An Updated Annotated Bibliography on Arc Routing Problems

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The number of arc routing publications has increased significantly in the last decade. Such an increase justifies a second annotated bibliography, a sequel to Corberán and Prins (Networks 56 (2010), 50–69), discussing arc routing studies from 2010 onwards. These studies are grouped into three main sections: single vehicle problems, multiple vehicle problems and applications. Each main section catalogs problems according to their specifics. Section 2 is therefore composed of four subsections, namely: the Chinese Postman Problem, the Rural Postman Problem, the General Routing Problem (GRP) and Arc Routing Problems (ARPs) with profits. Section 3, devoted to the multiple vehicle case, begins with three subsections on the Capacitated Arc Routing Problem (CARP) and then delves into several variants of multiple ARPs, ending with GRPs and problems with profits. Section 4 is devoted to applications, including distribution and collection routes, outdoor activities, post-disaster operations, road cleaning and marking. As new applications emerge and existing applications continue to be used and adapted, the future of arc routing research looks promising. **© 2017 Wiley Periodicals, Inc.**
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1. INTRODUCTION

The number of arc routing publications has increased significantly in the last decade. It is conceivable that the publication of books especially devoted to arc routing problems, including the most recent from [64], has boosted this research area. Such an increase justifies the second annotated bibliography here proposed. A sequel to [70], this work focuses primarily on arc routing studies published from 2010 onwards. With this in mind, and to avoid duplication, those studies published prior to 2010 may be found in [70] or in [64].

It is well known that Arc Routing Problems (ARPs) aim to identify the best way to traverse the links of a network within some constraints. Depending on the defined objective (to maximize or minimize), the function to be optimized, the identified constraints, the network characteristics, and even the number of service visits required for each link, a different variant of the problem should be defined.

Plenty of water has passed under the bridge since the first known ARP, involving the Königsberg bridges and their possible traversal routes, was identified and solved by Euler. An interesting historical perspective on arc routing is found in [65]. Most recently, the ongoing evolution of computers has been playing an important role in the study of ARPs, not only because they are increasingly capable of solving more realistic instances, but also by permitting the development of ever more sophisticated methodologies capable of handling such difficult problems. Furthermore, these emerging methodologies contribute to a steep increase in the number of studies with real-world applications, such as post-disaster operations, waste collection, snow plowing or salt spreading, cash-in-transit, and meter reading, to name but a few.

Among these new methodologies, the so-called matheuristics emerge, combining integer programming models, even those that used to be hardly solvable, with (meta)heuristics. A recent survey of such approaches for routing problems was conducted by Archetti and Speranza [16].

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TABLE 1. Initials and problem names.

Initials	Problem name	Initials	Problem name
#-IF	# with Intermediate Facilities	**L#**	Location #
#-MD	# with Multiple Depots	**M#**	Mixed #
#-RP	# with Refill Points	**MBPRPP**	Minimum Bound collected Profit RPP
#-TW	# with Time Windows	**MMK-#**	Min-Max K-Vehicles #
ARCP	Arc Routing for Connectivity Problem	**MO-#**	Multi Objective
ARP	Arc Routing Problem	**O-#**	Open #
BCARP	Bound overlapping mixed CARP	**OARP**	Orienteering ARP
C#	Capacitated #	**P#**	Periodic #
CARP	Capacitated ARP	**P-#**	Profitable #
CARPDD	CARP with Deadheading Demands	**P-ARPM**	Profitable ARP with multiple services
CARPSD	CARP with Stochastic Demands	**PC-ARCP**	Prize Collecting ARCP
CPP	Chinese Postman Problem	**PL-ARP**	Profitable Location ARP
CE#	Close-Enough #	**RPP**	Rural Postman Problem
Cl#	Clustered #	**SARP**	Sectoring ARP
D#	Directed #	**SCP**	Stacker Crane Problem
D-ARP	Dissimilar ARP	**St#**	Stochastic #
DC#	Distance Constrained #	**SyARP**	Synchronized ARP
Dy#	Dynamic #	**TD#**	Time-Dependent #
GRP	General Routing Problem	**TOARP**	Team Orienteering ARP
GRPP	Generalized RPP	**U#**	Undirected #
H#	Hierarchical #	**W#**	Windy #
K-#	Multiple Vehicle #		

Beginning with the simpler case, we identify the single vehicle problems with references from 2010 onwards. These problems involve only one vehicle, which must visit all the links of a network, starting and ending at the same point, the depot, within a minimum time (or cost or length). This well-known problem, referred to as the Chinese Postman Problem (CPP), is easily solvable over either undirected or directed networks, whereas the mixed and windy cases are NP-hard.

Although few CPP papers published since 2010 can be found, many new ARPs are defined and presented, usually inspired by real-world applications. This is the case of the Close-Enough ARP, which identifies tours for meter readings from a close enough distance by means of a handheld or portable device; the Dissimilar ARP, suggested for a cash collection application where dissimilarity is introduced to prevent possible robberies; post-disaster operations to restore network connectivity by removing debris from routes on the aftermath of a catastrophe; and other applications such as road marking and route planning for automated guided vehicles.

Problems with maximization objectives are also usually motivated by real case studies, as they better represent the objectives of private companies where more service signifies increased revenue, given some pre-defined constraints.

In considering multiple vehicles, each with a given capacity, we then tackle the capacitated counterpart of the problem. Although these multiple vehicle problems are difficult to solve in comparison to the single vehicle versions, they may better describe reality. This may well be the justification for both the number of new studies and the imbalance between single and multiple vehicle cases.

In an attempt to avoid ambiguity, Table 1 presents the different names and acronyms for the problems addressed in this annotated bibliography.

To distinguish between the characteristics of problems, we will henceforth adopt a standard notation as detailed below.

All problems may be defined on a graph $G = (N, A \cup E)$ with a set of nodes N and a set of links $A \cup E$. Service demanding nodes, that is, required nodes, if any, are represented by node set $N_R \subseteq N$, while $A_R \subseteq A$ and $E_R \subseteq E$ are, respectively, the set of required arcs and edges, also called arc tasks or edge tasks. If needed to guarantee connectivity, links in $A \cup E$ may be deadheaded, that is, traversed without being serviced. A homogeneous fleet of K vehicles is available at a depot node $0 \in N$, with a given capacity of W. We define a vehicle *route* or *tour* as a closed walk, starting and ending at the depot, and including services from some required links of a network.

Figure 1 depicts different classes of ARPs and highlights the unique characteristics that may have led to the different problem types studied since 2010. Different colors and borders denote different sets of characteristics. Data usually referring to network type, depot, the number or type of vehicles is marked as such; the remaining groupings show additional characteristics.

To assess the performance of methodologies and compare results, benchmark instances have been published for several problems, some of them available online. The corresponding links are displayed in the Appendix (Table 9), together with open source software links and references to instance generators.

Our paper is organized as follows. Three major sections classify studies into single vehicle problems, multiple vehicle problems or applications. Single vehicle problems, in

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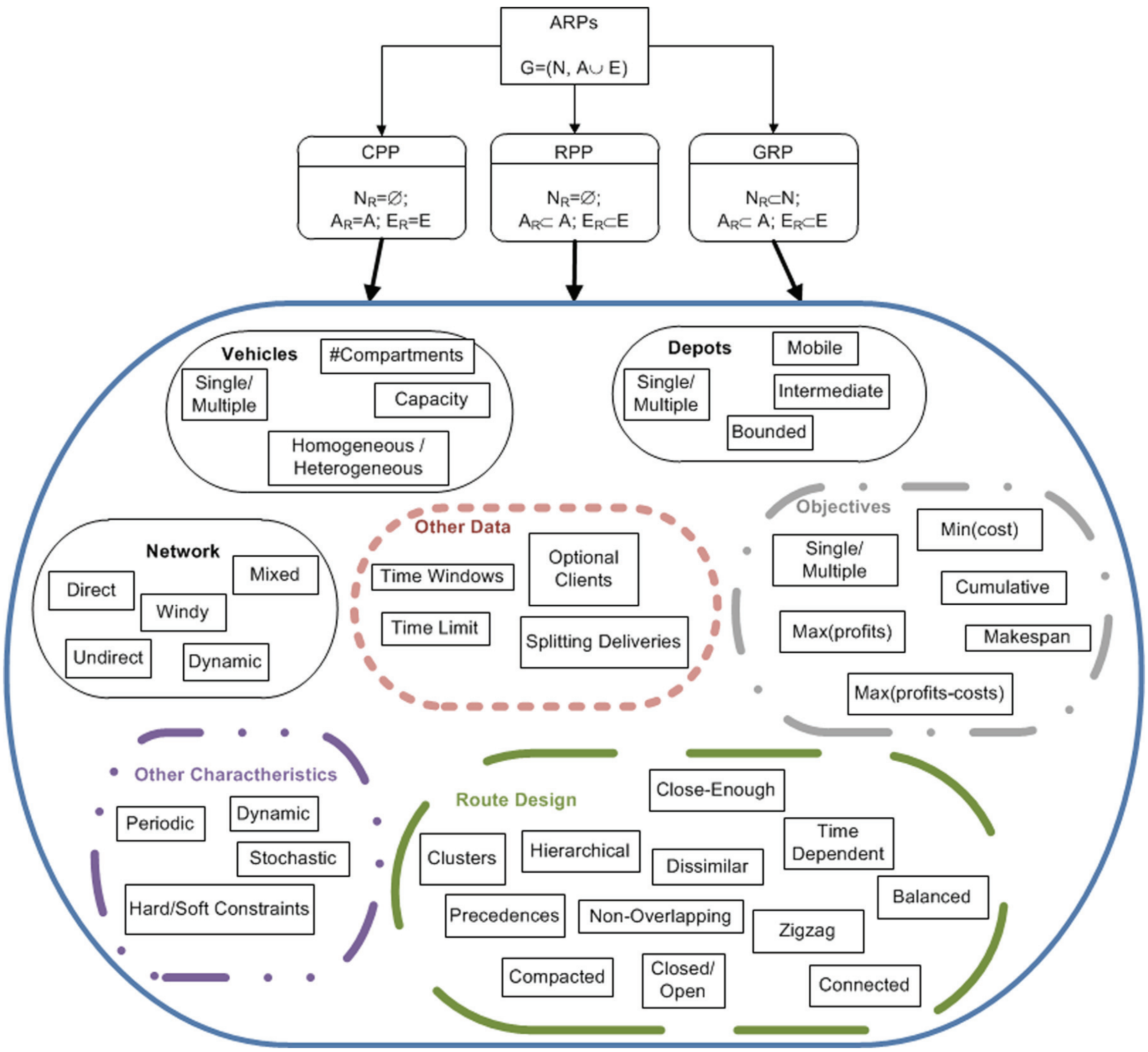

FIG. 1. Problems' characteristics. [Color figure can be viewed at wileyonlinelibrary.com]

Section 2, are grouped according to the type of service and are thus classified as a CPP, Rural Postman Problem (RPP), General Routing Problem (GRP) or ARPs with profits. Section 3 surveys multiple vehicle problems, beginning with three subsections on the Capacitated ARP (CARP), namely the Undirected CARP, the Mixed CARP and other variants. Then, several multiple ARPs are discussed, with their ensuing tactical and strategic problems. The GRP and ARPs with profits conclude this section. Section 4 is devoted to applications: distribution and collection routes, outdoor activities, post-disaster operations, road cleaning and marking.

Consistent with the names depicted in Table 1, Fig. 2 summarizes the problems here considered.

2. SINGLE VEHICLE ARC ROUTING PROBLEMS

In this section we consider ARPs with a single vehicle. They consist of identifying a tour for a single vehicle, its service beginning and ending at a single depot, and satisfying some demand for the identified clients, which are continuously spread along some links of a network. Part I of the recent arc routing book by Corberán and Laporte [64] is devoted to ARPs with a single vehicle, as will be noted henceforward.

This section splits surveyed works into four subsections, namely, CPP, RPP, GRP and Arc Routing with Profits. The main differences among these classes of problems are related to the set of clients to be visited.

In a CPP, all links of a network must be visited at least once, that is, $E_R \cup A_R = E \cup A$, to represent the route of a postal carrier who needs to distribute the mail at all addresses continuously spread along all the streets of a city network. The RPP also originated for mail distribution, in this case in rural areas where small, connected villages may each be similar to a CPP. Alas, the postal carrier needs to distribute mail at several villages, and therefore must travel among them without distributing mail. Such links with no demand for service are called deadheaded links. The RPP includes more general cases, where links to be serviced are mixed with deadheading links. The main difference between RPP and CPP is that in RPP the set of required links does not coincide with the set of links in the network, that is, $E_R \cup A_R \subset E \cup A$. The third subsection is devoted to the GRP, where clients are

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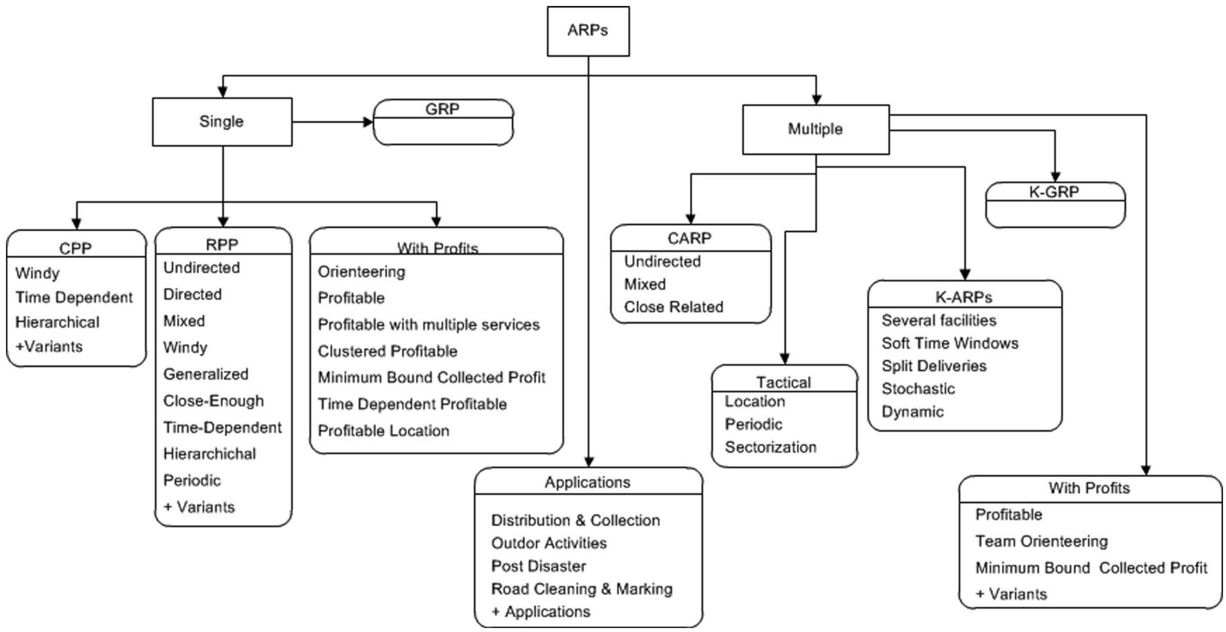

FIG. 2. Problems surveyed in this paper.

spread on edges and nodes, that is, $E_R \cup A_R \subset E \cup A$ and $N_R \subset N$. Finally, the last subsection focuses on ARPs that in some way include the profits obtained from performing the services.

