this line are approved of and all candidates right of this line are disapproved of (e.g., “ $a \mid b c$ ” means that  $a$  is approved of, while both  $b$  and  $c$  are disapproved of by this voter). In our constructions, we sometimes also insert a subset  $B \subseteq C$  into such approval rankings, where we assume some arbitrary, fixed order of the candidates in  $B$  (e.g., “ $a \mid B c$ ” means that  $a$  is approved of, while all  $b \in B$  and  $c$  are disapproved of by this voter).

## 2.2 Control Problems for Preference-Based Approval Voting

The control problems considered here were introduced by Bartholdi, Tovey, and Trick [12] for constructive control and by Hemaspaandra, Hemaspaandra, and Rothe [13] for destructive control. In constructive control scenarios the chair's goal is to make a favorite candidate win, and in destructive control scenarios the chair's goal is to ensure that a despised candidate does not win. As is common, the chair is assumed to have complete knowledge of the voters' preference rankings and approval strategies,<sup>8</sup> and as in most papers on electoral control we define the control problems in the unique-

<sup>5</sup>Brams and Sanver [1] define an AV strategy to be admissible if it is not dominated in a game-theoretic sense [15], and note that “admissible strategies under AV involve always voting for a most-preferred candidate and never voting for a least-preferred candidate.” Since we do not focus on the game-theoretic aspects of AV strategies, we define admissibility as in Definition 2.1.

<sup>6</sup>Brams and Sanver actually preclude only the case  $S_v = C$  for sincere voters  $v$  by stating that “sincere strategies are always admissible if we exclude ‘vote for everybody’” [1]. However, an AV strategy that disapproves of all candidates obviously is sincere, yet not admissible according to Definition 2.1, which is why we also exclude the case of  $S_v = \emptyset$ .

<sup>7</sup>Note that an AV strategy is never admissible for less than two candidates.

<sup>8</sup>A detailed discussion of this assumption can be found in [13]. In a nutshell, one justification of this assumption is that it is realistic in many (though certainly not in all) situations, particularly in those involving small-scale private

winner model.<sup>9</sup> In this model, the chair seeks to, via the control action, either make a designated candidate the unique winner (in the constructive case) or to prevent a designated candidate from being a unique winner (in the destructive case).

To achieve his or her goal, the chair modifies the structure of a given election via adding/deleting/partitioning either candidates or voters. Such control actions—specifically those with respect to control via deleting or partitioning candidates or via partitioning voters—may have an undesirable impact on the resulting election in that they might turn admissible AV strategies into inadmissible ones. That is why we define the following rule that coerces admissibility (even under such control actions):

**Rule 1** (AV Strategy Rewrite Rule). *If in an election  $(C, V)$  with  $\|C\| \geq 2$  we have  $S_v = \emptyset$  or  $S_v = C$  for some voter  $v \in V$ , then each such voter’s AV strategy is adjusted to approve of his or her top candidate and to disapprove of his or her bottom candidate.*

This rule coerces  $\emptyset \neq S_v \neq C$  for each  $v \in V$  whenever there are at least two candidates. That is, though it is legal for a voter to cast an inadmissible vote, the SP-AV system will rewrite this vote to make it admissible. In Section 2.3 below, we will briefly discuss the SP-AV system and, in particular, some subtle points regarding Rule 1.

We now formally define our control problems, where each problem is defined by stating the problem instance together with two questions, one for the constructive and one for the destructive case. These control problems are tailored to sincere-strategy preference-based approval voting by requiring every election occurring in these control problems (be it before, during, or after a control action—so, in particular, this also applies to the subelections in the partitioning cases) to have a sincere AV strategy profile. Note that when the number of candidates is reduced (due to deleting candidates or partitioning candidates or voters), approval lines may have to be moved in accordance with Rule 1.

To avoid unnecessary repetition, when defining the 22 control scenarios and problems considered in this paper, we will omit (or only very briefly sketch) the motivation of these control scenarios. Note, however, that each scenario considered has a natural real-world interpretation—ranging from “get-out-the-vote” drives (control by adding voters) over vote suppression or disenfranchisement (control by deleting voters) to gerrymandering (control by partitioning voters) for voter control, and similarly natural real-world interpretations have been discussed in detail for the single cases of candidate control. These real-world interpretations and motivating examples have been described at length in a number of previous papers on control, such as [12, 13, 2, 20]. (Note that the journal version of [20] appears in the same special issue as the present paper.)

---

elections and in those involving large-scale elections among software agents that cooperate in a multiagent environment and have an incentive to reveal their preferences over the given alternatives. Another justification is that this paper focuses on proving control resistances of (i.e., NP-hardness results for) SP-AV, and an NP-hardness result in the more restrictive setting of complete knowledge clearly implies the corresponding NP-hardness result in the more flexible setting of partial knowledge (see [13] for more discussion of this point).

<sup>9</sup>Exceptions are, e.g., [21, 2, 3, 14], where [2, 3, 14] consider both the unique-winner model and the nonunique-winner model.

### 2.2.1 Control by Adding Candidates

In this control scenario, the chair seeks to reach his or her goal by adding to the election, which originally involves only “qualified” candidates, some new candidates who are chosen from a given pool of spoiler candidates. In their study of control for plurality, Condorcet, and approval voting, Hemaspaandra, Hemaspaandra, and Rothe [13] considered only the case of adding an *unlimited* number of spoiler candidates (which is the original variant of this problem as defined by Bartholdi, Tovey, and Trick [12]). We consider the same variant of this problem here to make our results comparable with those established in [13], but for completeness we in addition consider the case of adding a *limited* number of spoiler candidates, where the prespecified limit is part of the problem instance. This variant of this problem was introduced by Faliszewski et al. [2, 14, 3] in analogy with the definitions of control by deleting candidates and of control by adding or deleting voters. They showed that, for the election system Copeland <sup>$\alpha$</sup>  they investigate, the complexity of these two problems can drastically change depending on the parameter  $\alpha$ , see [2, 3].

We first define the unlimited variant of control by adding candidates.

**Name** Control by Adding an Unlimited Number of Candidates.

**Instance** An election  $(C \cup D, V)$  and a designated candidate  $c \in C$ , where the set  $C$  of qualified candidates and the set  $D$  of spoiler candidates are disjoint.

**Question (constructive)** Is it possible to choose a subset  $D' \subseteq D$  such that  $c$  is the unique winner of election  $(C \cup D', V)$ ?

**Question (destructive)** Is it possible to choose a subset  $D' \subseteq D$  such that  $c$  is not a unique winner of election  $(C \cup D', V)$ ?

The problem Control by Adding a Limited Number of Candidates is defined analogously, with the only difference being that the chair seeks to reach his or her goal by adding at most  $\ell$  spoiler candidates, where  $\ell$  is part of the problem instance.

### 2.2.2 Control by Deleting Candidates

In this control scenario, the chair seeks to reach his or her goal by deleting (up to a given number of) candidates. Here it may happen that inadmissible AV strategies are created by the control action, but Rule 1 will coerce admissibility again (by moving the line between some voter’s acceptable and unacceptable candidates to behind the top candidate or to before the bottom candidate whenever necessary).

**Name** Control by Deleting Candidates.

**Instance** An election  $(C, V)$ , a designated candidate  $c \in C$ , and a nonnegative integer  $\ell$ .

**Question (constructive)** Is it possible to delete up to  $\ell$  candidates from  $C$  such that  $c$  is the unique winner of the resulting election?

**Question (destructive)** Is it possible to delete up to  $\ell$  candidates (other than  $c$ ) from  $C$  such that  $c$  is not a unique winner of the resulting election?

### 2.2.3 Control by Partition and Run-Off Partition of Candidates

There are two partition-of-candidates control scenarios. In both scenarios, the chair seeks to reach his or her goal by partitioning the candidate set  $C$  into two subsets,  $C_1$  and  $C_2$ , after which the election is conducted in two stages. In control by partition of candidates, the election's first stage is held within only one group, say  $C_1$ , and this group's winners that survive the tie-handling rule used (see the next paragraph) run against all members of  $C_2$  in the second and final stage. In control by run-off partition of candidates, the election's first stage is held separately within both groups,  $C_1$  and  $C_2$ , and the winners of both subelections that survive the tie-handling rule used run against each other in the second and final stage.

We use the two tie-handling rules proposed by Hemaspaandra, Hemaspaandra, and Rothe [13]: ties-promote (TP) and ties-eliminate (TE). In the TP model, all the first-stage winners of a subelection,  $(C_1, V)$  or  $(C_2, V)$ , are promoted to the final round. In the TE model, a first-stage winner of a subelection,  $(C_1, V)$  or  $(C_2, V)$ , is promoted to the final round exactly if he or she is that subelection's unique winner.