A recent paper on the complexity of ARPs is attributable to van Bevern et al. [194]. The focus is on surveying several computational complexity aspects for three base ARPs, namely the CPP, the RPP, and the CARP, and also some variants thereof. According to [194], the classical complexity classification concerning NP-hardness is enhanced with aspects of polynomial time approximation as well as with parameterized complexity. This latter aspect is still a much unplowed field in arc routing, with numerous and challenging open research questions.

Leaving the discussion of complexity aside, each section begins with the purer problem version, that is, with no extra constraints, followed by problem extensions studied from 2010 onwards. Such extensions may point to special network characteristics (such as windy networks), extra constraints (e.g., constraints on time or on the order of services), or even different objectives (e.g., maximization or multiple objectives).

The characteristics of each problem, including some problems that are defined in Section 4, are summarized in Table 2, and the papers addressing each problem are also listed.

2.1. *Chinese Postman Problem (CPP)*

The Chinese Postman Problem (CPP) consists in the identification of a shortest closed tour that traverses all links of a network at least once. This problem is polynomial-time solvable in the case of completely directed or undirected graphs, while the mixed and windy cases are NP-hard.

Further to the problem definitions and complexity questions, van Bevern et al. [194] present the evolution of CPP complexity studies. Beginning with classical complexity, including some polynomial solvable cases, and moving on to approximation algorithms, they reach parameterized complexity. Challenges involving this latter issue were left open and some of them were later answered by Gutin et al. [100], proving the mixed CPP parameterized by the number of arcs to be fixed-parameter tractable. Very recently, Gutin et al. [101] have also proven the CPP on an Edge-Colored (multi)Graph (ECG) to be polynomially solvable. In such multigraphs each edge has a color, and it is said that a tour is properly colored if no two consecutive edges share the same color. A CPP tour on an ECG is therefore a properly colored tour traversing each edge at least once.

Undirected CPP, that is, CPP over undirected networks, is surveyed in [124]. Two models as well as a description of the problem's main features are presented. As it is well known, on connected graphs and in the case of a non-Eulerian graph, an optimal solution is identified by the set of edges that need to be traversed twice. For Eulerian connected graphs, that is, graphs where all nodes are of even degree, the CPP solution coincides with the Eulerian closed tour including all the graph edges. Directed and mixed CPP, that is, CPP on respectively directed or mixed graphs, are considered in [68], where authors describe models, heuristics and exact algorithms. Linear programming (LP) relaxations of two detailed models are compared.

Although the CPP itself is easy, there is still ongoing research involving the number of extra edges or the best way to find them. Recently, Suil and West [186] studied some properties of CPP solutions on 3-regular graphs (multigraphs). A 3-regular graph is a graph where all nodes share the same degree, it being equal to 3. Moreover, solving

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TABLE 2. Single vehicle problems' characteristics and related published papers.

Problem		Objective	Network $G=(N,A\cup E)$				Other characteristics	Papers
			U/D/M/W	A_R	E_R	N_R		
CPP	UCPP	Min(cost)	U	$A_R=\emptyset$	$E_R=E$	$N_R=\emptyset$		[45, 124]
							Including TW constraints	[124]
							Generalized or close-enough	[124]
							CPP solutions on 3-regular (multi)graphs	[186]
							With load-dependent costs – the cost of traversing an edge depends on its length and on the weight of the vehicle's cargo	[61]
							Edge-colored multigraph – each edge has a color, and a tour is called properly colored if no two consecutive edges share the same color	[101]
		Min(service completion)	U	$A_R=\emptyset$	$E_R=E$	$N_R=\emptyset$	Cumulative problem – aims to minimize the service completion time on all edges	[124]
	DCPP	Min(cost)	D	$A_R=A$	$E_R=\emptyset$	$N_R=\emptyset$	–	[45, 68, 194]
	MCPP	Min(cost)	M	$A_R=A$	$E_R=E$	$N_R=\emptyset$	–	[68, 100, 194]
							Lower and upper bounds on the number of times each link is traversed	[76]
	WCPP	Min(cost)	W	$A_R=\emptyset$	$E_R=E$	$N_R=\emptyset$	Costs on links depend on the travel direction	[66, 68, 194]
	HCPP	Min(cost)	U	$A_R=\emptyset$	$E_R=E$	$N_R=\emptyset$	Links in a higher hierarchic level need to be serviced before links in lower levels	[124]
	TDCPP	Min(cost)	D	$A_R=A$	$E_R=\emptyset$	$N_R=\emptyset$	Time-dependent – the time needed to service a link depends on the service start time	[187, 190]

(Continued)

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TABLE 2. (Continued)

Problem		Objective	Network $G = (N, A \cup E)$				Other characteristics	Papers
			U/D/M/W	A_R	E_R	N_R		
RPP	URPP	Min(cost)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	–	[74, 78, 93, 110, 168, 194]
							Re-optimizing after some perturbation of an instance occurred	[14]
							Deadline classes – where the service for each class must be completed in a given time	[93]
							Dynamic RPP – the base graph changes dynamically	
	DRPP	Min(cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	–	[74, 78, 102, 184, 194]
							Including turn penalties	[46, 69]
	MRPP	Min(cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Including turn penalties	[69, 194]
	WRPP	Min(cost)	M/W	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Mixed and windy graphs with meander option (zigzag)	[95]
			U/W	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Including zigzag services	[69]
	CEARP	Min(cost)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$		[8, 105]
			D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$		[23, 69, 85, 104, 105]
							No fixed depot	[80]
			M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$		[105]
	ClRPP	Min(cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Each connected component of task must be completely serviced before moving to another component	[69]
	D-ARP	Min(cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Routes as dissimilar as possible to prevent robberies; Time horizon	[60]
	HRPP	Min(cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Links in a higher hierarchic level need to be serviced before links in lower levels	[56, 57]
	PRPP	Min(cost)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Multiple services per edge during the time horizon	[93]
	StCEARP		D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Stochastic CEARP – uncertainty of collecting data due to failed transmissions	[163]
	SCP	Min(cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	To schedule the movements of a crane at minimum cost	[21, 69]
	RPPSD	Min(cost)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Stochastic demands	[147]
	TDRPP	Min(cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Time-dependent travel times	[190]
			U/W	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Time-dependent and zigzag services	[152]

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TABLE 2. (Continued)

Problem		Objective	Network $G = (N, A \cup E)$				Other characteristics	Papers
			U/D/M/W	A_R	E_R	N_R		
GRP	DGRP	Min(cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R \subset N$	Stacker Crane Problem	[21]
	GRP	Min(total effort to clean debris)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R \subset N$	Debris Removal with: critical nodes (N_R) – areas such as schools, hospitals, etc. tasks – blocked roads linking those areas	[171]
		Min(time to reach N_R)						[37]
		Min(weighted sum of visiting times)					Weights being used to prioritize critical nodes	
ARPs with Profits	OARP	Max(profit or attractiveness)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$		[17]
			D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$		[11, 136, 185, 198]
			M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$		[32]
	P-ARP	Max(profit-cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Considered mandatory and optional clients; A penalty cost for not serving a client is charged	[15, 58]
			M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$		[32]
			U/W	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$		[20, 175]
	ClP-ARP	Max(profit-cost)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Edges are partitioned into clusters, and in the same cluster all edges or none are serviced	[7]
			U/W	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$		[62]
	MBPARP	Min(cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$		[32]
	P-ARPM	Max(profit-cost)	U	$A_R = \emptyset$	$E_R = E$	$N_R = \emptyset$	Or Maximum Benefit CPP/ARP (MBCPP/MBARP) – the profit of an edge is paid every time it is serviced, within a given upper limit	[17, 67]
	PL-ARP	Max(profit-cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Integrates P-ARP with facility location, at the end nodes of profitable selected arcs	[9]
	TDP-ARP	Max(profit-cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Time-dependent – the time needed to service a link depends on the service start time	[38, 208]

U/D/M/W – Undirected/Directed/Mixed/Windy.

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the CPP in a multigraph where all nodes have the same odd degree value is equivalent to finding the shortest spanning subgraph in which all nodes have an odd degree value. Suil and West [186] prove the validity of some tight upper bounds for the number of additional edges needed for 3-regular graphs and for 3-regular multigraphs, identifying some cases where the equality holds.

The CPP with start and end times imposed for servicing the links is known as the CPP with Time Windows (CPP-TW). This problem models some real-world applications related to garbage collection or street sweeping, where services usually may not be carried out at just any time of day. The survey by Laporte [124] seems to be the only reference to this problem from 2010 to date.

2.1.1. Windy CPP. Windy problems denote situations where travel direction is an issue, affecting the edge travel cost or time over undirected networks. A windy edge is thus an edge with two different associated travel times, one for each direction. These may be directly applied to traveling times against or with the wind, or even uphill versus downhill directions. Although usually NP-hard, the Windy CPP has some polynomial special cases, as over Eulerian networks (e.g., Refs. 66 and 194).

Corberán et al. [68] spotlight the Windy CPP (WCPP), which may be seen as a generalization of the Mixed CPP (MCPP). Formulations, algorithms and some inequalities that may be useful in branch-and-cut algorithms are detailed in [68]. This chapter also summarizes the previous work of Corberán et al. [66] with a branch-and-cut algorithm for the WCPP. These authors present two new families of facet-inducing inequalities and methodologies to identify and separate violated odd-cut zigzag inequalities, to add to the problem through a branch-and-cut procedure. A polynomial time algorithm is also developed to identify maximal violated inequalities. Upper bounds are computed via a new heuristic method. The new sets of inequalities, together with the previously known ones, are proven to be Chvátal-Gomory with a rank of up to two. The new inequalities lead to the complete description of the WCPP polyhedron on graphs with up to four nodes and ten edges. The algorithm proved to generate good computational results with WPP and MCPP instances with up to 3,000 nodes and 9,000 edges (and arcs, in the case of MCPP).

2.1.2. Time-Dependent and Hierarchical CPP. In time-dependent routing problems, the time needed to service a link depends on the time of the day it is serviced, that is, of its service start time. On some streets, for instance, collecting parking meter cash takes longer during rush hour than at other times of the day. The Time-Dependent CPP (TDCPP) aims to identify a minimum time closed tour traversing each link at least once.

Sun et al. [187], based on Tan et al. [190], present a mixed integer model, which is nonlinear due to the time-dependent functions. As previously, if travel time may be represented by piecewise linear functions, the model can be linearized through some additional variables. Solutions are generated via a cutting plane algorithm, and if needed, the final LP solution is transformed into an integer one. A real-world example based on the inspection of digital networks, as well as 90 randomly generated test instances divided into three groups, is used to assess the quality of feasible solutions generated. In these instances, the number of nodes varies between 10 and 25, while the number of arcs varies between 20 and 60. The effect of the valid inequalities added through the cutting plane algorithm is also analyzed.

Hierarchical routing problems are slightly different and of extreme importance in real applications such as snow plowing or debris removal. Imposing constraints on the order of service for some links (or clusters of links), for example, that more important streets should be handled first, leads to better handling of these problems. The objective of the Hierarchical CPP is therefore to identify a minimum cost tour, visiting each of the links of a network at least once and in such a way that a link in a higher hierarchical level is visited before all links in lower hierarchical levels.

Panchamgam et al. [154] analyze the hierarchical CPP, as well as the hierarchical traveling salesman problem, in terms of worst-case behavior. Laporte [124] surveys undirected Hierarchical CPP by presenting two algorithms and a transformation into a RPP. New studies focusing on these issues are frequently related to real-world applications and will be more thoroughly discussed in Section 4. Among these studies, we will emphasize those involving precedence constraints such as post-disaster operations (Section 4.3) and road cleaning and marking (Section 4.4).

2.1.3. Other CPP Variants. Ding et al. [76] study a generalization of the mixed CPP that introduces lower and upper bounds on the number of times each link is traversed. The goal is to identify a minimum cost tour, beginning and ending at the depot, traversing each link within its previously specified bounds. Denoting by n the number of nodes, m the number of links (all links, arcs and edges, are required), and by ℓ_0 the minimum lower bound limit among all edge traversals, the authors design an $\left(1+\frac{1}{\ell_0}\right)$-approximation algorithm in time $O\left(n^2m^3\log n\right)$ for instances with edges' lower limits strictly smaller than their respective upper limits, but allowing lower and upper limits for arcs to coincide. Moreover, for problems on undirected and directed networks, optimal combinatorial algorithms with a complexity of $O\left(n^3\right)$ and $O\left(nm^2\log n\right)$ are presented.

Laporte [124] surveys the generalized and the cumulative CPP, both defined on undirected graphs. In the generalized problem, the set of edges is partitioned into several subsets, and the aim is to find a minimum cost CPP tour containing at least one element from each partition. The cumulative problem consists of determining an Eulerian tour, beginning and ending at the depot, which minimizes the sum of service completion on all edges. Thus, each edge traversed more than once must be serviced during the first passage. Laporte [124] details the best known model for this problem to date.

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Corberán et al. [61] define and study a CPP with load-dependent costs where the cost of traversing an edge depends not only on the edge's length but also on the weight of the vehicle's cargo. This variant makes sense when the main concern is to minimize pollution, given that at constant speed, the most important measurable variables to explain pollution emissions are distance and vehicle weight. The authors prove the problem to be NP-hard and identify some polynomially solvable cases. Two formulations are provided, one exploring the nature of arc routing and another based on a transformation into node routing. Two metaheuristics are proposed, an iterated local search and a variable neighborhood search, including a constructive greedy heuristic to initialize both. Computational experiments on three sets of instances generated or adapted by the authors are reported, and they attest the extreme difficulty of this variant. The first set includes 18 Eulerian instances with 7, 10, and 20 nodes and 12, 18, and 32 edges, respectively. The second set of 18 instances has 11, 14, 17 nodes and 13, 32, and 35 edges, respectively. The third set consists of 24 instances with up to 27 nodes and up to 33 edges. The models fail to obtain the optimum in reasonable computational time despite the small instance size. The problem's difficulty plainly justifies the development of the proposed metaheuristics. These methods were then able to produce good solutions in short computational times for larger instances, with up to 30 nodes and 232 edges.

2.2. *Rural Postman Problem (RPP)*

Consider a connected undirected graph $G = (N, E)$ and a subset of required edges $E_R \subset E$ demanding service. RPP consists of finding a minimum cost tour traversing all the required edges at least once. The problem is generally NP-hard, being polynomially solvable if the graph induced by E_R is connected. Ghiani and Laporte [93] survey undirected RPP, while Corberán et al. [69] dedicate a chapter to the directed, the mixed and the windy RPP.

The complexity of the RPP is analyzed by van Bevern et al. [194]. The problem consists of identifying whether there is a tour traversing all the required links at least once, while not exceeding a given fixed given cost, $c_{\max}$. The complexity of the RPP is therefore obtained from the following parameters: number of required links; number of connected components induced by required links; number of imbalance nodes; $c_{\max}$; and number of non-required links in the RPP solution.

Archetti et al. [14] study the problem of re-optimizing an RPP solution after some perturbation of an instance occurs. Two perturbations are considered and are shown to be NP-hard: (i) addition of a new required edge; and (ii) removal of an edge (required or not). A heuristic with a worst-ratio of 2 is presented for (i), while a tighter ratio of 3/2 may be achieved for (ii) through simpler algorithms. Procedures dealing with partial solutions, as well as perturbations in the initial instances after a solution is found, may take advantage of such procedures. Perturbation is very common in some real-world applications, for instance, when orders are made or canceled. Based on their computational experiments, over three sets of instances for cases (i) and (ii) (derived from instances CPS_ARP in Table 9), the authors claim that "results show the nodal importance of re-optimization both in terms of quality of the solution and efficiency of the computation" Instance dimensions vary from 298 to 1,002 nodes and 206 to 2,305 required edges.

2.2.1. Undirected RPP. Holmberg [110] proposes heuristics for undirected RPP while addressing a snow removal problem (see Section 4.4.2). These methods begin with connecting the graph induced by the set of required edges at minimum cost, then finding an Eulerian tour. The generated tour is then improved by means of some post-processing procedures.

Ghiani and Laporte [93] examine some properties of undirected RPP solutions, as well as mathematical models, exact and heuristic algorithms. Some computational results are also summarized.

2.2.2. Directed and Mixed RPP. Directed and mixed RPP (DRPP and MRPP) are well detailed in [69]. With a substantial background in these problems, the authors offer a vital survey of MRRP models, followed by a study of the corresponding polyhedra in which the authors revise and explain several inequalities that are facet-inducing for the MRPP models. A comparison between LP relaxation bounds is also described. A transformation into a generalized traveling salesman problem (GTSP) is discussed and was able to solve instances with up to 500 nodes, 1,000 arcs (200 required), and 1,000 edges (25 required) in the original graph. A cutting plane algorithm optimally solved instances with up to 100 nodes, 220 edges (150 required), and 350 arcs (200 required). A branch-and-cut algorithm found the optimum in instances with up to 1,000 nodes, 2,000 edges, and 2,000 arcs within an average computational time of 30 minutes.

Further parameterized complexity analysis concerning the RPP is attributable to Refs. 184 and 102. These studies are based on Eulerian and matching extensions, as these problems are closely related to the RPP. Sorge et al. [184], expanding from Refs. 182 and 183, employ a parameterized equivalence between the RPP and an Eulerian extension, studying a variant of the minimum weight perfect matching on bipartite graphs. The parameterization is related to the number of weakly connected components in the graph induced by required arcs. To this end, works such as [77] (revised in [78]) or [73] (revised in [74]) are essential. Gutin et al. [102] answered some open questions posed by Sorge et al. [184] and raised some new questions concerning the parameterized complexity of this problem.

2.2.3. Windy RPP. As previously mentioned, windy networks are undirected networks in which traversal time (cost) associated with each required edge differs with the direction of its traversal. The Windy RPP (WRPP) consists of identifying a minimum time tour that travels from and back to the depot while servicing all the required edges of a windy graph.

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Corberán et al. [69] present a formal definition of the problem, an integer LP model, and heuristic and exact procedures. A summary of the principal results over some benchmark instances with up to 1,000 nodes, 4,000 edges, and 200 connected required components is also provided. Within a time limit of 10 hours, 510 out of the 522 instances were optimally solved with a branch-and-cut algorithm. A branch-and-price was tested on smaller instances, with up to 100 nodes and 184 edges, optimally solving all but three of the 198 instances. Heuristic results are also reported for the larger instances, producing good gap values in short computational times.

A new generalization of this problem, christened arc routing with meander option, was suggested by Golden [95]. Golden considered that narrow streets with light traffic may be serviced (and often it is desirable to do so) along both sides with a single pass in a zigzag fashion, that is, a meander, in one direction. On the contrary, on wide streets with heavy traffic, each side must be serviced on a different pass, and thus meandering is not allowed. The mixed windy RPP with the meander option is then defined. Some real-world applications are identified, such as garbage collection, meter reading, and local delivery.

In a later paper, Nossack et al. [152] tackle a similar problem, which they name the WRPP with time-dependent zigzag services. The authors consider that during certain times of the day some edges may be serviced in a zigzag fashion. This option may be applied to narrow streets with low traffic intensity. Larger streets or streets with high traffic intensity must be serviced on two single traversals. Thus, it is imposed that some streets may be zigzagged at specific times of the day, for example, in the early morning when not much traffic exists, and TW are set for these cases. Garbage collection may represent one real application of such situations. Nossack et al. [152] discuss two mixed integer programming formulations, suggest exact solution approaches, and state that "this is the first paper to present models and solution approaches for an ARP that combines zigzag options and time-dependencies". Formulations are based on a transformation into an equivalent node routing problem on a directed graph. Sensitivity of the solution value to zigzag and time window options is also analyzed in the computational results. The authors transform a street map from Columbia, Maryland, into a directed network with 171 nodes and 994 arcs (the initial network had 407 nodes and 1,578 arcs), from which they derive real-world instances supported by real data. These 20-node instances have an average of 56 arcs (24 required). Zigzag options, which were not pre-existing, were randomly generated, and three types were considered, namely: (i) edges with a required zigzag that may be serviced in any direction but only once by zigzagging; (ii) links with a zigzag option that can be serviced once if zigzagging or on two normal traversals otherwise; and (iii) links with a zigzag time window that may only be serviced by zigzagging during the predefined time window. For all segments with a zigzag option, it is assumed that it is better to zigzag in at least one direction. Nossack et al. [152] conclude that "the zigzag time can be relatively large and yet still offer cost-saving opportunities" and that "time windows dramatically increase the difficulty level of the problem." Since no optimums were found for all instances, the authors emphasize the need for a deeper study involving one of the models, which could improve the efficiency of cutting plane algorithms.

2.2.4. Generalized and Close-Enough RPPs. The Generalized RPP (GRPP) aims to identify a minimum cost tour, beginning and ending at the depot, which traverses at least one arc from each subset of a family of previously defined arc subsets. This family of arc subsets needs not be disjointed or induce connected subgraphs. This problem, introduced in the 2007 PhD thesis by Drexl and defined on directed graphs with no fixed depot, gave rise to the work published in [80]. Since the problem reduces to the DRPP when each subset of arcs includes only one arc, Drexl refers to it as a strong NP-hard problem.

The GRPP is equivalent to the Close-Enough ARP (CEARP), which was first defined for the directed case (see Refs. 105 and 69). In the CEARP, instead of needing to service every customer at its link location, a vehicle may also service customers residing outside the network as long as the customers are reachable from a link at a "close-distance," that is, if an arc within a distance up to a fixed given value is traversed by the working vehicle. The CEARP then consists of finding a minimum cost tour, beginning and ending at the depot, such that every customer is covered by the tour, that is, lies within a limited distance from at least one arc in the tour. We observe that the CEARP is equivalent to the GRPP (see Ref. 105) if we consider the arcs covering each customer in the CEARP as a subset (or a cluster) of the family of arc subsets in the GRPP.

This problem has a direct application for the routing of meter readers that use new technology, such as handheld devices with radio frequency identification technology, allowing utility meters to be read from a maximum distance. Thus, there is no need to actually visit every customer. Rather, if the meter reader gets within a given radius of a customer, that is, if it gets close enough to the customer, it may read the customer's meter. Meter readers then only need to visit the links that are close enough. According to Hà et al. [105], an "effective radius, also called the read range, is normally between 150 and 300m but may be as high as 381m." As a related new application of the CEARP, Fernández [85] quotes the quality control standard for network maintenance in which only a small subset of the edges of a network must be traversed.

Drexl [80] explains how to model several types of single vehicle problems as GRRPs on directed networks (referred to as GDRPP). These are relevant for practical applications. Drexl considers several real-world constraints, such as turn penalties, street segment sides, zigzag servicing, deadheading and service costs within windy graphs, open tours, and delivery sections. An integer LP formulation is presented. Solutions are identified using CPLEX to solve the relaxed model without subtour elimination constraints. The violated constraints are then identified and added to the model, using

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maximum flow algorithms. A second heuristic procedure resorts to a transformation into a GTSP and applies a memetic heuristic with a competitive behavior for the GTSP, according to [80]. Performance of the procedures was assessed over two sets of instances: (i) 420 GDRP instances that resemble real-world mail delivery problems in structure and size, with up to 420 nodes, 1,776 arcs and 196 r-groups, that is, groups of arcs where at least one arc must be visited; and (ii) 850 well-known benchmark instances for the undirected, mixed and windy RPP and for the GRP (instances CPS_ARP in Table 9), with up to 1,001 nodes, 5,046 arcs, and 1,332 r-groups. The results, although not competitive against the best known, lead to some interesting points of research. As the author states, "the aim here was not to try to compete with the high sophisticated algorithm developed (...), but to evaluate the general usefulness of both the exact and the heuristic algorithm. The reported results verify this usefulness and suggest further study of the GDRPP. Possible research avenues are outlined."

Hà et al. [104] propose an integer linear programing model for the CEARP, which is solved using a cutting plane method. This model first solves the relaxed problem without connectivity constraints, which are iteratively added while needed, that is, while still violated. Several instances imitating real street networks are randomly generated to assess the quality of the method. Authors claim their algorithm is capable of solving realistically sized instances and that its performance improves as the number of customers increases. In later work, the same authors [105] compare a newly proposed model with two other existing models created by Hà et al. [104] and Drexl [80]. Some valid inequalities are derived, and the proposed branch-and-cut is evaluated with instances based on directed, undirected, and mixed graphs. Instance sets include: (i) randomly generated for directed graphs by Hà et al. [104], with 500 nodes and 1,000 or 1,500 arcs; (ii) undirected (with up to 196 nodes and 316 edges) and two mixed (one with 500 nodes, 364 edges and 476 arcs; the other with 351 edges and 681 arcs) graphs based on benchmark graphs (instances CPS_ARP in Table 9). The authors conclude that their algorithm is the best choice in the case of directed and undirected graphs, while [80] seems to be the best option for the mixed case. In fact, better bounds are generated through enormous branch-and-bound trees. Ávila et al. [23] introduce two new formulations for the GRPP as well as several new families of valid inequalities that are essential for inclusion in a new branch-and-cut procedure. The polyhedron of solutions associated with one of the models is also studied. This study provides and analyzes better results than those of [105]. Instance sets, as well as the obtained results, are available online (see instances CPS_ARP in Table 9). Five sets of instances were used and are characterized by: (i) directed networks with 500 nodes, up to 1,500 arcs, and 500 to 15,000 customers as proposed by Hà et al. [105]; (ii) four instances for each of the two mixed networks of [105]; (iii) with the number of nodes varying between 298 and 499 and the number of arcs between 597 and 1,526; (iv) with the number of nodes varying between 452 and 749 and the number of arcs between 915 and 2,314; and (v) with the number of nodes varying between 605 and 1,000 and the number of arcs between 2,289 and 3,083. All instances in the first and third sets, all but one in the second set, and all but two in the fourth were optimally solved in short computational times. In the fifth set, however, only two out of twelve were optimally solved. The authors conclude that their "algorithm outperforms the existing exact methods." The final contribution of this paper is a new formulation with fewer variables, which deserves further attention.

Aráoz et al. [8] also focus on the GRPP on undirected networks. This paper culminates the work that has been developed and presented in some worldwide conferences, from ROUTE 2011 or IFORS 2011 to WARP1 or TRISTAN 2013 [85]. An integer programming model with only binary variables is suggested. Some dominance conditions with respect to the solutions allow improvement of the formulation. The associated polyhedron is tackled via facets and families of valid inequalities, and the separation problem for the families of valid inequalities is also studied. Solutions are generated through a two-phase algorithm. In the first phase, a branch-and-cut is applied to solve the LP relaxation of the improved model. Then, at each iteration, the LP model is reinforced with the addition of valid violated inequalities. The second phase is applied only when the first phase fails to identify a probable optimal solution. It then resorts to CPLEX to solve the improved exact model. Performance of the method was assessed over three sets of instances: two previously known sets (instances CPS_ARP in Table 9), and one new set of 20 instances with 300 nodes and up to 613 required edges from a total of 43,000–45,000 edges. The impact of the instance dimensions is reflected in the CPU run times. The authors conclude that a deeper analysis of the relationship between the number of edges and clusters would be helpful.

One new and interesting variant of the CEARP is its stochastic version, recently introduced by Renaud et al. [163], in which "the stochasticity lies in the uncertainty of collecting data due to failed transmissions." The probability of a successful meter reading is computed as a function of the distance between the customer and the tour assigned to the corresponding operator. An integer nonlinear formulation for the Stochastic CEARP (StCEARP) is presented, where probabilities associated with each arc/customer pair indicate whether a customer's meter may be read from a given arc (or its reversal arc). The CEARP tour should be chosen to guarantee a minimum fixed probability of a successful reading for each customer. An exact method based on a cutting plane algorithm is developed, involving two modules: a preprocessing phase to reduce the size of the formulation, and several primal heuristics to search for feasible solutions in each iteration of the cutting plane. The study builds 240 instances that are similar to those of Hà et al. [104] and Hà et al. [105] (with 300, 400 or 500 nodes and 625 to 10,000 customers, each serviced by 4.5 to 15 arcs out of 450 to 1,500 total arcs), and some new instances that consider larger reading ranges for customers. Renaud et al. [163] conclude that the preprocessing

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step is important, not only for reducing the instance size but also for producing better lower bound values.

2.2.5. Time-Dependent and Hierarchical RPP. The Time-Dependent RPP (TDRPP) is an RPP variant that involves time-dependent service times, that is, the service time for a link depends on the time its service initiates. A vehicle tour must then be defined so as to minimize the total time of servicing all the required links.