Note that partitioning the candidate set  $C$  into  $C_1$  and  $C_2$  is, in some sense, similar to deleting  $C_2$  from  $C$  to obtain subelection  $(C_1, V)$  and to deleting  $C_1$  from  $C$  to obtain subelection  $(C_2, V)$ . Also, the final stage of the election may have a reduced number of candidates (which depends on the tie-handling rule used). So, in the partitioning cases, it may again happen that inadmissible AV strategies are created by the control action, but Rule 1 will coerce admissibility again.

**Name** Control by Partition of Candidates.

**Instance** An election  $(C, V)$  and a designated candidate  $c \in C$ .

**Question (constructive)** Is it possible to partition  $C$  into  $C_1$  and  $C_2$  such that  $c$  is the unique winner of the final stage of the two-stage election in which the winners of subelection  $(C_1, V)$  that survive the tie-handling rule run against all candidates in  $C_2$  (with respect to the votes in  $V$ )?

**Question (destructive)** Is it possible to partition  $C$  into  $C_1$  and  $C_2$  such that  $c$  is not a unique winner of the final stage of the two-stage election in which the winners of subelection  $(C_1, V)$  that survive the tie-handling rule run against all candidates in  $C_2$  (with respect to the votes in  $V$ )?

**Name** Control by Run-Off Partition of Candidates.

**Instance** An election  $(C, V)$  and a designated candidate  $c \in C$ .

**Question (constructive)** Is it possible to partition  $C$  into  $C_1$  and  $C_2$  such that  $c$  is the unique winner of the final stage of the two-stage election in which the winners of subelection  $(C_1, V)$  that survive the tie-handling rule run (with respect to the votes in  $V$ ) against the winners of subelection  $(C_2, V)$  that survive the tie-handling rule?

**Question (destructive)** Is it possible to partition  $C$  into  $C_1$  and  $C_2$  such that  $c$  is not a unique winner of the final stage of the two-stage election in which the winners of subelection  $(C_1, V)$  that survive the tie-handling rule run (with respect to the votes in  $V$ ) against the winners of subelection  $(C_2, V)$  that survive the tie-handling rule?

### 2.2.4 Control by Adding Voters

In this control scenario, the chair seeks to reach his or her goal by introducing new voters into a given election. These additional voters are chosen from a given pool of voters whose preferences and approval strategies over the candidates from the original election are known. Again, the number of voters that can be added is prespecified.

**Name** Control by Adding Voters.

**Instance** An election  $(C, V)$ , a collection  $W$  of additional voters with known preferences and approval strategies over  $C$ , a designated candidate  $c \in C$ , and a nonnegative integer  $\ell$ .

**Question (constructive)** Is it possible to choose a subset  $W' \subseteq W$  with  $\|W'\| \leq \ell$  such that  $c$  is the unique winner of election  $(C, V \cup W')$ ?

**Question (destructive)** Is it possible to choose a subset  $W' \subseteq W$  with  $\|W'\| \leq \ell$  such that  $c$  is not a unique winner of election  $(C, V \cup W')$ ?

### 2.2.5 Control by Deleting Voters

The chair here seeks to reach his or her goal by suppressing (up to a prespecified number of) voters.

**Name** Control by Deleting Voters.

**Instance** An election  $(C, V)$ , a designated candidate  $c \in C$ , and a nonnegative integer  $\ell$ .

**Question (constructive)** Is it possible to delete up to  $\ell$  voters from  $V$  such that  $c$  is the unique winner of the resulting election?

**Question (destructive)** Is it possible to delete up to  $\ell$  voters from  $V$  such that  $c$  is not a unique winner of the resulting election?

### 2.2.6 Control by Partition of Voters

In this scenario, the election again is conducted in two stages, and the chair now seeks to reach his or her goal by partitioning the voters  $V$  into two subcommittees,  $V_1$  and  $V_2$ . In the first stage, the subelections  $(C, V_1)$  and  $(C, V_2)$  are held separately in parallel, and the winners of each subelection who survive the tie-handling rule move forward to the second and final stage in which they compete against each other.

As in the candidate-deletion and the candidate-partition cases, also in control by partition of voters it may happen that inadmissible AV strategies are created by the control action, since the final stage of the election may have a reduced number of candidates. However, if that happens then Rule 1 will again coerce admissibility.

**Name** Control by Partition of Voters.

**Instance** An election  $(C, V)$  and a designated candidate  $c \in C$ .

**Question (constructive)** Is it possible to partition  $V$  into  $V_1$  and  $V_2$  such that  $c$  is the unique winner of the final stage of the two-stage election in which the winners of subelection  $(C, V_1)$  that survive the tie-handling rule run (with respect to the votes in  $V$ ) against the winners of subelection  $(C, V_2)$  that survive the tie-handling rule?

**Question (destructive)** Is it possible to partition  $V$  into  $V_1$  and  $V_2$  such that  $c$  is not a unique winner of the final stage of the two-stage election in which the winners of subelection  $(C, V_1)$  that survive the tie-handling rule run (with respect to the votes in  $V$ ) against the winners of subelection  $(C, V_2)$  that survive the tie-handling rule?

### 2.3 A Brief Discussion of SP-AV

The notion of SP-AV, as defined here, slightly differs from the definition proposed in this paper’s precursors [4, 25]. For example, [4] specifically required for single-candidate elections that each voter must approve of this candidate. In the present paper, we drop this requirement, as it in fact is not needed (because the one candidate in a single-candidate election will always win—even with zero approvals, i.e., SP-AV is a “voiced” voting system).

The other definitional change is more subtle. In [4, 25], we adopted Brams and Sanver’s assumption that voters  $v$  are required to have admissible AV strategies (i.e., only AV strategies  $S_v$  with  $\emptyset \neq S_v \neq C$  were allowed).<sup>10</sup> By this assumption, any vote with an inadmissible AV strategy was considered void, and we applied our rule of rewriting inadmissible AV strategies to coerce admissibility only when a control action had turned an originally admissible vote into an inadmissible one. One problem with this approach was that this rule depended on (and could be viewed as redefining) control rather than being an integral part of the voting system itself. In contrast, we now allow voters to cast inadmissible votes, and Rule 1 will turn them into admissible votes the same way it will coerce admissibility for votes that became inadmissible in the course of a control action. The in-preparation bookchapter [26] elaborates on this point and on other points regarding the definitional changes SP-AV has undergone in the course of its development up to its final form in the present paper. We stress that none of the two changes mentioned above has a severe impact on our findings or their proofs.

Another issue to be addressed is that the choice of Rule 1 might seem to be purely a matter of taste, at first glance. For example, given an inadmissible AV strategy of the form  $| a b c d$  (respectively,  $a b c d |$ ), why don’t we change it into an admissible vote of the form, say,  $a b | c d$  rather than, according to Rule 1, into  $a | b c d$  (respectively,  $a b c | d$ )? The reason is that, once we have agreed that it is desirable to coerce admissibility, our choice of Rule 1 is the most sensible way, as this is the minimally invasive rule to coerce admissibility among all possible such rules: We do change the voters’ approval strategies, but we wish to do this in the least harmful way.

### 2.4 Immunity, Susceptibility, Vulnerability, and Resistance

The following notions—which are due to Bartholdi, Tovey, and Trick [12] (see also [13, 20, 2, 3, 14])—will be central to our complexity analysis of the control problems for SP-AV.

**Definition 2.2.** *Let  $\mathcal{E}$  be an election system and let  $\Phi$  be some given type of control.*

1.  *$\mathcal{E}$  is said to be immune to  $\Phi$ -control if*

---

<sup>10</sup>Except that [1] excludes only the case  $S_v \neq C$ , see Footnote 6.

- (a)  $\Phi$  is a constructive control type and it is never possible for the chair to turn a designated candidate from being not a unique winner into being the unique winner via exerting  $\Phi$ -control, or
  - (b)  $\Phi$  is a destructive control type and it is never possible for the chair to turn a designated candidate from being the unique winner into being not a unique winner via exerting  $\Phi$ -control.
2.  $\mathcal{E}$  is said to be susceptible to  $\Phi$ -control if it is not immune to  $\Phi$ -control.
  3.  $\mathcal{E}$  is said to be vulnerable to  $\Phi$ -control if  $\mathcal{E}$  is susceptible to  $\Phi$ -control and the control problem associated with  $\Phi$  is solvable in polynomial time.
  4.  $\mathcal{E}$  is said to be resistant to  $\Phi$ -control if  $\mathcal{E}$  is susceptible to  $\Phi$ -control and the control problem associated with  $\Phi$  is NP-hard.