Although it shares some similarities with established RPP variants, namely, the RPP with TW (RPP-TW) and the RPP with Deadline Classes (RPP-DC), the TDRPP introduced by Tan et al. [190] is a new problem. It differs from the RPP-TW in that although service times are time-dependent, service is not restricted to TW. The TDRPP also differs from the RPP-DC, as the TDRPP does not have deadlines for service completion or divide required links into deadline-based classes.

Tan et al. [190] present a mixed integer model that is nonlinear due to time-dependent functions. If travel time may be represented by piecewise linear functions, the model can be linearized through some additional variables. Solutions are generated via a cutting plane algorithm that begins with the resolution of a relaxed LP and then iteratively identifies violated constraints to be added to the model. Lastly, if needed, the LP solution is transformed into an integer solution. Randomly generated test instances (with up to 25 nodes and 50 arcs, and a percentage of required arcs ranging from 30 to 70%) were used to assess the quality of the feasible solutions obtained. Although poor gap values are reported, the effect of the facet-inducing inequalities is analyzed.

Although this work is discussed in the Windy RPP Section (2.2.3), it also bears mentioning here that Nossack et al. [152] contribute to the ongoing work on time-dependent functions by linking the issue of time dependency to zigzag options and windy graphs.

Another RPP variant related to traversal times or orders is the Hierarchical RPP (HRPP), where services must respect some priority levels, that is, previously identified hierarchic orders. The inclusion of these priority levels adds new challenges to the basic RPP. The required links are divided into clusters that must be serviced in a hierarchical order, that is, a vehicle tour must service all required links in a higher hierarchical level before servicing those in lower levels. In an HRPP, each link has three traversal-associated costs: (i) costs prior to servicing; (ii) the service cost; and (iii) costs after servicing. As usual, problems are classified based on whether they involve directed, undirected or mixed networks. Applications for the HRPP include, for instance, street cleaning or snow plowing, where main streets need to be cleaned prior to secondary ones and only a subset of the links requires service. In this case, deadheading a street after cleaning has a relatively low associated traversal cost, while cleaning the street takes more time, and traversing the street before cleaning takes the most time.

Other recent work with the Hierarchical RPP comes from Refs. 56 and 57, who address the mixed case, which is NP-hard as it generalizes the mixed RPP. Colombi et al. [56] propose a new mathematical model, a matheuristic and a tabu search algorithm. The matheuristic wraps the solution of a variant of the mixed RPP, the open mixed RPP, for each hierarchy. The solution for each hierarchy involves finding the minimum cost path that services all links in the hierarchy, starting at the depot and ending at a fictitious sink node. All nodes in the hierarchy are linked to this sink, with costs computed from the shortest paths between this hierarchic level and the subsequent one. The tabu search approach initiates with the solutions provided by the matheuristic, then innovates on the improvement and on the diversification strategies, which are analyzed in the computational tests. The solutions generated by this procedure are compared with those from a branch-and-cut algorithm, using the proposed model without connectivity constraints. These are then iteratively identified and added to the model, while needed. Computational tests with CPS_ARP instances (Table 9), obtained by modifying benchmark instances for the MRPP and the MGRP, show the relationship between the number of hierarchies and the hardness of instances. The matheuristic "deserves to be considered as a stand-alone method providing very good results in all classes of instances."

In [57], a deep polyhedral analysis dealing with the formulation of Colombi et al. [56] gives rise to an improved branch-and-cut procedure. This study has led to the identification of several classes of valid inequalities and conditions under which the induce facets of the associated polyhedron. The use of such inequalities in a branch-and-cut procedure led to very good results, finding 198 optimal values out of 264 benchmark instances of [56], with optimal solutions for instances with up to 999 nodes, 2,678 links and five hierarchic levels obtained in a CPU time of less than one hour. As authors have reason to conclude, "...producing an effective exact method for a hard combinatorial optimization problem benefits from a good understanding of its associated polyhedron."

2.2.6. Other RPP Variants. The Periodic RPP (PRPP) is the RPP defined for a time horizon where services must be performed with a given periodicity. The PRPP may then be considered as a repetition of several RPPs, one per period in the time horizon. It has a natural application for repetitive actions such as weekly garbage collection. Since it combines several RPPs, the PRPP is also NP-hard.

Ghiani and Laporte [93] and Corberán et al. [69] discuss some variants of the RPP. Ghiani and Laporte [93] surveyed: the RPP with multiple edge services, including a period horizon where edges must be serviced more than once, which may be seen as a PRPP; the RPP with deadline classes, where service on the edges in each class must be completed within a given time limit; and the dynamic RPP, where the base graph changes dynamically. This last variant was suggested by a real case study related to industrial cutting.

Corberán et al. [69] discuss the following RPP variants: the stacker crane problem, related to the movements of a crane during service; the RPP with turn penalties, where some turns

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are dangerous and must be penalized if not forbidden; the clustered RPP, where the edges are divided into clusters and each cluster must be serviced jointly, that is, the edges of each cluster must be all serviced before moving to another cluster; and the windy RPP with zigzag services, where edge costs may be different depending on direction and some can be serviced using a single zigzag passage.

Another RPP variant recently studied in the literature is the stochastic Eulerian tour problem, which concerns a tour traversing each link exactly once, of minimum expected length, on an undirected, connected, Eulerian and stochastic graph. The stochasticity is related to a probability associated with the need to service each edge. Thus, the number of required edges on a specific day is represented by a random variable. This may be seen as an ARP on undirected and Eulerian graphs, that is, graphs where an Eulerian tour with stochastic demands may be identified. For non-Eulerian graphs, the first step in solving an ARP is often to transform the graph into an Eulerian one. Mohan et al. [147] develop three constructive heuristics for this NP-hard problem. The first heuristic iteratively adds to the end of the tour in construction the required non-visited edge that least increases the expected total length of the tour. The second proceeds in a similar way, but it adds only non-visited edges that are linked with the last edges included in the tour. The third constructs several small subtours that are iteratively mixed using an expected savings criterion. A post-optimization procedure is also adapted. Mohan et al. [147] cite mail delivery by Canada Post as an application of this problem, since the postal carrier first collects the mail to be delivered from relay boxes at various points along the route. Thus, the required edges where mail must be distributed are known only after the postal carrier begins traveling the route, and starting routes should incorporate the stochasticity of the problem. The postal carrier follows the route but has some flexibility to skip streets, if needed. Two sets of randomly generated instances (on grid networks with up to 174 required edges and on Euclidean graphs with up to 153 required edges) were used to determine which heuristic to choose (the third is better for some instances, and the second is better for the remainder).

2.3. General Routing Problem

General Routing Problems (GRPs) generalize ARPs as well as Node Routing Problems (NRPs) by considering both required links and required nodes. That is, some services need to be performed along the links of a network, while others are to be performed on nodes. One example may be a waste collection problem mixing door-to-door household refuse collection with the collection of larger containers that are distant from each other and from neighborhoods with door-to-door collection. The objective of the GRP, then, is to identify a minimum cost tour, beginning and ending at the depot, servicing all required links and all required nodes. This problem, when defined on a mixed graph, is also known as the Node, Edge, and ARP (NEARP).

Ávila et al. [21] study the Stacker Crane Problem (SCP) as a special Directed GRP (DGRP) case on the original graph, that is, with no need to remove all the non-required nodes. The objective of the SCP is to schedule the maneuvers of a crane that starts at an initial point, performs a set of movements, and returns to its initial position, at minimum total cost. The study derives a polyhedral description and some large families of facet-inducing inequalities for the DGRP, and it also details a branch-and-cut algorithm for both problems. This algorithm provides optimal solutions for large-sized instances, which justifies the authors' statement that "our algorithm is among the best solution procedures proposed for both problems." DGRP instances were generated to try to imitate street networks, with 500 nodes and 1,166–1,535 arcs; 700 nodes and 1,788–2,376 arcs; or 1,000 nodes and 2,342–3,177 arcs. New SCP instances were also generated and are available online (see instances CPS_ARP in Table 9). These are grid instances, 50×50 or 100×100, where the number of jobs varies from 100 to 3,000.

2.4. Arc Routing Problems with Profits

Maximization problems are increasingly used as an aid to vehicle tour design for private companies, which aim to maximize profit given a minimum desired quality of service. The recent proliferation of such studies justifies their treatment in their own section.

In Chapter 12 of [64], Archetti and Speranza [17] survey ARPs with profits. These problems differ in their objective functions as well as in some side constraints. Taken from real-world studies for private companies, the scenario involves the company receiving a premium for each link serviced. Since vehicles have either a limited capacity or an imposed time limit, not all demand links can be serviced. The goal then becomes to identify which links to service and determine the associated tour (including the depot) that optimizes a given objective function, considering the collected profit and/or travel cost. Thus, and contrary to what usually happens in an RPP, the customers to be serviced must be chosen from a larger set of potential customers.

Archetti and Speranza [17] identify two classes of the RPP with profits: the Profitable RPP (P-RPP), also known as the Prize-Collecting RPP or Privatized RPP, and the Orienteering ARP (OARP). All of these problems aim to generate a tour beginning and ending at the depot and including some required links. In the P-RPP, the objective is to maximize net profit, given as the difference between the collected profit and the traversal cost. Profit may be collected once, at most, from any given link. The objective of the OARP is to maximize the total collected profit for a tour within a given, limited traveling time. In this survey, Archetti and Speranza [17] present a formulation and refer to some studies regarding solution procedures for both the P-RPP and OARP classes.

In a later study, Benavent et al. [32] introduce the problem of minimizing total travel cost while guaranteeing a minimum collected profit. Although these authors name this the Prize-Collecting RPP, we are renaming it the Minimum Bound

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TABLE 3. Terminology for arc routing problems with profits.

	Problem	Objective functions			Constraints			
Initials	Name proposed	Max (profit)	Max (profit-cost)	Min (cost)	Multiple services	Travel Time Limit	Min. Collected Profit	Observation
OARP	Orienteering ARP	✓				✓		
P-ARP	Profitable ARP		✓					
ClP-ARP	Clustered P-ARP		✓					Tasks are clustered, and for a given cluster all tasks or none are serviced
MBPARP	Minimum Bound collected Profit ARP			✓			✓	
P-ARPM	Profitable ARP with multiple services		✓		✓			A limit is given for the number of times that profit can be collected
PL-ARP	Profitable Location ARP		✓					Includes facilities location at arc end nodes
TDP-ARP	Time-Dependent P-ARP		✓		✓			Multigraph to allow multiple services

Undirected, Directed, Mixed or Windy graphs may be emphasized by the respective initial (U, D, M, W) before the name (for instance, the mixed P-ARP is represented as MP-ARP).

collected Profit RPP (MBPRPP), for the sake of coherence with previous classification of node routing problems and in keeping with what seems to be the most common classification of arc routing works. In fact, some confusion still exists regarding the terminology of arc routing with profits, as pointed out by Archetti and Speranza [17]. In Table 3, we make a new attempt to build a common terminology by adding some new names to the set used by Archetti and Speranza [17] for the single vehicle case. We construct a similar table for the multiple vehicle case (Section 3.9).

In addition to discussing the MBPRPP, the recent paper by Benavent et al. [32] also proposes flow-based models for several ARPs with profits. While sharing the same set of constraints, these models differ in the objective functions considered, as well as in some sets of additional constraints.

It should be noted that whenever profits are being collected, the set of tasks is often partitioned in two: the mandatory tasks, which must be completed; and the optional ones, which may be completed if it pays off, that is, if it is profitable. Moreover, whenever optional tasks are considered, a penalty cost may be incurred if no services are ultimately provided.

2.4.1. Orienteering Arc Routing Problem (OARP). With the aim of studying bicycle trips within a target distance, Souffriau et al. [185] embraced the Orienteering ARP (OARP), in which profit is measured by the number of arcs and the limit constraint refers to an upper bound on the total cost (or distance) of the tour. A model is provided, as well as a metaheuristic capable of producing good feasible solutions in quite short computational times. With the central focus on two real cases, namely, “an on-line cycle route planning application that offers personalized cycle routes based on user preferences, and an SMS service that provides cyclists ‘in the field’ with routes on demand,” this study is further detailed in Section 4.2.

Benavent et al. [32] propose a flow-based compact model for the mixed OARP (MOARP), including time limit and/or capacity constraints for the vehicle. Benchmark instances from the literature have led to 476 adapted instances (with up to 428 nodes; 789 arcs, 149 of which are mandatory and 160 optional; and 223 edges, 168 of which are mandatory and 150 optional) assessing the model performance. With 406 out of 476 optimum values, and with very small gap values, the model behaved well. Moreover, even its linear relaxation was able to provide good bounds.

The directed problem is also studied by Archetti et al. [11] using large families of facet-inducing inequalities in a branch-and-cut algorithm. Solutions generated by the linear relaxation of the model are converted into feasible OARP solutions by means of a minimum cost network flow problem. This heuristic is used through the branch-and-cut method and provides good lower bounds that help prune the nodes of the branch-and-bound tree, allowing for its faster convergence. Several instances (with 100–2,000 nodes and 7,000–14,000 arcs) were generated, and small benchmark instances for the TOARP (multiple OARP) were also tested. This methodology allows for the optimal solving of the larger instances within one hour, and many were solved within a few minutes.

2.4.2. Profitable Arc Routing Problem (P-ARP). Archetti et al. [15] address the problem of including some nearby customers on a route only when it is worthwhile to do so. In this scenario, some customers are fixed and must be serviced, while others are to be chosen within the imposed constraints and with the goal of maximizing the difference between the profit and the travel cost. This travel cost also includes a penalty cost when a customer is not serviced. The directed case, that is, the Directed Profitable ARP (DP-ARP), is considered, and an integer LP formulation is given. Feasible

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solutions are generated within an ILP-refined tabu search procedure. Beginning with a standard tabu search, a greedy heuristic is first used to find an initial solution. Then, iteratively, it moves to neighbor solutions, where re-optimization and intensification procedures are proposed and employed to explore the neighborhoods. A diversification strategy is also applied whenever a neighborhood has been thoroughly explored, after which an ILP-refinement is applied. Based on the characteristics of the solutions identified through the tabu search procedure, the set of vertices is partitioned into: good (likely to be visited); bad (not to be visited); and ambiguous (remaining vertices). A tour including all the good vertices is then identified, and ambiguous vertices are inserted in this tour via the resolution of an ILP problem that chooses positions that minimize the insertion cost. A branch-and-cut with a fixed time limit is also used to obtain lower bounds for the problem. From a benchmark set of instances, 172 instances were generated and are available online (see Brescia_Inst, Table 9). In these instances, the set of nodes ranges from 7 to 140 and the number of required arcs from 3 to 203. The corresponding results "show the effectiveness of the proposed solution method and the positive impact of the ILP model on the solution quality."

For this same problem, Colombi and Mansini [58] propose inequalities that are added to the existing model of [15] within a branch-and-cut algorithm. These new inequalities strengthen both the LP relaxation of the problem and the relaxed version without the connectivity constraints. A matheuristic followed by an improvement heuristic is also developed. At each iteration of the matheuristic, a DRPP is solved to optimality for a new subset of required arcs. Then, as in [15], a refinement procedure involving a branch-and-cut algorithm tries to improve the best solution generated by the matheuristic, which quickly converges from imposing the connectivity constraints. These two different constraints are referred to as the standard way (via the resolution of the max-flow separation problem), and the lazy way (where subtours are eliminated only when an integer solution is found). The authors stress that this refinement heuristic may be used by itself. From the results obtained over the same set of instances as [15], the authors conclude "that heuristic methods are extremely effective outperforming existing algorithms." The best choice is to combine the reinforced model with the lazy separation of the connectivity constraints within the branch-and-cut. The authors also emphasize that the "exact method is able to close, in less than one hour, all the 22 benchmark instances that have not been solved to optimality yet," which is a promising result.

As for the OARP, Benavent et al. [32] propose a flow-based compact model for the mixed P-ARP (MP-ARP), including time limit and/or capacity constraints for the vehicle. Benchmark instances from the literature have led to 486 adapted instances (with up to 428 nodes; 789 arcs, 149 of which are mandatory and 160 optional; and 223 edges, 168 of which are mandatory and 150 optional) to assess the performance of the model. With 438 out of 486 optimum values, in short computational times, and with very small gap values for the remaining instances, the model seems to be a good option in medium-sized instances. However, the upper bounds of the linear relaxation are generally not good.

Windy undirected graphs within a P-ARP are studied by Schaeffer et al. [175] and Ávila et al. [20]. Schaeffer et al. [175] offer an ant colony algorithm starting with an empty tour. This is iteratively extended following some preferred neighbors and according to characteristics including the number of visits to links and nodes and the computed pheromone per link. Four sets of RPP and GRP benchmark instances described by Corberán et al. [62] are adapted to this Windy Profitable ARP (WP-ARP). Computational results show that the heuristic usually generates profitable closed tours in less than half a minute in instances with up to 196 nodes and 632 edges. No comparison with other procedures is provided, as this is the first work on the WP-ARP.

More recently, Ávila et al. [20], pursuing the optimal solution of the WP-ARP, have developed a branch-and-cut methodology. It begins with the study of the polyhedron associated with the proposed model. Several families of facet-inducing inequalities are detailed and exploited through a branch-and-cut algorithm. The computational results outperform those of [175]. Large-sized instances were generated (see CPS_ARP in Table 9), and instances with up to 1,000 nodes and some with 1,500 nodes were optimally solved, and in reasonable computing times. As possibilities for future research, the authors point to the development of new valid inequalities, the design of more efficient separation algorithms for some of the inequalities derived, and the improvement of heuristic algorithms.

2.4.3. Profitable ARPs with Multiple Services (P-ARPM). The Maximum Benefit CPP (MBCPP) deals with profitable problems allowing multiple services and was tackled by Corberán et al. [67]. This study considers several benefits associated with each edge, one for each turn the edge is serviced, and several traversal costs per edge. A deadheading cost is also associated with each edge. The objective is to find a tour with maximum total net benefit. Having the RPP as a special case, the MBCPP is an NP-hard problem. The authors propose a formulation for the undirected case and a branch-and-cut algorithm from the description of its associated polyhedron. This description comes from several families of valid facet-inducing inequalities. Heuristics and exact procedures for the separation problems are also proposed. The performance of the procedure is analyzed over ARP instances (see CPS_ARP in Table 9) with up to 1,000 nodes and 3,000 edges. This problem was also referenced in [17] and is a particular case of the Profitable ARP with Multiple Services (P-ARPM).

2.4.4. Clustered Profitable Arc Routing Problem (ClP-ARP). Corberán et al. [62] introduce a new ARP variant, the Windy Clustered Profitable Postman Problem (WClP-ARP). As in the P-ARP, profit is collected only once if and when the edge is serviced, no matter the number of traversals on the edge. Edges are partitioned into clusters, and if a

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required edge is serviced, all the remaining edges in the same cluster must also be serviced. This study contains a mathematical model and some polyhedral results including several facet-defining and valid inequalities. The separation problem for the different families of inequalities is studied, and both exact and heuristic algorithms are provided to identify such inequalities. This paper is the first in the literature to deal with windy and clustered ARPs including profits. The cluster version in particular is worthy of more than just theoretical interest, since profit problems are key for private companies. In some real-world applications, such as those arising from privatization of garbage collection or postal services, it makes no sense to serve a customer if the whole neighborhood is not profitable. In the case of refuse collection, for instance, it is not reasonable to collect the refuse on alternate streets, or even in a small part of a neighborhood. This justifies the identification of clusters. Four sets of benchmark instances for the RPP and for the GRP were adapted to the WClP-ARP (with up to 196 nodes and 316 edges, of which up to 238 are required), and reported results show the effectiveness of the algorithm in solving medium- and large-sized instances in short computational times.

Also working in response to the clustered problem but on undirected graphs, Araoz et al. [7] propose a GRASP (Greedy Random Adaptive Search Procedure) and path relinking heuristic for the ClP-RPP. Despite the good results obtained by the branch-and-cut procedure of [62], heuristics are always important where NP-hard problems are concerned, as in the case of the ClP-RPP. In this paper, two different constructive heuristics are proposed, based on the resolution of an RPP for each cluster defined from different sets of selected customers and the corresponding clusters. The heuristics differ in the way that suitable clusters are selected for servicing. The first heuristic is bottom-up, as clusters are selected and inserted step by step. The second heuristic is top-down, as it starts by including all the clusters, which are then removed step by step. The path relinking phase works as a post-optimization procedure between pairs of solutions from a pool of elite solutions. Computational results over benchmark instances, including those of [62] (with up to 196 nodes and 316 edges, of which up to 238 are required), show these metaheuristics to be competitive and produce very good results in rather small computing times.

2.4.5. Minimum Bound Collected Profit Arc Routing Problem (MBPARP). Benavent et al. [32] introduce the Minimum Bound collected Profit ARP (MBPARP), which aims to minimize the total travel cost by imposing a minimum profit to be collected. This minimum profit is imposed for performing some service while minimizing cost. The objective, then, is to identify a tour, beginning and ending at the depot that minimizes total cost while servicing all mandatory links and some optional links in order to attain the profit threshold. Again, the name of the problem is herein changed in keeping with the terminology shown in Table 3. As for the remaining profitable problems studied in this paper and already reported on, a compact flow-based model is proposed. From the usual benchmark instances, 486 instances were adapted (with up to 428 nodes; 789 arcs, 149 of which are mandatory and 160 optional; and 223 edges, 168 of which are mandatory and 150 optional). Results point to weak LP bounds but to good model performance, since a majority of instances were optimally solved. As expected, higher values of the minimum imposed profit lead to worse results. Even so, if 50% profit for all optional links is imposed as the minimum value, then 439 out of the 486 instances are optimally solved, and gap values are very small.

2.4.6. Time-Dependent Profitable RPP (TDP-RPP). Time-dependent ARPs differ from standard ARPs in that links' traversal times depend on the time of the day they are traversed, that is, conditions change over time (as discussed in Section 2.1.2). This generalization fits, for instance, real-world situations where distribution/collection periods include rush hours. Black et al. [38] introduce the Time-Dependent Profitable ARP (TDP-ARP) for directed multigraphs (with parallel arcs) and propose a mathematical formulation and two metaheuristics to generate feasible solutions. The first metaheuristic is based on a Variable Neighborhood Search (VNS) that resorts to ten different, although similar, neighborhoods. The second adapts a method for the time-dependent vehicle routing problem. This work was motivated by a freight transport company with the need to select orders for full truckloads to be carried between pairs of pickup and destination points. It is assumed that customers' orders are known before the vehicle starts duty. Fulfillment of an order results in a known benefit for the company. The aim is to identify which orders to accept, then identify the tour that fulfills selected orders while maximizing the difference between the profit resulting from those orders and the total travel costs. Parallel arcs are used to allow multiple orders between the same origin/destination pairs. Computational results are provided for 41 reality-based instances generated from two distinct road networks: (i) a network in the northwest of England (with 25 nodes and 625 arcs, of which those with an associated prize vary from 50 to 600); and (ii) a network in the south-east of England in and around London (with 50 nodes and 2,500 arcs, of which those with an associated prize vary from 75 to 350). These are available online (see W_CARPs in Table 9). Results are considered good for both heuristics, since good quality solutions are generated even for the larger instances, with the VNS generally showing better performance.

The same problem was addressed by Yu and Lin [208], who proposed an iterated greedy heuristic that is validated over the benchmark instances of [38]. This heuristic begins with an initial solution, which is iteratively modified in two phases, the destruction and the subsequent construction. In the destruction phase, a given number of links are removed from the solution, and a partial solution is obtained. A greedy heuristic is then applied in the construction phase to repair the feasibility of the solution. This heuristic provided the best solutions for most of the benchmark instances.

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2.4.7. Profitable Location Arc Routing Problem (PL-ARP). The integration of facility location issues in a P-ARP was first explored by Arbib et al. [9] for the directed case. Each selected profitable arc leads to the selection of both facilities, which are located at the end points of the arc. The objective is to choose a tour that maximizes the difference between the profits collected along the required selected arcs and the total traveling and installation cost. Installation cost is a fixed cost incurred for each installed facility. The authors describe real-world applications of this problem as "mid/long range passenger/freight transportation, for example, long-distance coach services, airlines, and interstate trucking. The problem is to choose the connections on which to open a service." The study proves that the problem is NP-hard, and it presents an integer LP model as well as a branch-and-cut procedure. This begins with the relaxed model with no connectivity constraints, these being iteratively added if needed. Benchmark instances from the literature were adapted (generating new instances with 7–102 nodes and 10–200 arcs), and the algorithm performed well, optimally solving 487 instances out of 516.

3. MULTIPLE VEHICLE ARC ROUTING PROBLEMS (K-ARPS)

More challenging problems arise when a fleet of K vehicles is available to service the links of a network. Usually, a homogeneous fleet of capacitated vehicles is at the depot node, where all tours must start and end, to meet the demands on the required links at a minimum total cost. Such problems are known as multiple or K-vehicles ARPs (K-ARPs).

As in the case of single vehicle problems, introducing several particularities on K-ARPs to accommodate specific features leads to the definition of different problems within the larger category of K-ARPs. From 2010 onwards, new problems have indeed been defined, and researchers are also devoting further effort to the study of known problems. One key variant worth mentioning at the outset is the Capacitated ARP (CARP), which has received increasing attention in recent years, as reflected in the four chapters devoted to CARP in the recent arc routing book edited by [64]. Some additional variants are summarized in [151]. Within the larger category of K-ARPs, the many specific features and combinations make the clustering of problems a difficult task.

To highlight the differences among K-ARP variants, Tables 4, 5, 6 and 8 list problems, including some referred to in Section 4, and their characteristics, as well as the related studies.

The evaluation of K-ARP methods is usually based on the following benchmark instances:

(i) Six CARP benchmark instance sets referred to in (I_CARP):
  1. *kshs* - 6 instances with 6–10 nodes, 15 edges, all required, and 3 or 4 vehicles;
  2. *gdb* - 23 instances with 7–27 nodes, 11–55 edges, all required, and 4–10 vehicles;
  3. *val* - 34 instances with 24–50 nodes, 34–97 edges, all required, and 2–10 vehicles;
  4. *egl* - 24 instances with 77–140 nodes, 98–190 edges (51–190 required), and 5–35 vehicles;
  5. *eglL* - 23 instances with 255 nodes, 375 edges (347 or 375 required), and 20–42 vehicles;
  6. *bmcv* - 100 instances with 26–97 nodes, 35–142 edges (28–121 required), and 212 vehicles.

(ii) Two MCARP benchmark instance sets referred to in (B_MCARP):
  1. *mval* - 34 instances with 24–50 nodes, 43–136 links, including up to 44 required arcs and 106 required edges, and 4–12 vehicles;
  2. *lpr* - 15 instances with 28–401 nodes, 52–1,056 links, including up to 764 required arcs and 387 required edges, and 2–29 vehicles.

This section is organized as follows: the first subsection is devoted to the CARP, including exact, lower bound methods and heuristics; the two ensuing subsections refer to the mixed CARP and a group of problems closely related to CARP; different multiple ARPs are then detailed in the following four subsections, focusing on problems with several facilities, soft TW and split deliveries; finally, the stochastic and dynamic cases are considered. Tactical and strategic extensions precede the last two subsections, which deal with multiple GRPs and multiple ARPs with profits, respectively.

3.1. *Undirected CARP*

The Undirected Capacitated ARP (UCARP), usually referred to simply as CARP, is defined on an undirected graph. Each edge has an associated deadheading cost, and each task has also a known demand and service cost. A fleet of homogenous vehicles is available at the depot and is enough to satisfy all demand. Vehicles should begin and end their routes at the depot and service link demands within their capacity (W). The aim is to find a set of minimum cost routes, not exceeding the number of available vehicles.

It is well known that CARP is an NP-hard problem. So is its $3/2$-approximation, that is, the problem of finding a feasible solution with a value of $3/2$ the optimum value at most. Fortunately, a polynomial algorithm with a $(7/2 - 3/W)$-approximation is known even if the triangle inequality is not satisfied [195]. A discussion on complexity issues related to CARP can be found in [194].

3.1.1. Exact and Lower Bound Methods. Exact CARP methods and lower bound procedures were recently surveyed by Belenguer et al. [28] and Ahr and Reinelt [2], respectively. Regardless of CARP's proven complexity, a number of recently developed exact methods can be found. One possible approach consists of transforming the problem into a node routing problem and then applying existing VRP methodologies to solve it. The results obtained depend on the transformation used so that instance dimension does not increase too much. Recently, Foulds et al. [87] propose a compact transformation, ensuring the number of nodes in

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TABLE 4. Multiple vehicle problems' characteristics and related published papers.