For example, approval voting is known to be immune to eight of the twelve types of candidate control considered in [13]. The proofs of these results crucially employ the equivalences and implications between immunity/susceptibility for various control types shown in [13] and the fact that approval voting satisfies the unique version of the Weak Axiom of Revealed Preference (denoted by Unique-WARP, see [13, 12]): If a candidate  $c$  is the unique winner in a set  $C$  of candidates, then  $c$  is the unique winner in every subset of  $C$  that includes  $c$ . In contrast with approval voting, sincere-strategy preference-based approval voting does not satisfy Unique-WARP, and we will see later in Section 3.1 that it indeed is susceptible to each type of control considered here.

**Proposition 2.3.** *Sincere-strategy preference-based approval voting does not satisfy Unique-WARP.*

**Proof.** Consider the election  $(C, V)$  with candidate set  $C = \{a, b, c, d\}$  and voter collection  $V = \{v_1, v_2, v_3, v_4\}$ . Removing candidate  $d$  changes the profile as follows according to Rule 1:

$$\begin{array}{rcl}
 v_1 : & b & c & a & | & d & & & b & c & | & a \\
 v_2 : & c & | & a & d & b & & & c & | & a & b \\
 v_3 : & a & b & c & | & d & & & a & b & | & c \\
 v_4 : & b & a & c & | & d & & & b & a & | & c
 \end{array}$$

is changed to  
(by removing  $d$ ):

Note that the approval/disapproval line has been moved in voters  $v_1$ ,  $v_3$ , and  $v_4$  according to Rule 1. Although  $c$  was the unique winner of  $(C, V)$ ,  $c$  is not a winner of  $(\{a, b, c\}, V)$  (in fact,  $b$  is the unique winner of  $(\{a, b, c\}, V)$ ). Thus, SP-AV does not satisfy Unique-WARP.  $\square$

### 3 Results for Sincere-Strategy Preference-Based Approval Voting

Theorem 3.1 below (see also Table 2) shows the complexity results regarding control of elections for SP-AV. As mentioned in the introduction, with 19 resistances and only three vulnerabilities, this system has more resistances and fewer vulnerabilities to control (for our 22 control types) than is currently known for any other natural voting system with a polynomial-time winner problem.

Control by	Plurality		SP-AV		AV	
	Constr.	Destr.	Constr.	Destr.	Constr.	Destr.
Adding an Unlimited Number of Candidates	R	R	<b>R</b>	<b>R</b>	I	V
Adding a Limited Number of Candidates	R	R	<b>R</b>	<b>R</b>	I	V
Deleting Candidates	R	R	<b>R</b>	<b>R</b>	V	I
Partition of Candidates	TE: R TP: R	TE: R TP: R	<b>TE: R</b> <b>TP: R</b>	<b>TE: R</b> <b>TP: R</b>	TE: V TP: I	TE: I TP: I
Run-off Partition of Candidates	TE: R TP: R	TE: R TP: R	<b>TE: R</b> <b>TP: R</b>	<b>TE: R</b> <b>TP: R</b>	TE: V TP: I	TE: I TP: I
Adding Voters	V	V	<b>R</b>	<b>V</b>	R	V
Deleting Voters	V	V	<b>R</b>	<b>V</b>	R	V
Partition of Voters	TE: V TP: R	TE: V TP: R	<b>TE: R</b> <b>TP: R</b>	<b>TE: V</b> <b>TP: R</b>	TE: R TP: R	TE: V TP: V

Table 2: Overview of results. Key: I means immune, R means resistant, V means vulnerable, TE means ties-eliminate, and TP means ties-promote. Results for SP-AV are new; their proofs are either new or draw on proofs from [13]. Results for plurality and AV, stated here to allow comparison, are due to Bartholdi, Tovey, and Trick [12] and to Hemaspaandra, Hemaspaandra, and Rothe [13]. (The results for control by adding a limited number of candidates for plurality and approval voting, though not stated explicitly in [12, 13], follow immediately from the proofs of the corresponding results for the “unlimited” variant of the problem, see Footnote 4.)

**Theorem 3.1.** *Sincere-strategy preference-based approval voting has the resistances and vulnerabilities to the 22 types of control defined in Section 2.2 that are shown in Table 2.*

### 3.1 Susceptibility

By definition, all resistance and vulnerability results in particular require susceptibility. In the following two lemmas, we prove that sincere-strategy preference-based approval voting is susceptible to the 22 types of control defined in Section 2.2. To this end, we will make use of Theorems 4.1, 4.2, and 4.3 of Hemaspaandra, Hemaspaandra, and Rothe [13] that provide susceptibility equivalences and implications for various control types.<sup>11</sup> For the sake of self-containment, we give these results below, stated essentially word-for-word as in [13]. In particular, Theorem 3.2 (which is [13, Thm. 4.1]) gives four equivalences between susceptibility to constructive/destructive control by adding/deleting candidates/voters; Theorem 3.3 (which is [13, Thm. 4.2]) gives four implications that link susceptibility to control by (run-off) partition of candidates/voters with susceptibility to control by deleting candidates/voters; and Theorem 3.4 (which is [13, Thm. 4.3]) states that every “voiced” voting system is susceptible to constructive control by deleting candidates and to destructive control by adding candidates, and that for each voiced voting system susceptibility to destructive control by partition of voters (in model TE or TP) implies susceptibility to destructive control by deleting voters. A voting system is said to be *voiced* if in every one-candidate election, this candidate wins.

<sup>11</sup>Although [13] does not consider the case of control by adding a limited number of candidates explicitly, it is immediate that all proofs for the “unlimited” case in [13] work also for this “limited” case.

**Theorem 3.2** (Thm. 4.1 of [13]). *1. A voting system is susceptible to constructive control by adding (either a limited or an unlimited number of) candidates if and only if it is susceptible to destructive control by deleting candidates.*

*2. A voting system is susceptible to constructive control by deleting candidates if and only if it is susceptible to destructive control by adding (either a limited or an unlimited number of) candidates.*

*3. A voting system is susceptible to constructive control by adding voters if and only if it is susceptible to destructive control by deleting voters.*

*4. A voting system is susceptible to constructive control by deleting voters if and only if it is susceptible to destructive control by adding voters.*

**Theorem 3.3** (Thm. 4.2 of [13]). *1. If a voting system is susceptible to constructive control by partition of voters (in model TE or TP), then it is susceptible to constructive control by deleting candidates.*

*2. If a voting system is susceptible to constructive control by partition or run-off partition of candidates (in model TE or TP), then it is susceptible to constructive control by deleting candidates.*

*3. If a voting system is susceptible to constructive control by partition of voters in model TE, then it is susceptible to constructive control by deleting voters.*

*4. If a voting system is susceptible to destructive control by partition or run-off partition of candidates (in model TE or TP), then it is susceptible to destructive control by deleting candidates.*

**Theorem 3.4** (Thm. 4.3 of [13]). *1. If a voiced voting system is susceptible to destructive control by partition of voters (in model TE or TP), then it is susceptible to destructive control by deleting voters.*

*2. Each voiced voting system is susceptible to constructive control by deleting candidates.*

*3. Each voiced voting system is susceptible to destructive control by adding (either a limited or an unlimited number of) candidates.*

We start with susceptibility to candidate control for SP-AV.

**Lemma 3.5.** *SP-AV is susceptible to constructive and destructive control by adding candidates (in both the “limited” and the “unlimited” variant of the problem), by deleting candidates, and by partition of candidates (with or without run-off and for each in both tie-handling models, TE and TP).*

**Proof.** From Theorem 3.4 and the obvious fact that SP-AV is a voiced voting system, it immediately follows that SP-AV is susceptible to constructive control by deleting candidates and to destructive control by adding candidates (in both the “limited” and the “unlimited” variant of the problem).

Consider the election  $(C, V)$  with candidate set  $C = \{a, b, c, d, e, f\}$  and voter collection  $V = \{v_1, v_2, \dots, v_6\}$  and the following partition of  $C$  into  $C_1 = \{a, c, d\}$  and  $C_2 = \{b, e, f\}$ :

	$(C, V)$	is partitioned into	$(C_1, V)$	and	$(C_2, V)$
$v_1 :$	$a \ b \ c \   \ d \ e \ f$		$a \ c \   \ d$		$b \   \ e \ f$
$v_2 :$	$b \ c \   \ a \ d \ e \ f$		$c \   \ a \ d$		$b \   \ e \ f$
$v_3 :$	$a \ c \   \ b \ d \ e \ f$		$a \ c \   \ d$		$b \   \ e \ f$
$v_4 :$	$b \ a \ c \   \ d \ e \ f$		$a \ c \   \ d$		$b \   \ e \ f$
$v_5 :$	$a \ b \ d \ e \ c \   \ f$		$a \ d \   \ c$		$b \ e \   \ f$
$v_6 :$	$a \ e \ d \ f \ c \   \ b$		$a \ d \   \ c$		$e \ f \   \ b$

With six approvals,  $c$  is the unique winner of  $(C, V)$ . However,  $a$  is the unique winner of  $(C_1, V)$ , which implies that  $c$  is not promoted to the final stage, regardless of whether we use the TE or TP tie-handling rule and regardless of whether we employ a partition of candidates with or without run-off. Thus, SP-AV is susceptible to destructive control by partition of candidates (with or without run-off and for each in both tie-handling models, TE and TP). By Theorem 3.3, SP-AV is also susceptible to destructive control by deleting candidates. By Theorem 3.2 in turn, SP-AV is also susceptible to constructive control by adding candidates (in both the “limited” and the “unlimited” variant of the problem).

Note that  $a$  is not the unique winner of  $(C, V)$ , as  $a$  loses to  $c$  by 5 to 6. However, if we partition  $C$  into  $C_1 = \{a, c, d\}$  and  $C_2 = \{b, e, f\}$ , then  $a$  is the unique winner of  $(C_1, V)$  and  $b$  is the unique winner of  $(C_2, V)$ . Since both subelections have a unique winner, it does not matter whether the TE rule or the TP rule is applied. The final-stage election is  $(\{a, b\}, V)$  in the case of run-off partition of candidates, and it is  $(\{a, b, e, f\}, V)$  in the case of partition of candidates. Since  $a$  wins against  $b$  in the former case by 4 to 2 and in the latter case by 5 to 4 (and  $e$  and  $f$  do even worse than  $b$  in this case),  $a$  is the unique winner in both cases. Thus, SP-AV is susceptible to constructive control by partition of candidates (with or without run-off and for each in both models, TE and TP).  $\square$

We now turn to susceptibility to voter control.

**Lemma 3.6.** *SP-AV is susceptible to constructive and destructive control by adding voters, by deleting voters, and by partition of voters in both tie-handling models, TE and TP.*

**Proof.** Consider the election  $(C, V)$  with candidate set  $C = \{a, b, c, d, e, f\}$  and voter collection  $V = \{v_1, v_2, \dots, v_8\}$  and partition  $V$  into  $V_1 = \{v_1, v_2, v_3, v_4\}$  and  $V_2 = \{v_5, v_6, v_7, v_8\}$ . Thus, we

change:

	$(C, V)$	into	$(C, V_1)$	and	$(C, V_2)$
$v_1$ :	$a b c \mid d e f$		$a b c \mid d e f$		
$v_2$ :	$a c \mid b d e f$		$a c \mid b d e f$		
$v_3$ :	$c b a d \mid e f$		$c b a d \mid e f$		
$v_4$ :	$a b \mid d e c f$		$a b \mid d e c f$		
$v_5$ :	$a d c \mid b e f$				$a d c \mid b e f$
$v_6$ :	$e b c d \mid a f$				$e b c d \mid a f$
$v_7$ :	$d e c f \mid b a$				$d e c f \mid b a$
$v_8$ :	$d f \mid b a c e$				$d f \mid b a c e$

With six approvals,  $c$  is the unique winner of  $(C, V)$ . However,  $a$  is the unique winner of  $(C, V_1)$  and  $d$  is the unique winner of  $(C, V_2)$ , which implies that  $c$  is not promoted to the final stage, regardless of whether we use the TE or TP tie-handling rule. (In the final-stage election  $(\{a, d\}, V)$ ,  $a$  wins by 5 to 3.) Thus, SP-AV is susceptible to destructive control by partition of voters in models TE and TP. By Theorem 3.4 and since SP-AV is a voiced system, SP-AV is also susceptible to destructive control by deleting voters. Finally, by Theorem 3.2, SP-AV is also susceptible to constructive control by adding voters.

Now, if we let  $a$  and  $c$  change their roles in the above election and argument, we see that SP-AV is also susceptible to constructive control by partition of voters in models TE and TP. By Theorem 3.3, susceptibility to constructive control by partition of voters in model TE implies susceptibility to constructive control by deleting voters. Again, by Theorem 3.2, SP-AV is also susceptible to destructive control by adding voters.  $\square$

### 3.2 Candidate Control

Theorems 3.7 and 3.10 below show that sincere-strategy preference-based approval voting is fully resistant to candidate control. This result should be contrasted with that of Hemaspaandra, Hemaspaandra, and Rothe [13], who proved immunity and vulnerability for all cases of candidate control within approval voting (see Table 2). In fact, SP-AV has the same resistances to candidate control as plurality, and we will show that the construction presented in [13] to prove plurality resistant also works for sincere-strategy preference-based approval voting in all cases of candidate control except one—namely, except for constructive control by deleting candidates. Theorem 3.10 establishes resistance for this one missing case.

All resistance results in this section follow via a reduction from the NP-complete problem Hitting Set (see, e.g., Garey and Johnson [27]):

**Name** Hitting Set.

**Instance** A set  $B = \{b_1, b_2, \dots, b_m\}$ , a nonempty collection  $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$  of subsets  $S_i \subseteq B$ ,<sup>12</sup> and a positive integer  $k \leq m$ .

---

<sup>12</sup>Our assumption that  $\mathcal{S}$  be nonempty (i.e., that  $n \geq 1$ ) is not explicitly specified in Garey and Johnson [27]. However, it is clear that requiring  $n \geq 1$  does not change the complexity of the problem.

**Question** Does  $\mathcal{S}$  have a hitting set of size at most  $k$ , i.e., is there a set  $B' \subseteq B$  with  $\|B'\| \leq k$  such that for each  $i$ ,  $S_i \cap B' \neq \emptyset$ ?

Note that some of our proofs for SP-AV are based on constructions presented in [13] to prove the corresponding results for approval voting or plurality, whereas some other of our results require new insights to make the proof work for SP-AV. For completeness, we will present each construction here (even if the modification of a previous construction is rather straightforward), explicitly stating whether it is based on a previous construction from [13], and if so, we will state in each case on which construction it is based and what the differences to the related previous construction are.

**Theorem 3.7.** *SP-AV is resistant to all types of constructive and destructive candidate control defined in Section 2.2 except for constructive control by deleting candidates (which will be handled separately in Theorem 3.10).*

Resistance of SP-AV to constructive control by deleting candidates, which is the missing case in Theorem 3.7, will be shown as Theorem 3.10 below.

The proof of Theorem 3.7 is based on a construction for plurality in [13], except that only the arguments for *destructive* candidate control are given there (simply because plurality was shown resistant to all cases of constructive candidate control already by Bartholdi, Tovey, and Trick [12] via different constructions). We now provide a short proof sketch of Theorem 3.7 and the construction from [13] (slightly modified so as to formally conform with the SP-AV voter representation) in order to (i) show that the same construction can be used to establish all but one resistances of SP-AV to *constructive* candidate control, and (ii) explain why constructive control by deleting candidates (which is missing in Theorem 3.7) does *not* follow from this construction.

**Proof Sketch of Theorem 3.7.** Susceptibility holds by Lemma 3.5 in each case. The resistance proofs are based on a reduction from Hitting Set and employ Construction 3.8 below, slightly modified so as to formally conform with the SP-AV voter representation.

**Construction 3.8** (Hemaspaandra et al. [13]). *Let  $(B, \mathcal{S}, k)$  be a given instance of Hitting Set, where  $B = \{b_1, b_2, \dots, b_m\}$  is a set,  $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$  is a nonempty collection of subsets  $S_i \subseteq B$ , and  $k \leq m$  is a positive integer. Define the election  $(C, V)$ , where  $C = B \cup \{c, w\}$  is the candidate set and where  $V$  consists of the following voters:*

1. *There are  $2(m - k) + 2n(k + 1) + 4$  voters of the form:  $c \mid w \ B$ .*
2. *There are  $2n(k + 1) + 5$  voters of the form:  $w \mid c \ B$ .*
3. *For each  $i$ ,  $1 \leq i \leq n$ , there are  $2(k + 1)$  voters of the form:  $S_i \mid c \ w \ (B - S_i)$ .*
4. *For each  $j$ ,  $1 \leq j \leq m$ , there are two voters of the form:  $b_j \mid w \ c \ (B - \{b_j\})$ .*

Since

$$\begin{aligned}
 & \text{score}_{(\{c,w\}, V)}(c) - \text{score}_{(\{c,w\}, V)}(w) \\
 &= (2(m - k) + 2n(k + 1) + 4 + 2n(k + 1)) - (2n(k + 1) + 5 + 2m) \\
 &= 2k(n - 1) + 2n - 1
 \end{aligned}$$

is positive (because of  $n \geq 1$ ),  $c$  is the unique winner of election  $(\{c, w\}, V)$ . The key observation is the following proposition, which can be proven as in [13].