Problem	Objective	Network $G=(N,A\cup E)$				Other characteristics	Papers
		U/D/M/W	A_R	E_R	N_R		
ARCP (Arc Routing for Connectivity)	Min(time to reconnect)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Clear blocked arcs to recover connectivity; Deadheading allowed after cleaning	[19, 117]
	Max(benefit of reconnecting)						[117]
						Time limit; Prize – takes into account the number of people it benefits; Prizes may prioritize components – higher weights to airports, schools, hospitals, etc.	[4]
	Min(cost)					Multiple days; Depot – 1^{st} day starting point; No fixed depots to end/start each day's routes; Hierarchic services defined by priority sets	[199]
	Min(Max(tour cost))					Coordinated routes	[3, 5]
	Min(unblocking times); Min(Max (unblocking time))					Dynamic networks; Time-dependent travel times	[153]
BCARP	Min(cost)	M	$A_R\subset A$	$E_R\subset E$	$N_R=\emptyset$	Upper bound on the number of nodes shared by different routes	[59]
CARP	Min(cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$		[2, 27, 28, 40, 41, 42, 55, 87, 128, 129, 138, 139, 140, 142, 143, 157, 160, 161, 174, 178, 191, 194, 195, 200]
						With split deliveries – an edge demand can be serviced by several vehicles	[29]
						Multi-compartment vehicles (MCV) – tasks have a demand for different products and are serviced by MCV	[150]
						Multi-compartment vehicles; Multi-days; Semi-Periodic; With/without split deliveries; Coordinated vehicles	[118]
		M	$A_R\subset A$	$E_R\subset E$	$N_R=\emptyset$	–	[94, 96, 141, 193]
		D	$A_R\subset\emptyset$	$E_R=\emptyset$	$N_R=\emptyset$	2 types of vehicles: a small one with a finite capacity to service tasks (SV) and another one to refill it (RV); Multiple loads are allowed	[6]
		M	$A_R\subset A$	$E_R\subset E$	$N_R=\emptyset$		[134]
	Min(travel distance)	D	$A_R\subset A$	$E_R=\emptyset$	$N_R=\emptyset$	Snow plowing operations	[126]
	Min(road maintenance costs; pollution)	D	$A_R\subset A$	$E_R=\emptyset$	$N_R=\emptyset$	Routing of deicing vehicles; Starting points; Vehicle types; IF – number, location, type; Load balancing; Characteristics of road networks – boundaries	[90, 206]
CARPDD	Min(cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Deadheading demands, like deadheading an edge, also uses vehicles' capacity, although at a smaller rate than serving it	[27, 119, 181]

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TABLE 4. (Continued)

Problem	Objective	Network $G=(N,A\cup E)$ U/D/M/W	A_R	E_R	N_R	Other characteristics	Papers
CARP-MD	Min(cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$		[114, 121]
						Heterogeneous vehicles' fleet	[130]
CARP-IF	Min(cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Intermediate Facilities (IF) – where vehicles may be emptied to respect capacity; There is a time limit per tour	[92]
		M	$A_R\subset A$	$E_R\subset E$	$N_R=\emptyset$		[202–205]
						IF with limited capacity; Heterogeneous fleet	[170]
		U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Debris collection and transportation; New constraints – accessibility between different areas; 2 task types: blocked roads with debris to remove & transport; cleaned roads with debris to transport; Blocked arcs can be traversed only after being serviced	[158, 159]
CARPSD	Min(cost + expected cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Stochastic demands – possibility of route failures whenever the realized demand exceeds the vehicle capacity	[125]
	Min(deadheading cost; K)	D	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Road maintenance including visual checking on roads; Stochastic services – normal distribution; Time limit – from the probability of each tour duration; Risks regarding extreme outcomes	[54]
	Min(worst-case value)						[53]
CARP-TW	Min(#tours + TW violations)	M	$A_R\subset A$	$E_R\subset E$	$N_R=\emptyset$	Soft Time Windows (TW) – a penalty is due when service of an arc is outside its TW	[1, 197]
	Min(cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Road resurfacing; Precedence between road treatments	[111]
DyCARP	Min(cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Dynamic network; Several depots; Heterogeneous fleet – different capacities; Re-planning due to obstacles in a planned tour, vehicle breakdown or added/canceled demands	[127, 207]
		D/M	$A_R\subset A$	$E_R=\emptyset$/ $E_R\subset E$	$N_R=\emptyset$		[127]
K-ARP	Min(disruption costs)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Rescheduling K-ARP – due to a vehicle breakdown; Need to reschedule tasks not serviced due to the tour disruption; Several starting points – ending task nodes in initial plan	[149]
	Min(cost)	U	$A_R=\emptyset$	$E_R\subset E$	$N_R=\emptyset$	Snow plowing operations; Primary roads served 1^{st} to reach secondary ones	[110]
	Min(traversal and penalty cost)					Deicing – priority nodes; Penalties – delays in reaching priority nodes	[120]
	Min(cost)	D	$A_R\subset A$	$E_R=\emptyset$	$N_R=\emptyset$	Sweeping routes; Task services: as many times as the number of street sides; Time-dependent – traffic flow; Turn constraints	[39]
	Min(K)					Winter road maintenance: depot location; sector design; vehicle scheduling and fleet configuration; time limit constraint	[116]

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TABLE 4. (Continued)

Problem	Objective	Network $G = (N, A \cup E)$ U/D/M/W	A_R	E_R	N_R	Other characteristics	Papers
K-ARP	Min(cost)	W	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Plowing with precedence constraints; 1^{st} plow crucial streets; Steep streets should not be plowed uphill	[81, 82]
	Min(total deadheading)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Railroad track inspection to prevent failures; Time limit; heterogeneous fleet; track outages; Capacity can be recovered at IF, $\leq$ once per shift	[122, 123]
K-CERPP	Min(cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	or K-GRPP – clients are served from a close enough distance	[22]
MMK-CPP	Min(longest route)	U	$A_R = \emptyset$	$E_R = E$	$N_R = \emptyset$	Seeks the minimization of the longest route	[34]
MMK-RPP	Min(longest route)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Seeks the minimization of the longest route	[34]
		U/W					[31, 33, 35]
						With aesthetic issues, i.e., solution shape characteristics	[63, 135]
						Patrol routes design; Unpredictable routes; Balanced workload	[201]
MO-CARP	Min(cost; longest route)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	3^{rd} objective – minimize the number of vehicles	[97]
							[177, 179, 180]
		M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$		[145]
O-CARP	Min(cost)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	No fixed depot; Tours need not form cycles	[192]
						Fixed depot to start with no fixed end; Maximum distance span	[88]
SyARP	Min(longest route)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Snow plowing – synchronized vehicle routes	[172]
SyCGRP	Min(time)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R \subset N$	2 types of vehicles with synchronized routes	[173]
TDCARP	Min(service and travel total cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Service costs computed with a piecewise linear function, which includes intervals where service costs are minimum	[188]
						Dynamic networks – due to weather report updates; Service costs computed with a piecewise linear function	[189]

A fleet of K homogeneous vehicles with fixed capacity is assumed.
U/D/M/W – Undirected/Directed/Mixed/Windy.

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the resulting graph exceeds the number of required edges by only one. Resorting to an algorithm for the capacitated vehicle routing problem, the authors reported computational experiments on CARP benchmark instances, and succeeded in solving some open instances.

A branch-and-cut-and-price is attributed to Martinelli et al. [138]. This algorithm is based on a formulation that does not identify elementary tours, as only one variable is associated with each edge. Hence, integer solutions might not be feasible for CARP. In fact, the method produces valid lower bounds but not necessarily optimum values. This motivates Martinelli et al. [139] to formulate a problem to separate capacity constraints. This model makes it possible to identify violated capacity cuts for any number of vehicles at once. However, as exact separation is suitable only for medium-sized instances, the authors also propose a dual-ascent heuristic to deal with the larger instances. Additionally, they use an initial set of cuts on the one index formulation to reduce the number of times the exact procedure is applied. This strategy has obtained impressive lower bounds on some benchmark instances.

Bartolini et al. [26] use a transformation into node routing, where each required edge is represented by a cluster of two nodes, and present an extended formulation for set partitioning. They use four lower bounding procedures solving increasingly stronger relaxations, with the last one often providing a lower bound quite close to the optimum. Relaxations are obtained by allowing non-elementary tours. The corresponding linear relaxations are strengthened with valid inequalities. Usually the integer problem is not solved. However, should an upper bound be known, the lower bounding procedures may be complemented and an attempt is made to explicitly solve the last extended formulation. To generate tours with small reduced costs, a two-phase algorithm is developed. Firstly, and by means of a dynamic programming procedure, forward and backward paths are built, that is, paths that begin or end at the depot; secondly, these paths are combined in feasible tours within a controlled reduced cost. The effectiveness of this procedure is attested in a computational study on CARP benchmark instances.

In several works, Bode and Irnich explore the sparsity of the original graph to design exact algorithms for the CARP. First, Bode and Irnich [40] develop a branch-and-price algorithm that is preceded by a cutting phase. The computational results evidence the effectiveness of the direct approach versus the node routing approach. Later, Bode and Irnich [41] and Bode and Irnich [42] concentrate on the pricing problem and work on its relaxations, while providing an efficient labeling procedure for their resolution. Computational experiments display improved achievements.

Very recently, Porumbel et al. [157] propose a method to solve the CARP by combining Column Generation (CG) with Iterated Local Search (ILS). Both run in parallel. On one hand, the best routes found by CG are included in the current ILS solution during the perturbation phase, so that dual information is conveyed to the ILS. On the other hand, the CG improvement operator works on current ILS solutions and thus helps to increase diversification of the solutions visited by ILS and avoids getting stuck in local optima. Extensive computational experiments on benchmark instances are reported and prove the competitiveness of the method.

The approach of Foulds et al. [87] was tested on set 4 (*egl*) and provided optimal solutions for four instances, the largest with 140 nodes, 190 edges (75 required), and 14 vehicles. Martinelli et al. [139] performed computational tests on all six benchmark sets, obtaining the first lower bounds for set 5 (*eglL*) and improving upper bounds. The authors report good figures for the small/medium instances as well. Bartolini et al. [26] display results for sets 1–4 (*kshs*, *gdb*, *val*, *egl*) and set 6 (*bmcv*), having solved 27 instances for the first time (6 from *egl* and 21 from *bmcv*). Bode and Irnich [41] report 19 best lower bounds over the 33 unsolved instances of *bmcv*, proving the optimality of 15 instances for the first time. They obtained three new best lower bounds for *egl* instances. Interestingly, as noted by Bode and Irnich [41], the two works' combined results by Bartolini et al. [26] and Bode and Irnich [41] leave only 12 *egl* and 10 *bmcv* instances unsolved. Porumbel et al. [157] test their method over the six sets of instances, finding new upper bounds for half the *bmcv* instances.

Considering the latest developments in exact methods, we may conclude that the performance of a solution has a close relationship with the type of instance that it addresses. For sparse graphs, the direct arc routing approach seems to be the most suitable.

3.1.2. Heuristic Algorithms. The challenge posed by CARP and its relevant applications continuously attracts scholarly effort to find feasible solutions, and a number of heuristics, mostly metaheuristics in fact, have been developed for CARP in recent years. A state of the art analysis of this topic may be found in [160], who also gives an interesting comparative summary of the performance of several heuristics on benchmark instance sets (CPS_ARP in Table 9). A revision to references of order-first split-second methods, including arc and node routing cases, is presented by Prins et al. [161], with the purpose of convincing readers that "route-first cluster second heuristics compete with other heuristics."

Santos et al. [174] present an ant colony-based metaheuristic while introducing some modifications. Computational tests were performed on sets 1–4 (*kshs*, *gdb*, *val*, *egl*) and set 6 (*bmcv*) of CARP benchmark instances. The authors conclude: (i) best performance is achieved when the initial population contains only very good solutions; (ii) only the best solutions are needed to compute current pheromone amounts; (iii) best overall results are generated from only the ten best solutions generated so far; (iv) the newly proposed ant decision is better than the usual pseudo-random-proportional; and (v) modifications introduced in the local search improve the quality of solutions and/or reduce CPU time.

In [140], a biased Random Key Genetic Algorithm (RKGA) is combined with a local search. In an RKGA, randomly generated numbers (the keys) are used to represent chromosomes. A deterministic algorithm, called a decoder, is used to transform a vector of these random keys into a

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feasible solution to the original problem. The RKGA is introduced to overcome the problem of non-feasible offspring (a drawback in genetic algorithms). Population is divided into groups of elite and non-elite individuals according to a fitness measure, the elite being a small percentage of the population to be transferred to the next population. This variant is called biased random key because each new individual is obtained by combining a randomly selected element from the elite partition of the current population with another from the non-elite partition. Mate repetition is also allowed. Martinez et al. [140] report results of computational experiments performed on sets 1–3 (*kshs*, *gdb*, *val*) of CARP benchmark instances. Optimal or near optimal solutions are obtained with small computational times. According to the authors, the successful results are due to: (i) the manner in which the initial population is generated, allowing intensification of the search procedure in different regions; (ii) the parameterized uniform crossover operator employed to mate elite with non-elite individuals; and (iii) the effectiveness of the improvement phase after crossover and mutation. The neighbors list and classical local search methods also permit the intensification of the search process in attractive regions.

Martinelli et al. [139] transform the original CARP instances and implement an iterated local search procedure for a node routing problem. They describe extensive computational experiments on the six sets of CARP benchmark instances, improving the known upper bounds.

A greedy randomized adaptive search procedure with path-relinking is proposed in [191]. Some features of the method are highlighted, namely: "(i) a reactive parameter tuning, where the parameter value is stochastically selected biased in favor of those values which historically produced the best solutions in average; (ii) a statistical filter, which discard initial solutions if they are unlikely to improve the incumbent best solution; (iii) infeasible local search, where high-quality solutions, though infeasible, are used to explore the feasible/infeasible boundaries of the solution space; (iv) evolutionary path-relinking, a recent trend where the pool of elite solutions is progressively improved by successive relinking of pairs of elite solutions." Computational results on sets 2–4 (*gdb*, *val*, *egl*) of CARP benchmark instances are reported as providing better bounds than previous metaheuristics, although they are more time-consuming.

A memetic algorithm with iterated local search is proposed by Liu et al. [129]. The method incorporates a new crossover operator (the longest common substring crossover), an iterated local search and a perturbation mechanism. Infeasible solutions are allowed during the local search process and may be present during the perturbation mechanism. The method is also evaluated on sets 2–4 (*gdb*, *val*, *egl*) of CARP benchmark instances, evidencing a competitive performance.

To tackle the largest CARP benchmark instances, *eglL*, Mei et al. [142] present a method to find promising decompositions to derive solutions by means of a divide and conquer method. Later, Mei et al. [143] improve on the decomposition procedure by incorporating information about the quality of the best solution found in the search. Experimental studies, conducted on sets 2–6 (*gdb*, *val*, *egl*, *eglL*, *bmcv*) of CARP benchmark instances, confirm intuition: it is a good approach, particularly for large instances.