**Proposition 3.9** (Hemaspaandra et al. [13]). *1. If  $\mathcal{S}$  has a hitting set  $B'$  of size  $k$ , then  $w$  is the unique SP-AV winner of election  $(B' \cup \{c, w\}, V)$ .*

*2. Let  $D \subseteq B \cup \{w\}$ . If  $c$  is not the unique SP-AV winner of election  $(D \cup \{c\}, V)$ , then there exists a set  $B' \subseteq B$  such that*

- (a)  $D = B' \cup \{w\}$ ,*
- (b)  $w$  is the unique SP-AV winner of election  $(B' \cup \{c, w\}, V)$ , and*
- (c)  $B'$  is a hitting set of  $\mathcal{S}$  of size less than or equal to  $k$ .*

As an example, the resistance of SP-AV to constructive and destructive control by adding candidates (both in the limited and the unlimited version of the problem) now follows immediately from Proposition 3.9, via mapping the Hitting Set instance  $(B, \mathcal{S}, k)$  to the set  $\{c, w\}$  of qualified candidates and the set  $B$  of spoiler candidates, to the voter collection  $V$ , and by having  $c$  be the designated candidate in the destructive case and by having  $w$  be the designated candidate in the constructive case.

The other cases of Theorem 3.7 can be proven similarly. □ Theorem 3.7

Turning now to the missing case mentioned in Theorem 3.7 above: Why does Construction 3.8 not work for constructive control by deleting candidates? Informally put, the reason is that  $c$  is the only serious rival of  $w$  in the election  $(C, V)$  of Construction 3.8, so by simply deleting  $c$  the chair could make  $w$  the unique SP-AV winner, regardless of whether  $\mathcal{S}$  has a hitting set of size  $k$ . However, via a different construction, we can prove resistance also in this case.

**Theorem 3.10.** *SP-AV is resistant to constructive control by deleting candidates.*

**Proof.** Susceptibility holds by Lemma 3.5. To prove resistance, we provide a reduction from Hitting Set.<sup>13</sup> Let  $(B, \mathcal{S}, k)$  be a given instance of Hitting Set, where  $B = \{b_1, b_2, \dots, b_m\}$  is a set,  $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$  is a nonempty collection of subsets  $S_i \subseteq B$ , and  $k < m$  is a positive integer.<sup>14</sup>

Define the election  $(C, V)$ , where  $C = B \cup \{w\}$  is the candidate set and  $V$  is the collection of voters. We assume that the candidates in  $B$  are in an arbitrary but fixed order, and for each voter below, this order is also used in each subset of  $B$ . For example, if  $B = \{b_1, b_2, b_3, b_4\}$  (where the elements of  $B$  are ordered as  $b_1, b_2, b_3, b_4$ ) and some subset  $S_i = \{b_1, b_3\}$  of  $B$  occurs in some voter then this voter prefers  $b_1$  to  $b_3$ , and so does any other voter whose preference list contains  $S_i$ .

$V$  consists of the following  $4n(k+1) + 4m - 2k + 3$  voters:

1. For each  $i$ ,  $1 \leq i \leq n$ , there are  $2(k+1)$  voters of the form:  $S_i \mid (B - S_i) \ w$ .
2. For each  $i$ ,  $1 \leq i \leq n$ , there are  $2(k+1)$  voters of the form:  $(B - S_i) \ w \mid S_i$ .

<sup>13</sup>In contrast, Bartholdi, Tovey, and Trick [12] gave a reduction from Exact Cover by Three-Sets (which is defined in the proof of Theorem 3.11) to prove that plurality is resistant to constructive control by deleting candidates.

<sup>14</sup>Note that if  $k = m$  then  $B$  is always a hitting set of size at most  $k$  (provided that  $\mathcal{S}$  contains only nonempty sets—a requirement that doesn't affect the NP-completeness of the problem), and we thus may require that  $k < m$ .

3. For each  $j$ ,  $1 \leq j \leq m$ , there are two voters of the form:  $b_j \mid w \ (B - \{b_j\})$ .
4. There are  $2(m - k)$  voters of the form:  $B \mid w$ .
5. There are three voters of the form:  $w \mid B$ .

Since for each  $b_j \in B$ , the difference

$$\text{score}_{(C,V)}(w) - \text{score}_{(C,V)}(b_j) = 2n(k+1) + 3 - (2n(k+1) + 2 + 2(m-k)) = 1 - 2(m-k)$$

is negative (due to  $k < m$ ),  $w$  loses to each member of  $B$  and so does not win election  $(C, V)$ .

We claim that  $\mathcal{S}$  has a hitting set  $B'$  of size  $k$  if and only if  $w$  can be made the unique SP-AV winner by deleting at most  $m - k$  candidates.

From left to right: Suppose  $\mathcal{S}$  has a hitting set  $B'$  of size  $k$ . Then, for each  $b_j \in B'$ ,

$$\text{score}_{(B' \cup \{w\}, V)}(w) - \text{score}_{(B' \cup \{w\}, V)}(b_j) = 2n(k+1) + 2(m-k) + 3 - (2n(k+1) + 2 + 2(m-k)) = 1,$$

since the approval line is moved for  $2(m - k)$  voters of the third group according to Rule 1, thus transferring their approvals from members of  $B - B'$  to  $w$ . It is easy to see that the approval line is not moved in any of the other voters according to Rule 1; in particular, the approval line is not moved in any of the voters from the first and second group, since  $B' \cap S_i \neq \emptyset$  for each  $i$ ,  $1 \leq i \leq n$ . So  $w$  is the unique SP-AV winner of election  $(B' \cup \{w\}, V)$ . Since  $B' \cup \{w\} = C - (B - B')$ , it follows from  $\|B\| = m$  and  $\|B'\| = k$  that deleting  $m - k$  candidates from  $C$  makes  $w$  the unique SP-AV winner.

From right to left: Let  $D \subseteq B$  be any set such that  $\|D\| \leq m - k$  and  $w$  is the unique SP-AV winner of election  $(C - D, V)$ . Let  $B' = (C - D) - \{w\}$ . Note that  $B' \subseteq B$  and that we have the following scores in  $(B' \cup \{w\}, V)$ :

$$\begin{aligned} \text{score}_{(B' \cup \{w\}, V)}(w) &= 2(n - \ell)(k+1) + 2(m - \|B'\|) + 3, \\ \text{score}_{(B' \cup \{w\}, V)}(b_j) &\leq 2n(k+1) + 2\ell(k+1) + 2 + 2(m-k) \quad \text{for each } b_j \in B', \end{aligned}$$

where  $\ell$  is the number of sets  $S_i \in \mathcal{S}$  that are not hit by  $B'$ , i.e.,  $B' \cap S_i = \emptyset$ . Recall that for each  $i$ ,  $1 \leq i \leq n$ , all of the  $2(k+1)$  voters of the form  $S_i \mid (B - S_i) \mid w$  in the first voter group have ranked the candidates in the same order. Thus, for each  $i$ ,  $1 \leq i \leq n$ , whenever  $B' \cap S_i = \emptyset$  one and the same candidate in  $B'$  benefits from moving the approval line according to Rule 1, namely the candidate occurring first in our fixed ordering of  $B'$ . Call this candidate  $b$  and note that

$$\text{score}_{(B' \cup \{w\}, V)}(b) = 2n(k+1) + 2\ell(k+1) + 2 + 2(m-k).$$

Since  $w$  is the unique SP-AV winner of  $(B' \cup \{w\}, V)$ ,  $w$  has more approvals than any candidate in  $B'$  and in particular more than  $b$ . Thus, we have

$$\begin{aligned} &\text{score}_{(B' \cup \{w\}, V)}(w) - \text{score}_{(B' \cup \{w\}, V)}(b) \\ &= 2(n - \ell)(k+1) + 2(m - \|B'\|) + 3 - 2n(k+1) - 2\ell(k+1) - 2 - 2(m-k) \\ &= 1 + 2(k - \|B'\|) - 4\ell(k+1) > 0. \end{aligned}$$

Solving this inequality for  $\ell$ , we obtain

$$0 \leq \ell < \frac{1 + 2(k - \|B'\|)}{4(k+1)} < \frac{4 + 4k}{4(k+1)} = 1.$$

Thus  $\ell = 0$ . It follows that  $1 + 2(k - \|B'\|) > 0$ , which implies  $\|B'\| \leq k$ . Thus,  $B'$  is a hitting set of size at most  $k$ .  $\square$

### 3.3 Voter Control

Turning now to control by adding and by deleting voters, it is known from [13] that approval voting is resistant to constructive control and is vulnerable to destructive control (see Table 2).<sup>15</sup> Their proofs can be modified so as to also apply to sincere-strategy preference-based approval voting. We here provide only proof sketches; more details of the proofs are provided in the technical report version [25].

**Theorem 3.11.** *SP-AV is resistant to constructive control by adding voters and by deleting voters and is vulnerable to destructive control by adding voters and by deleting voters.*

**Proof Sketch of Theorem 3.11.** Susceptibility holds by Lemma 3.6 in all cases. To prove resistance to constructive control by adding voters (respectively, by deleting voters), the construction of [13, Thm. 4.43] (respectively, of [13, Thm. 4.44]) works, modified only by specifying voter preferences consistently with the voters' approval strategies (and, in the deleting-voters case, by adding a dummy candidate who is disapproved of and ranked last by every voter in the construction to ensure an admissible AV strategy profile). These constructions provide polynomial-time reductions from the NP-complete problem Exact Cover by Three-Sets (denoted by X3C; see, e.g., Garey and Johnson [27]), which is defined as follows:

**Name** Exact Cover by Three-Sets (X3C).

**Instance** A set  $B = \{b_1, b_2, \dots, b_{3m}\}$ ,  $m > 1$ ,<sup>16</sup> and a collection  $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$  of subsets  $S_i \subseteq B$  with  $\|S_i\| = 3$  for each  $i$ .

**Question** Does  $\mathcal{S}$  have an exact cover for  $B$ , i.e., is there a subcollection  $\mathcal{S}' \subseteq \mathcal{S}$  such that every element of  $B$  occurs in exactly one set in  $\mathcal{S}'$ ?

The polynomial-time algorithms showing that approval voting is vulnerable to destructive control by adding voters and by deleting voters [13, Thm. 4.24] can be straightforwardly adapted to also work for sincere-strategy preference-based approval voting, since no approval lines are moved according to Rule 1 in these control scenarios.  $\square$

<sup>15</sup>Meir et al. [21] proved in their interesting “multi-winner” model (which generalizes Bartholdi, Tovey, and Trick’s model [12] by adding a utility function and some other parameters) that approval voting is resistant to constructive control by adding voters. According to Footnote 13 of [13], this resistance result immediately follows from the corresponding resistance result in [28, 13], essentially due to the fact that lower bounds in more flexible models are inherited from more restrictive models.

<sup>16</sup>Our assumption that  $m > 1$  is not explicitly specified in Garey and Johnson [27]. However, it is clear that requiring  $m > 1$  does not change the complexity of the problem.

We now prove that, just like plurality, sincere-strategy preference-based approval voting is resistant to constructive and destructive control by partition of voters in model TP. In fact, the proof presented in [13] for plurality in these two cases also works for SP-AV with minor modifications. In contrast, approval voting is vulnerable to the destructive variant of this control type [13].

**Theorem 3.12.** *SP-AV is resistant to constructive and destructive control by partition of voters in model TP.*

**Proof Sketch of Theorem 3.12.** The proof is again based on Construction 3.8, but the reduction is now from Restricted Hitting Set, which is defined just as Hitting Set (see Section 3.2) except that  $n(k+1)+1 \leq m-k$  is required in addition. Restricted Hitting Set is also NP-complete [13]. Now, the key observation is the following proposition, which can be proven as in [13].

**Proposition 3.13** (Hemaspaandra et al. [13]). *Let  $(B, \mathcal{S}, k)$  be a given Restricted Hitting Set instance, where  $B = \{b_1, b_2, \dots, b_m\}$  is a set,  $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$  is a nonempty collection of subsets  $S_i \subseteq B$ , and  $k \leq m$  is a positive integer such that  $n(k+1)+1 \leq m-k$ . If  $(C, V)$  is the election resulting from  $(B, \mathcal{S}, k)$  via Construction 3.8, then the following three statements are equivalent:*

1.  $\mathcal{S}$  has a hitting set of size less than or equal to  $k$ .
2.  $V$  can be partitioned such that  $w$  is the unique SP-AV winner in model TP.
3.  $V$  can be partitioned such that  $c$  is not the unique SP-AV winner in model TP.

The theorem now follows immediately from Proposition 3.13. □ Theorem 3.12

Finally, we turn to control by partition of voters in model TE. For this control type, Hemaspaandra et al. [13] proved approval voting resistant in the constructive case and vulnerable in the destructive case. We have the same results for sincere-strategy preference-based approval voting. Our resistance proof in the constructive case (see the proof of Theorem 3.14) is similar to the corresponding proof of resistance in [13]. However, while our polynomial-time algorithm showing vulnerability for SP-AV in the destructive case (see the proof of Theorem 3.15) is based on the corresponding polynomial-time algorithm for approval voting in [13], it extends their algorithm in a nontrivial way.

**Theorem 3.14.** *SP-AV is resistant to constructive control by partition of voters in model TE.*

**Proof.** Susceptibility holds by Lemma 3.6. The proof of resistance is based on the construction of [13, Thm. 4.46] with only minor changes. Let an X3C instance  $(B, \mathcal{S})$  be given, where  $B = \{b_1, b_2, \dots, b_{3m}\}$ ,  $m > 1$ , is a set and  $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$  is a collection of subsets  $S_i \subseteq B$  with  $\|S_i\| = 3$  for each  $i$ ,  $1 \leq i \leq n$ . Without loss of generality, we may assume that  $n \geq m$ . Define the value  $\ell_j = \|\{S_i \in \mathcal{S} \mid b_j \in S_i\}\|$  for each  $j$ ,  $1 \leq j \leq 3m$ .

Define the election  $(C, V)$ , where  $C = B \cup \{w, x, y\} \cup Z$  is the candidate set with the distinguished candidate  $w$ ,  $Z = \{z_1, z_2, \dots, z_n\}$ , and where  $V$  is defined to consist of the following  $4n + m$  voters:

1. For each  $i$ ,  $1 \leq i \leq n$ , there is one voter of the form:  $y \ S_i \mid w \ ((B - S_i) \cup \{x\}) \cup Z$ .

2. For each  $i$ ,  $1 \leq i \leq n$ , there is one voter of the form:  $y \ z_i \mid w \ (B \cup \{x\} \cup (Z - \{z_i\}))$ .
3. For each  $i$ ,  $1 \leq i \leq n$ , there is one voter of the form:  $w \ (Z - \{z_i\}) \ B_i \mid x \ y \ z_i \ (B - B_i)$ , where  $B_i = \{b_j \in B \mid i \leq n - \ell_j\}$ .
4. There are  $n + m$  voters of the form:  $x \mid y \ (B \cup \{w\} \cup Z)$ .

Note that  $score_{(C,V)}(b_j) = n$  for each  $b_j \in B$ . Since the above construction is only slightly modified from the proof of [13, Thm. 4.46], so as to formally conform with the SP-AV voter representation, the same argument as in that proof shows that  $\mathcal{S}$  has an exact cover for  $B$  if and only if  $w$  can be made the unique SP-AV winner by partition of voters in model TE. Note that, in the present control scenario, approval voting and SP-AV can differ only in the run-off, but the construction ensures that they don't differ there.

From left to right, if  $\mathcal{S}$  has an exact cover for  $B$  then partition the set of voters as follows:  $V_1$  consists of the  $m$  voters of the form  $y \ S_i \mid w \ ((B - S_i) \cup \{x\} \cup Z)$  that correspond to the sets in the exact cover, of the  $n + m$  voters who approve of only  $x$ , and of the  $n$  voters who approve of  $y$  and  $z_i$ ,  $1 \leq i \leq n$ . Let  $V_2 = V - V_1$ . It follows that  $w$  is the unique SP-AV winner of both subelection  $(C, V_2)$  and the run-off, simply because no candidate proceeds to the run-off from the other subelection,  $(C, V_1)$ , in which  $x$  and  $y$  tie for winner with a score of  $n + m$  each.