Liu et al. [128] describe a memetic algorithm for CARP, tested on three instances: an undirected graph with 100 nodes and 167 edges, an oriented graph with 200 nodes and 647 edges, and a mixed graph with 600 nodes and 999 links. Instances were obtained from the instance generator proposed, which is a major contribution of the work and is available online (see LIU in Table 9). The purpose of this instance generator is to add instances with realistic features for specific applications to CARP benchmark sets.

A rank-based memetic algorithm is presented in [200]. The essence of this algorithm is the newly introduced rank-based neighborhood search operator. Experiments reported on sets 2–4 (*gdb*, *val*, *egl*) and set 6 (*bmcv*) of the CARP benchmark instances show impressive performance. The authors claim the success of their approach resides in four major components: local search procedure yielding to promising local optima; a rank-based neighborhood search operator; evaluation of edge position in a solution; and good pruning strategies.

Recently, Shang et al. [178] propose another metaheuristic, more precisely, an immune clonal selection algorithm. This algorithm is inspired by the clonal selection theory, which is proposed to account for the behavior and capabilities of antibodies in the acquired immune system. An improved constructive heuristic is used to initialize the antibody population, which helps accelerate the algorithm's convergence. The authors then stress that the immune clonal selection algorithm decides in favor of high-quality antibodies by adopting a variety of different strategies for different clones of the same antibody. It not only promotes cooperation and information exchange among antibodies, but also increases diversity and speeds up convergence. Shang et al. [178] also propose two different antibody repair operations for various types of infeasible solutions. These infeasible solutions are allowed in order to permit a faster movement towards global optima. Experimental studies on sets 2–4 (*gdb*, *val*, *egl*) of CARP benchmark instances display improved performance.

A hybrid metaheuristic approach is presented in [55]. The method incorporates an effective local refinement procedure into a memetic algorithm, coupling a randomized tabu thresholding procedure with an infeasible descent procedure. The approach includes a specially designed route-based crossover operator for solution recombination and a distance-and-quality-based replacement criterion for pool updating. Extensive experimental studies are reported on sets 2–6 (*gdb*, *val*, *egl*, *eglL*, *bmcv*) of CARP benchmark instances. In 15 out of the 191 tested, an improved best result is found; moreover, 9 of the best results are found in the same set of 10 large instances.

3.2. *Mixed CARP*

The mixed CARP (MCARP) is addressed in [96], where compact models are proposed. Two mixed integer

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programming flow-based formulations, a valid one and a relaxation, are presented. The valid formulation enables solving small-sized instances to optimality, while the relaxation provides good lower bounds for large instances. These conclusions were drawn from computational experiments using the models on MCARP benchmark instances, which include two sets: one with 34 instances with 24–50 nodes and 43–138 links, all required; and another set with 15 instances, containing larger instances, with 28–401 nodes and 50–1,056 links, not all required.

Very recently, van Bevern et al. [193] prove the mixed and windy CARP (MWCARP) to be 35-approximable in $O\left(2^C C^2 + n^3 \log n\right)$, C being the number of weakly connected components of the subgraph induced by the tasks in the original graph. The polynomial time algorithm begins with computing a giant tour, then splitting it into subtours of demand up to W. The algorithms accomplishing both phases are basically those used in the complexity study proofs. To assess the quality of the produced upper bounds, the authors use the MCARP benchmark instances and compare results with those obtained by other polynomial algorithms. Their manner of computing the giant tour seems to be advantageous. Large benchmark CARP instances (I_CARP in Table 9) were also tried, and for all ten instances better upper bounds were found. Computational results over a new set of instances (RvB_MCARP in Table 9) with 255–401 nodes and 362–1,025 links, generated to have data sets with $C > 1$ (tried for C ranging from 2 to 5), are also reported. The computational times are rather short, and results can be used for an initial solution for local search algorithms.

Multiple ARPs on mixed and windy graphs have been also presented by other authors and will be mentioned in the next sections, since they deal with additional constraints or have other specific characteristics.

3.3. Other CARP Variants

The CARP with Deadheading Demand (CARPDD) was studied by Sipahioglu et al. [181], Kirlik and Sipahioglu [119] and Bartolini et al. [27]. The problem is a slight variation of the CARP where deadheading an edge also uses vehicles' capacity, although capacity declines at a lower rate than service is accomplished. An example of an application is multi-robot sensor-based coverage path planning, where vehicles still consume energy while deadheading links. Kirlik and Sipahioglu [119] propose an integer linear model, which enables them to solve small-sized instances to optimality, as well as a constructive heuristic and two improving procedures. The methods are tested on instances obtained by modifying sets 2–4 (*gdb*, *val*, *egl*) of CARP benchmark instances. Bartolini et al. [27] introduce a new family of valid inequalities to strengthen the lower bounds, after showing that the strongest CARP lower bounds can be weak for the CARPDD. These authors develop an exact branch-and-cut-and-price algorithm, reporting extensive computational results on a large set of benchmark instances (very sparse new instances generated from benchmark instances—see BCL_CARPDD in Table 9—and those of [119]). The same exact algorithm is also tested on classical CARP benchmark instances (*kshs*, *gdb*, *val*, *egl*, *bmcv*) and shown to improve on the best known solutions achieved in the authors' previous work [26] and the work of Bode and Irnich [40].

Considering only one tour, the Distance Constrained Generalized Directed RPP was previously dealt with in the single vehicle section and are named GRPP or CERPP. To simplify and lend coherence, the multiple case may be named K-GRPP (multiple generalized RPP) or K-CERPP (multiple close-enough RPP). The problem arises when a vehicle with a receiver is used to register gas, water, or electricity consumption. The vehicle travels through a series of neighborhoods and needs to get close enough to read each meter. In the multiple vehicle case, the problem deals with finding a set of routes, one for each vehicle and not exceeding a time limit, and traversing at least one arc from each of the subsets of a defined arc subset collection, so that total length is minimized. This collection is such that all clients are reachable from a close enough distance. Ávila et al. [22] presents several formulations with some families of valid inequalities and developed branch-and-cut algorithms. Formulations and algorithms are compared over a large set of instances to show that two of the proposed algorithms perform well and are able to solve instances with up to five vehicles, 196 nodes, 450 arcs, and 150 customers.

Constantino et al. [59] observe that the shape of the solutions obtained when solving ARPs very often fails in practical terms for being not sufficiently attractive in the eyes of practitioners, and they define a problem to help lessen that failure rate. These authors define the Bounded overlapping mixed CARP (BCARP) as a mixed CARP with an additional constraint imposing an upper bound on the number of nodes shared by different routes. Integer models are defined to find the limit to be imposed, and one of them selected as the best one. A two-step approach is also proposed. This involves first solving an integer model to find the bound to be imposed on the overlapping nodes, then using an integer solver to find an MCARP solution bearing the additional constraint. As the integer solver makes this approach inappropriate for medium-sized instances, a heuristic procedure is also presented. Computational results are provided using MCARP benchmark instances to evaluate the performance of the methods in terms of solution cost and shape, and problems with up to 53 nodes and 160 links are optimally solved. One important conclusion is that the additional requirement to prevent overlapping routes from impacting the total cost has only a small effect. One other contribution is the proposal of three different measures to quantify shape quality. These are also evaluated and compared in tests.

3.3.1. Open CARP. The Open CARP (O-CARP) releases the imposition that tours must begin and end at the depot, and so in this CARP variant tours may not form cycles. In [192], an integer LP formulation is given and the properties of the problem are discussed. A reactive path-scanning heuristic, guided by cost-demand edge-selection and ellipse rules, is

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proposed to find upper bounds on the optimal value. Extensive computational experiments were performed on instances derived from CARP benchmarks, resulting in instances with 7–140 nodes and 15–190 edges. In 32% of the instances, optimality was found and proved. Fung et al. [88] consider that only the end of each tour is open, that is, may occur outside the depot. They use an asymmetric graph and impose, in addition to vehicle capacity, a maximum distance span. A memetic algorithm and a lower bound obtained through a transformation into a node routing problem are presented. Computational experiments were run on randomly generated instances (with 200–500 nodes and 20–300 arc tasks), on adapted CARP instances (20 from gdb and val instances, with 21–92 required arcs) and on instances of the Open Capacitated Vehicle Routing Problem (with 50–199 customers), each and all attesting to the quality of the proposed methods.

3.3.2. CARP with Other Objectives Within the framework of ARPs with multiple vehicles, a number of researchers have focused on different objectives other than minimizing the total cost. Some studies are also concerned with combinations of diverse objectives; thus multi-objective approaches are also part of the landscape. Benavent et al. [34] present the most important studies of ARPs with min-max objectives.

An example of a different objective is the minimization of the length of the longest route, as in the Min-Max K-vehicles Windy RPP (MMK-WRPP). The problem is defined on a windy graph and aims to find a set of routes to service all tasks in the network. The objective is the minimization of the length of the longest route, as this hopefully leads to finding a set of balanced routes. Benavent et al. [35] describe a metaheuristic based on the combination of a multi-start procedure with an iterated local search. Computational experiments on a large set of instances reveal the method to be capable of producing high-quality solutions in reasonable computing times. A year later, the same authors [33] present several new facet-inducing inequalities and improve on a previously developed branch-and-cut. The first polyhedral study, by the same authors, is well surveyed in [34]. The aforementioned metaheuristic is used to obtain high-quality feasible solutions. The algorithm has been tested on the same set of instances as the first version of the branch-and-cut. Computational results are very good, underlining the contribution of the new inequalities. Later on, Benavent et al. [31] propose a branch-and-price-and-cut with the goal of developing a method able to deal with a larger number of vehicles. Computational experiments were conducted on a set of 144 instances, with 7–50 nodes, 10–184 edges, 4–78 required, with 2–6 vehicles for all instances. The branch-and-price-and-cut improved the number of optimums found, from 113 to 124 for 5 vehicles and from 101 to 121 for 6 vehicles. These tests show that the goal of dealing with a larger number of vehicles was thus achieved. The branch-and-cut proposed by Benavent et al. [33] still represents the best approach when 2, 3, or 4 vehicles are used.

In an effort to address the shape problem mentioned above, Corberán et al. [63] introduce aesthetic considerations into the MMK-WRPP. The goal is to obtain routes that are acceptable to practitioners. Several formulations for the problem are proposed, incorporating measures developed by Constantino et al. [59] to control the solution's shape quality. From those models, and based on computational tests on generated small-sized instances, a bi-objective function is selected. This outperforms functions that minimize the length of the maximum route constrained by a threshold for the shape quality measure, and it even outperforms functions that optimize the shape quality measure by imposing a tour length limit. Computational experiments on generated larger-sized instances (having the largest set 100–255 nodes, 300–620 edges, 100–400 required and 2–5 vehicles) were then conducted using both a branch-and-cut and a heuristic. The heuristic is an extension of the cluster-first procedure for the MMK-WRPP proposed by Lum et al. [135] and will be detailed in the Sectoring Problems Section (3.7.3).

Concerning Multi-Objective CARP (MO-CARP), Grandinetti et al. [97] present an optimization-based heuristic to find the non-dominated solutions. The method evidenced a competitive behavior when tested on CARP benchmark instances (*kshs*, *gdb*, *val*, *egl*), considering that three objectives are being simultaneously minimized: the total transportation cost, the longest route cost, and the number of vehicles. Other heuristics are developed and tested for the same purpose, namely: Mei et al. [145] derived a decomposition-based memetic algorithm; Shang et al. [180] a multi-population cooperative coevolutionary algorithm; Shang et al. [179] a decomposition-based memetic algorithm; and Shang et al. [177] an immune clonal algorithm based on directed evolution. The authors report test results for various sets of CARP benchmark instances, claim good achievements, and point to possibilities for future research.

3.4. *K-ARPs with Several Facilities*

In this subsection, we report the recent research conducted on multiple vehicle ARPs with multiple depots, which usually consist of intermediate facilities, refill vehicles, and multiple landfills with a limited capacity.

The case of multi-depot CARP (CARP-MD), as opposed to the single depot considered in the CARP definition, was recently studied by Krushinsky and Woensel [121]. They work on an asymmetric graph and present a two-index mixed integer LP formulation, as well as valid inequalities to strengthen the LP relaxation. A branch-and-cut is also described and tested over three different sets of instances: randomly generated (with 20–243 nodes, 32–521 arcs, all required, and 3–8 vehicles); derived from real networks (with 30–298 nodes, 106–629 arcs, 92–473 required, and 3–9 vehicles); and some derived from CARP benchmark instances (with 24–140 nodes, 68–380 arcs and 3–6 vehicles). New heuristics for CARP-MD can also be found in [114], where a hybrid genetic algorithm with perturbation is proposed, and in a paper by Liu et al. [130], who develop a genetic local

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search algorithm where the fleet is also considered to be heterogeneous. The tests were performed on a set of instances obtained from the CARP benchmark instances.

Ghiani et al. [92] present an ant colony optimization procedure for the ARP with Intermediate Facilities under capacity and length restrictions (CARP-IF). This variant of the CARP, defined on an undirected graph, also includes Intermediate Facilities (IF) where each vehicle must unload each time its capacity is reached. A tour, beginning, and ending at the depot, must respect a maximum length. The heuristic's performance is tested on instances (with 7–27 nodes and 11–92 edges) derived from medium sized CARP sets. Based on the results, the authors conclude that this approach is capable of providing substantial improvements over other known heuristics.

Willemse and Joubert work on the Mixed Capacitated ARP under time restrictions and with Intermediate Facilities (MCARP-IF), with application to household waste collection. A mixed graph is used to represent the road network, and in addition to the standard technical features found in CARP, a set of intermediate facilities is available where vehicles may unload if capacity becomes an issue. A time limit for each route is also imposed. In [202], the problem is defined and constructive heuristics, adapted from heuristics for the CARP or the MCARP, are tested to provide feasible solutions for the new problem. These solutions were designed to minimize not only the total cost, but also the fleet size. The authors highlight the observation that inconsistency in the heuristics' performance may be due to the fact that used instances (some of them adapted from CARP and MCARP instances) do not accurately mimic the application. Willemse and Joubert [203, 205] then describe and discuss new instances generated for the problem (see WJ_CARPIF in Table 9). Later, Willemse and Joubert [204] develop splitting procedures, and results on benchmark instances show that the constructive heuristic linked with the new optimal splitting algorithm performs better than the near-optimal versions.

Rodrigues and Ferreira [170] work with the MCARP in order to address scenarios with a heterogeneous fleet, multiple landfills with a limited capacity or a combination of both. They begin with a sectoring phase (which will be detailed in Section 3.7.3), and the routing phase is then executed by extending the mixed integer programming model of [96] to cope with the three above-mentioned extensions. The procedure was tested on instances derived from the literature and on real-world-based instances (see Section 4.1.3). The authors observe that integrating sectoring and routing is an effective way to deal with such applications.