From right to left, suppose  $w$  can be made the unique SP-AV winner by partition of voters in model TE. Let  $(V_1, V_2)$  be a partition of  $V$  such that  $w$  is the unique SP-AV winner of the run-off. According to model TE,  $w$  must also be the unique SP-AV winner of one subelection, say of  $(C, V_1)$ . Note that each voter of the form  $y \ z_i \mid w \ (B \cup \{x\} \cup (Z - \{z_i\}))$  has to be in  $V_2$  (otherwise, we would have  $score_{(C,V_1)}(w) = score_{(C,V_1)}(z_i)$  for at least one  $i$ , and so  $w$  would not be the unique SP-AV winner of  $(C, V_1)$  anymore). However, if there were more than  $m$  voters of the form  $y \ S_i \mid w \ ((B - S_i) \cup \{x\} \cup Z)$  in  $V_2$  then  $score_{(C,V_2)}(y) > n + m$ , and so  $y$  would be the unique SP-AV winner of the other subelection,  $(C, V_2)$ . But then, also in the SP-AV model,  $y$  would win the run-off against  $w$  because  $score_{(\{w,y\},V)}(y) = 3n + m > n = score_{(\{w,y\},V)}(w)$ , which contradicts the assumption that  $w$  has been made the unique SP-AV winner by the partition  $(V_1, V_2)$ . Hence, there are at most  $m$  voters of the form  $y \ S_i \mid w \ ((B - S_i) \cup \{x\} \cup Z)$  in  $V_2$ , and these  $m$  voters correspond to an exact cover of  $B$ , since otherwise there would be at least one  $b_j \in B$  with  $score_{(C,V_1)}(b_j) = n = score_{(C,V_1)}(w)$ .  $\square$

**Theorem 3.15.** *SP-AV is vulnerable to destructive control by partition of voters in model TE.*

**Proof.** Susceptibility holds by Lemma 3.6. To prove vulnerability, we describe a polynomial-time algorithm showing that (and how) the chair can exert destructive control by partition of voters in model TE for sincere-strategy preference-based approval voting. Our algorithm extends the polynomial-time algorithm designed by Hemaspaandra et al. [13] to prove approval voting vulnerable to this type of control. Specifically, our algorithm adds Loop 2 below to their algorithm, and we will explain below why it is necessary to add this second loop.

Let  $(C, V)$  be an election, and for each voter  $v \in V$ , let  $S_v \subseteq C$  denote  $v$ 's AV strategy. In each iteration of Loop 1 in the algorithm below, we will consider three candidates,  $a$ ,  $b$ , and  $c$ . Define the

following five numbers:<sup>17</sup>

$$\begin{aligned} W_c &= \|\{v \in V \mid a \notin S_v, b \notin S_v, c \in S_v\}\|, & L_c &= \|\{v \in V \mid a \in S_v, b \in S_v, c \notin S_v\}\|, \\ D_a &= \|\{v \in V \mid a \in S_v, b \notin S_v, c \notin S_v\}\|, & D_b &= \|\{v \in V \mid a \notin S_v, b \in S_v, c \notin S_v\}\|, \text{ and} \\ D_{ac} &= \|\{v \in V \mid a \in S_v, b \notin S_v, c \in S_v\}\|. \end{aligned}$$

In addition, we introduce the following notation. Given an election  $(C, V)$  and two distinct candidates  $x, y \in C$ , let  $\text{diff}(x, y)$  denote the number of voters in  $V$  who prefer  $x$  to  $y$  minus the number of voters in  $V$  who prefer  $y$  to  $x$ . Define  $B_c$  to be the set of candidates  $y \neq c$  in  $C$  such that  $\text{diff}(y, c) \geq 0$ .

The input to our algorithm is an election  $(C, V)$ , where each voter  $v \in V$  has a sincere AV strategy  $S_v$  (otherwise, the input is considered malformed and outright rejected), and a distinguished candidate  $c \in C$ . On this input, our algorithm works as follows.

1. **Checking the trivial cases:** can be done as in the case of approval voting, see the proof of [13, Thm. 4.21]. In particular, if  $C = \{c\}$  then output “control impossible” and halt, since  $c$  cannot help but win. If  $C$  contains more candidates than only  $c$  but  $c$  already is not the unique SP-AV winner of  $(C, V)$  then output the (successful) partition  $(V, \emptyset)$  and halt. Otherwise, if  $\|C\| = 2$  then output “control impossible” and halt, as  $c$  is the unique SP-AV winner of  $(C, V)$  in the current case and so, however the voters are partitioned,  $c$  must win—against the one rivaling candidate—at least one subelection and also the run-off.
2. **Loop 1:** For each  $a, b \in C$  such that  $\|\{a, b, c\}\| = 3$ , check whether  $V$  can be partitioned into  $V_1$  and  $V_2$  such that  $\text{score}_{(C, V_1)}(a) \geq \text{score}_{(C, V_1)}(c)$  and  $\text{score}_{(C, V_2)}(b) \geq \text{score}_{(C, V_2)}(c)$ . As shown in the proof of [13, Thm. 4.21], this is equivalent to checking

$$(3.1) \quad W_c - L_c \leq D_a + D_b.$$

If (3.1) fails, this  $a$  and  $b$  cannot prevent  $c$  from being the unique winner of at least one subelection and thus also of the run-off, so we move on to test the next  $a$  and  $b$  in this loop. If (3.1) holds, however, output the partition  $(V_1, V_2)$  and halt, where  $V_1$  consists of the voters contributing to  $D_a$ , of the voters contributing to  $D_{ac}$ , and of  $\min(W_c, D_a)$  voters contributing to  $W_c$ , and where  $V_2 = V - V_1$ .

3. **Loop 2:** For each  $d \in B_c$ , partition  $V$  as follows. Let  $V_1$  consist of all voters in  $V$  who approve of  $d$ , and let  $V_2 = V - V_1$ . If  $d$  is the unique winner of  $(C, V_1)$ , then output  $(V_1, V_2)$  as a successful partition and halt. Otherwise, go to the next  $d \in B_c$ .
4. **Termination:** If in no iteration of either Loop 1 or Loop 2 a successful partition of  $V$  was found, then output “control impossible” and halt.

---

<sup>17</sup>This notation is adopted from [13] and adjusted here to the SP-AV system.  $W_c$  is the number of votes in which  $c$  wins one approval against both  $a$  and  $b$ ,  $L_c$  is the number of votes in which  $c$  loses one approval against both  $a$  and  $b$ , and  $D_a$ ,  $D_b$ , and  $D_{ac}$  are the numbers of votes in which the candidate(s) in the subscript gain one approval against the candidate(s) not in the subscript (thus decreasing their deficit).

Let us give a short explanation of why Loop 2 is needed for SP-AV by stressing the difference with approval voting. As shown in the proof of [13, Thm. 4.21], if none of the trivial cases applied, then condition (3.1) holds for some  $a, b \in C$  with  $\|\{a, b, c\}\| = 3$  if and only if destructive control by partition of voters in model TE is possible for approval voting. Thus, for approval voting, if Loop 1 was not successful for any such  $a$  and  $b$ , we may immediately jump to the termination stage, where the algorithm outputs “control impossible” and halts. In contrast, if none of the trivial cases applied, then the existence of candidates  $a$  and  $b$  with  $\|\{a, b, c\}\| = 3$  who satisfy (3.1) is *not* equivalent to destructive control by partition of voters in model TE being possible for SP-AV: It is a sufficient, yet not a necessary condition. The reason is that even if there are no candidates  $a$  and  $b$  who can prevent  $c$  from winning one subelection (in some partition of voters) and from proceeding to the run-off, it might still be possible that  $c$  loses or ties the run-off due to moving the approval line according to Rule 1.

Indeed, if Loop 1 was not successful,  $c$  will lose or tie the run-off exactly if there exists a candidate  $d \neq c$  such that  $\text{diff}(d, c) \geq 0$  and  $d$  can win one subelection (for some partition of voters). This is precisely what is being checked in Loop 2. Indeed, note that the partition  $(V_1, V_2)$  chosen in Loop 2 for  $d \in B_c$  is the best possible partition for  $d$  in the following sense: If  $d$  is not a unique SP-AV winner of subelection  $(C, V_1)$  then, for each  $W \subseteq V$ ,  $d$  is not a unique SP-AV winner of subelection  $(C, W)$ . To see this, simply note that if  $d$  is not a unique SP-AV winner of  $(C, V_1)$ , then there is some candidate  $x$  with  $\text{score}_{(C, V_1)}(x) = \text{score}_{(C, V_1)}(d) = \|V_1\|$ , which by our choice of  $V_1$  implies  $\text{score}_{(C, W)}(x) \geq \text{score}_{(C, W)}(d)$  for each subset  $W \subseteq V$ .  $\square$

## 4 Conclusions and Open Questions

We have shown that Brams and Sanver’s sincere-strategy preference-based approval voting system [1], when adjusted so as to *coerce* admissibility (rather than excluding inadmissible votes *a priori*), combines the resistances of approval and plurality voting to procedural control: SP-AV is resistant to 19 of the 22 previously studied types of control. On the one hand, like Copeland voting [2, 3], SP-AV is fully resistant to constructive control, yet unlike Copeland it additionally is broadly resistant to destructive control. On the other hand, like plurality [2, 12, 13, 14], SP-AV is fully resistant to candidate control, yet unlike plurality it additionally is broadly resistant to voter control. In conclusion, for these 22 types of control, SP-AV has more resistances and fewer vulnerabilities to control than is currently known for any other natural voting system with a polynomial-time winner problem (see Table 1). However, when comparing approval voting and SP-AV, it should also be noted that the former is even immune to nine of these 22 control types, whereas the latter has no immunities at all. Since immunity may be seen as a perfect protection against control and resistance provides protection to control only in a computational sense, one should carefully evaluate the pros and cons of both systems. The result of such an evaluation will certainly depend on which particular types of control one wishes to be protected against.

As an interesting task for future research, we propose to expand the study of SP-AV with respect to other computational properties than its behavior regarding procedural control (see, e.g., [9, 26]), and to investigate also its social choice properties in more detail. In addition, we propose as an

interesting and extremely ambitious task for future work the study of SP-AV (and other voting systems as well) beyond the worst-case—as we have done here—and towards an appropriate typical-case complexity model; see, e.g., [29, 30, 31, 32, 33] for interesting results and discussion in this direction.

**Acknowledgments:** We are grateful to Edith and Lane A. Hemaspaandra for helpful comments and interesting discussions that are reflected in parts of Section 2.3. We thank the anonymous MLQ, MFCS-08, and COMSOC-08 referees for their helpful comments on preliminary versions of this paper.

## References

- [1] S. Brams and R. Sanver, Critical strategies under approval voting: Who gets ruled in and ruled out, *Electoral Studies* **25**(2), 287–305 (2006).
- [2] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Llull and Copeland voting computationally resist bribery and constructive control, *Journal of Artificial Intelligence Research*. To appear. Full version available as [34].
- [3] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Copeland voting fully resists constructive control, in: *Proceedings of the 4th International Conference on Algorithmic Aspects in Information and Management*, (Springer-Verlag *Lecture Notes in Computer Science* #5034, June 2008), pp. 165–176.
- [4] G. Erdélyi, M. Nowak, and J. Rothe, Sincere-strategy preference-based approval voting broadly resists control, in: *Proceedings of the 33rd International Symposium on Mathematical Foundations of Computer Science*, (Springer-Verlag *Lecture Notes in Computer Science* #5162, August 2008), pp. 311–322.
- [5] E. Ephrati and J. Rosenschein, Multi-agent planning as a dynamic search for social consensus, in: *Proceedings of the 13th International Joint Conference on Artificial Intelligence*, (Morgan Kaufmann, 1993), pp. 423–429.
- [6] R. Fagin, R. Kumar, and D. Sivakumar, Efficient similarity search and classification via rank aggregation, in: *Proceedings of the 2003 ACM SIGMOD International Conference on Management of Data*, (ACM Press, 2003), pp. 301–312.
- [7] S. Ghosh, M. Mundhe, K. Hernandez, and S. Sen, Voting for movies: The anatomy of recommender systems, in: *Proceedings of the 3rd Annual Conference on Autonomous Agents*, (ACM Press, 1999), pp. 434–435.
- [8] C. Dwork, R. Kumar, M. Naor, and D. Sivakumar, Rank aggregation methods for the web, in: *Proceedings of the 10th International World Wide Web Conference*, (ACM Press, 2001), pp. 613–622.
- [9] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, A richer understanding of the complexity of election systems, in: *Fundamental Problems in Computing: Essays in Honor of Professor Daniel J. Rosenkrantz*, edited by S. Ravi and S. Shukla, (Springer, 2009), chap. 14, pp. 375–406.
- [10] J. Bartholdi III, C. Tovey, and M. Trick, Voting schemes for which it can be difficult to tell who won the election, *Social Choice and Welfare* **6**(2), 157–165 (1989).

- [11] J. Bartholdi III, C. Tovey, and M. Trick, The computational difficulty of manipulating an election, *Social Choice and Welfare* **6**(3), 227–241 (1989).
- [12] J. Bartholdi III, C. Tovey, and M. Trick, How hard is it to control an election?, *Mathematical Comput. Modelling* **16**(8/9), 27–40 (1992).
- [13] E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Anyone but him: The complexity of precluding an alternative, *Artificial Intelligence* **171**(5–6), 255–285 (2007).
- [14] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Llull and Copeland voting broadly resist bribery and control, in: *Proceedings of the 22nd AAAI Conference on Artificial Intelligence*, (AAAI Press, July 2007), pp. 724–730.
- [15] S. Brams and P. Fishburn, Approval voting, *American Political Science Review* **72**(3), 831–847 (1978).
- [16] A. Gibbard, Manipulation of voting schemes, *Econometrica* **41**(4), 587–601 (1973).
- [17] M. Satterthwaite, Strategy-proofness and Arrow’s conditions: Existence and correspondence theorems for voting procedures and social welfare functions, *Journal of Economic Theory* **10**(2), 187–217 (1975).
- [18] J. Duggan and T. Schwartz, Strategic manipulability without resoluteness or shared beliefs: Gibbard–Satterthwaite generalized, *Social Choice and Welfare* **17**(1), 85–93 (2000).
- [19] P. Everaere, S. Konieczny, and P. Marquis, The strategy-proofness landscape of merging, *Journal of Artificial Intelligence Research* **28**, 49–105 (2007).
- [20] E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Hybrid elections broaden complexity-theoretic resistance to control, in: *Proceedings of the 20th International Joint Conference on Artificial Intelligence*, (AAAI Press, January 2007), pp. 1308–1314.
- [21] R. Meir, A. Procaccia, J. Rosenschein, and A. Zohar, Complexity of strategic behavior in multi-winner elections, *Journal of Artificial Intelligence Research* **33**, 149–178 (2008).
- [22] N. Betzler and J. Uhlmann, Parameterized complexity of candidate control in elections and related digraph problems, in: *Proceedings of the 2nd Annual International Conference on Combinatorial Optimization and Applications*, (Springer-Verlag *Lecture Notes in Computer Science* #5165, July 2008), pp. 43–53.
- [23] P. Faliszewski, E. Hemaspaandra, and L. Hemaspaandra, The complexity of bribery in elections, in: *Proceedings of the 21st National Conference on Artificial Intelligence*, (AAAI Press, July 2006), pp. 641–646.
- [24] S. Brams and R. Sanver, Voting systems that combine approval and preference, in: *The Mathematics of Preference, Choice, and Order: Essays in Honor of Peter C. Fishburn*, edited by S. Brams, W. Gehrlein, and F. Roberts, (Springer, to appear).
- [25] G. Erdélyi, M. Nowak, and J. Rothe, Sincere-strategy preference-based approval voting fully resists constructive control and broadly resists destructive control, *Tech. Rep. arXiv:0806.0535v4 [cs.GT]*, ACM Computing Research Repository (CoRR), June 2008, revised September 2008.
- [26] D. Baumeister, G. Erdélyi, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Computational aspects of approval voting, in: *Handbook of Approval Voting*, edited by J. Laslier and R. Sanver, (Springer, manuscript, to appear).

- [27] M. Garey and D. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*, (W. H. Freeman and Company, New York, 1979).
- [28] E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Anyone but him: The complexity of precluding an alternative, in: *Proceedings of the 20th National Conference on Artificial Intelligence*, (AAAI Press, 2005), pp. 95–101.
- [29] J. McCabe-Dansted, G. Pritchard, and A. Slinko, Approximability of Dodgson’s rule, *Social Choice and Welfare* **31**(2), 311–330 (2008).
- [30] A. Procaccia and J. Rosenschein, Junta distributions and the average-case complexity of manipulating elections, *Journal of Artificial Intelligence Research* **28**, 157–181 (2007).
- [31] V. Conitzer and T. Sandholm, Nonexistence of voting rules that are usually hard to manipulate, in: *Proceedings of the 21st National Conference on Artificial Intelligence*, (AAAI Press, July 2006), pp. 627–634.
- [32] C. Homan and L. Hemaspaandra, Guarantees for the success frequency of an algorithm for finding Dodgson-election winners, *Journal of Heuristics*. To appear. Full version available as Department of Computer Science, University of Rochester Tech. Rep. TR-881, September 2005, revised June 2007.
- [33] G. Erdélyi, L. Hemaspaandra, J. Rothe, and H. Spakowski, On approximating optimal weighted lobbying, and frequency of correctness versus average-case polynomial time, in: *Proceedings of the 16th International Symposium on Fundamentals of Computation Theory*, (Springer-Verlag *Lecture Notes in Computer Science* #4639, August 2007), pp. 300–311.
- [34] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe, Llull and Copeland voting computationally resist bribery and control, Tech. Rep. arXiv:0809.4484v2 [cs.GT], ACM Computing Research Repository (CoRR), September 2008.