The capacitated ARP with refill points and multiple loads is another extension of CARP. Two different types of vehicles are used: a small one, with a finite capacity to service demands on arcs (SV - Servicing Vehicle), and a second vehicle to refill the SV (RV - Refilling Vehicle). SVs can deliver multiple loads before having to return to the depot, while an RV has to return to the depot each time it meets an SV. The total demand of an SV path between two consecutive refill points cannot exceed its capacity. The problem lies in determining the routes for both kinds of vehicles in order to minimize the total cost. The problem is tackled in [6] in order to solve a road marking problem (see Section 4.4.4). A mixed integer programming formulation, a cutting plane algorithm and a route-first cluster-second heuristic are presented. The methods were tested on randomly generated instances. The exact method found the optimal solution for small-sized instances (with 20–70 nodes and an average number of arcs varying from 2.5 to 8.5 per node), and the heuristic behaved well when tested on a real road network in Canada with 140 nodes and 374 arcs. For the same problem, López-Santana et al. [134] propose a hybrid heuristic inspired by a scatter search procedure also involving an iterated local search and simulated annealing. Promising experimental results are reported on *val* CARP instances transformed according to a proposed procedure.

3.5. K-ARPs with Soft Time Windows and Split Deliveries

Sometimes an ideal time period exists for performing a service, and this may be relevant to its cost. One option for modeling such situations consists of charging a penalty whenever services are performed outside of the ideal time. Another issue possibly affecting cost is the possibility of split deliveries, that is, a service demand that is split by more than one vehicle, or the use of vehicles able to carry more than one commodity. These particularities are addressed in this subsection.

Capacitated arc routing with soft TW differs from CARP in that the network is mixed and a penalty is due when an arc is serviced outside its time window. The objective is to minimize a weighted sum of the number of days (equal to the number of tours) and the total time window violations. Vansteenwegen et al. [197] note that this problem arises in digital map construction (better detailed in Section 4.5.3). Their approach uses a transformation into node routing to derive a metaheuristic that combines a local search to decrease the number of tours with an iterated local search to minimize time window violations. Computational experiments were performed on a test set from the literature and on eight large real-life instances designed from Flemish bicycle networks with 93–891 nodes, 124–1,289 edges and 1–91 arcs. Afsar [1] uses a Dantzig-Wolfe decomposition and column generation approach to solve the problem to optimality. The method was tested on instances with up to 40 nodes and 69 required edges.

A related problem is the CARP with time-dependent service costs, where the service cost associated with a task, which is usually constant, is replaced by a piecewise linear function. This problem, motivated by winter gritting applications where the timing of each intervention is crucial, is addressed by Tagmoutiet al. [188]. A heuristic based on variable neighborhood descent is presented and provides fast and good results when tested over instances derived from *gdb* and *egl* CARP benchmark instances.

The split-delivery capacitated arc-routing problem is a variant of the CARP in which an edge demand can be serviced by several vehicles. Belenguer et al. [29] present a lower

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bound computed with a cutting-plane algorithm and an evolutionary local search reinforced by a multi-start procedure and a variable neighborhood descent. Computational results reported on instances derived from sets 2–4 (*gdb*, *val*, *egl*) of CARP benchmark instances attest to the quality of these methods.

Muyldermans and Pang [150] introduce a new extension of the CARP, the multi-compartment capacitated ARP, where the required edges have a demand for different products, and multi-compartment vehicles are available to co-distribute these commodities. The motivation for the study is the evaluation of savings in distribution or collection, should such vehicles be used. A local search algorithm is presented. It begins with a solution from a savings heuristic, which is successively improved by investigating a combination of local search moves. Neighbor lists and marking are used to speed up the searches, and the method is combined with a guided local search. The performance of the algorithm was tested in medium-sized instances (with one and two commodities and 77–140 nodes, 98–190 edges, 51–190 being required). The results were very good for the original instances but slightly worse in both solution quality and computation times for the transformed instances. Further experiments were conducted on a new set of generated instances (with 312 edges and 2–4 commodities) to gain more insight into the potential benefits of deploying multi-compartment vehicles. The authors conclude that there would be an improvement over separate collection strategies and that the improvement increases when: (i) the number of commodities is higher; (ii) the vehicle capacity increases; (iii) the items are less bulky; (iv) more edges have a demand for all commodities; (v) the required edge density is lower; and (vi) the depot is more centrally located on the road network.

3.6. *Stochastic and Dynamic K-ARPs*

Attention has also been given to the uncertainty involved in arc route planning. Stochastic factors are frequently present in tasks' demand, service or deadheading costs. Such issues impose considerations about the objective to be followed while looking for a set of routes. In these cases, minimization of the total average cost should probably be replaced by an objective that also promotes the construction of a set of routes that is robust to fluctuation. Laporte et al. [125] develop an adaptive large neighborhood search heuristic for the CARP with Stochastic Demands (CARPSD). The demand quantities are modeled by independent random variables with a known probability distribution. This reflects a situation in which route failure may occur whenever the effectively collected demand exceeds the vehicle capacity. Computational results over instances derived from CARP *gdb* benchmark instances are provided. The algorithm solutions' values are better than those obtained by first solving a deterministic CARP and then computing the expected cost while using random variables to describe demands.

Dynamic problems are often used to represent real situations where either data or the base network changes dynamically. Although stochastic variables could better represent such occurrences, the dynamic problems studied focus on deterministic cases. This approach is followed by Tagmouti et al. [189] in their definition of dynamic CARP with time-dependent service costs. Their aim is to get closer to real-world applications by taking updated weather reports into account while creating the routing plan. This challenge was previously addressed by the same authors [188], as mentioned in the previous subsection. For the dynamic problem, the authors propose an adaptation of the variable neighborhood descent heuristic that was developed for the static version. Computational experiments are reported over simulated instances (with 25–100 nodes and 36–162 required arcs) and reveal the heuristic to be a valuable tool to convey information updates to the system, although more research should be devoted to the time trade-off between optimization and the availability of new solutions.

In addition to weather, other types of events, such as traffic jams, traffic accidents or vehicle breakdowns, may render a planned route non-executable. Moreover, after the vehicles are on the road, sometimes new demands emerge. These situations require the ability to quickly reschedule, that is, draft another plan based on the current state of the system. In this context, Monroy-Licht et al. [149] introduce the rescheduling ARP, which considers adjustments to the initial routing itinerary when one or more vehicle failures occur during the execution stage and the original plan must be modified. They measure the disruption cost by accruing the number of edges moved to a different route in the modified schedule. Different policies considering operational and disruption costs are analyzed in the rerouting phase, and mixed integer programming formulations are presented to model these policies. A heuristic procedure is also given. Computational tests were conducted on two types of instances: those generated from some CARP benchmark instances (96 selected from *gdb*, *val*, *egl*, *eglL*, *bmcv*), and larger instances based on the real network of the Eastern Townships, Quebec (with 140 nodes and 187 edges, all required, and tried with 9–12 vehicles). They conclude that operational and disruption costs are conflicting objectives and that a good trade-off can be found by using a formulation that minimizes both.

Yazici et al. [207] address Dynamic path planning for CARP (DyCARP) with multi-robot sensor-based coverage. Disruption arises when, while executing a planned path, robots hit unexpected obstacles and require rapid re-planning. The proposed methods were tested with real instances with up to 90 nodes and 1–10 robots, in reduced computing times of under 5 seconds. A memetic algorithm with a split scheme for solving a Dynamic CARP (DyCARP) can also be found in [127]. This method was tested on examples with undirected, directed and mixed networks containing up to 100 nodes, 328 links, and 8 vehicles.

3.7. *Tactical and Strategic Extensions*

Tactical and strategic decisions are long-term decisions, that is, those that stay stable during long periods of time.

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Focusing on these long-term decisions in arc routing environments, we identify studies from 2010 to the present that involve location, periodic, and sectoring or districting problems. These problems' characteristics are summarized and the corresponding papers are listed in Table 5.

3.7.1. Location ARPs. Location ARPs (LARPs) consider both the location of facilities (e.g., depots and intermediate depots) and route planning, since separating the two problems may lead to suboptimal solutions. Usually the capacitated version is considered on a weighted, connected and undirected network with a homogenous fleet of vehicles. All vehicles may be assigned to any depot, but each begins and ends its only tour at the same depot. A fixed cost per vehicle may also be considered. The aim is to identify which depots should be opened and which clients each one should service so as to minimize the total cost. Additionally, the vehicles' capacity must be met; each client is serviced only once and no split deliveries are allowed; each vehicle performs one trip at most; and the vehicles assigned to a depot must be enough to meet clients' needs.

Not many works can be found linking location and arc routing. This much can be confirmed in the surveys of Lopes et al. [131] and Prodhon and Prins [162]. These surveys focus on state-of-the-art location routing, and their contents were categorized according to the type of methodology applied. In their conclusions, these authors suggest possible research directions for location routing, including the arc routing case.

A study by Doulabi and Seifi [79] is among the few references on LARP since 2010. These authors study a mixed capacitated LARP, thus combining depot location, fleet assignments and arc routing decisions. The potential depot locations have opening costs and are limited in number. Clients and their demands are given and must be met. The proposed methodology is based on two different integer programming models for the single and multi-depot cases, respectively, derived from one in [96]; some related relaxations; and finally, an insertion heuristic for the CARP jointly with a location-allocation heuristic, within a simulated annealing metaheuristic. The proposed relaxations are useful for providing good lower bounds. Computational results over adapted MCARP benchmark instances (*mval* and *lpr*) provide, according to the authors, "good quality solutions for multi-depot LARP in reasonable time." The paper also includes an analysis of the cost savings obtained by aggregating location with routing decisions.

Lopes et al. [132] present a model and constructive and improvement heuristics to use within different metaheuristics, namely, a tabu search, a GRASP, and a third approach that takes into consideration the advantages of the first two. New instances, derived from known CARP benchmark instances, were developed and are available online (see LPFS_LARP in Table 9), as is the related software. Lopes et al. [132] state that this new set of instances (with up to 10 potential facilities, 140 nodes and 190 required edges) seems to be representative of several different cost configurations and thus may be used to test LARP methodologies.

Although the issue is combined with a sectoring problem, Chen et al. [52] also address depot location, presenting a mathematical model, a branch-and-cut algorithm and a heuristic. Their work will be detailed in Section 3.7.3.

Riquelme-Rodriguez et al. [167] add inventory constraints to a periodic LARP in order to identify the best location for water depots, which are used by vehicles periodically spraying the roads to eliminate dust in an open-pit mine. The aim is to minimize the routing and the penalty costs incurred as a result of the lack of humidity on roads. Vehicles need to refill during their duty. The roads must be sprayed often during the given time horizon, which justifies the periodic aspect of the problem. A model adapted from one developed by Riquelme-Rodriguez et al. [166] is described, and a heuristic is also proposed. First, there is the assignment of edge services to vehicles (the allocation phase), completed in such a way that each edge is serviced by only one vehicle during the time horizon; second depots are assigned to nodes (the location phase), which is done in two different ways. Finally, the routing phase takes into account results obtained in the first two steps. Real mine instances (considering 3–5 vehicles, 21–51 nodes and 22–60 edges) from [166] were used to validate the methodology.

Huber [113] have recently proposed a bi-objective LARP. The first objective consists of the usual LARP minimization costs, that is, both the traveling and opening cost. The second objective is the minimization of the sum of lead times in servicing clients. The tradeoff between the two objectives is investigated over instances adapted from [132], with the same dimensions. A variable neighborhood search approach is designed, and results are reported and analyzed.

3.7.2. Periodic ARPs. Periodic ARPs represent a class of problems where repeated actions need to be performed while respecting some periodicity and/or along a time horizon. These are considered tactical since periodicity decisions in client servicing are usually done at a tactical level, that is, they should remain unchanged for long time periods. The aim is to construct arc routing tours to be performed during the time horizon while minimizing the total cost. Clients in the network links must be serviced with a given periodicity, or a fixed number of times during the time horizon. In each period required links are serviced once at most. A fleet of homogeneous and capacitated vehicles is available. Instance dimensions are referred by #/#/# to indicate the maximum values for nodes/edges/(number of required services during the time horizon).

Mei et al. [144] seek two objectives: the minimization of the number of vehicles and the usual minimization of the total routing cost. A memetic algorithm is developed, including a new solution representation scheme and a new crossover operator. A route-merging procedure is also embedded in the overall algorithm, yielding a positive impact on the results. Results obtained over some previously adapted arc routing instances (smaller ones, adapted from *gdb* and *val*, with 50/194/300; and real-world data set with 255/347/1,062 or 255/375/1,138) were used to successfully

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TABLE 5. Tactical and strategic extensions of ARP characteristics and related published papers.

Problem	Objective	Network $G = (N, A \cup E)$				Other characteristics	Papers
		U/D/M/W	A_R	E_R	N_R		
LARP	Min(Costs of (Tour + Opening depots + vehicles)) = (*)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Dp – potential depot locations; Depot location, fleet assignments and arc routing decisions; Potential depots – opening costs and limited in number	[52, 131, 132, 162]
		M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$ $Dp \subset N$		[79, 131, 162]
	(*); Min(sum of lead times of services)	U	$A_R = \emptyset$	$E_R \subset E$		Above LARP characteristics; Bi-objective	[113]
	Min(routing + penalty cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Time horizon and tasks are serviced more than once; Different scenarios leading to different parameter values; Penalty costs if quality of service is not guaranteed; Lower and upper limits on the number of depots; Inventory constraints	[167]
PCARP	Min(maximum K; total cost)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Multi-period time horizon; Multiple services per task; A frequency is imposed; Vehicles' capacity imposed per period; Assignment of tasks to periods could be a decision	[144, 210]
	Min(routing + penalty cost)					Including inventory constraints	[165, 166]
	Max(weight of services)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Adds irregular services; Hierarchy of task classes – defined with weights	[148]
PK-ARP-RP	Min (K; total length)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Periodic K-ARP; Refill points – used to restore vehicles' capacity	[112]
SARP	Min(weighted sum of all soft criteria)	U	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Connected sectors; Soft criteria: workload balance; small deadheading; local and global compactness	[48]
	Min(compactness)					Parity constraints on nodes; Balanced sectors within a given tolerance	[89]
	Min(total cost)	D	$A_R \subset A$	$E_R = \emptyset$	$N_R = \emptyset$	Location of depots & assignment of tasks to them; Time limit per vehicle	[52]
	Max(attractiveness)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Attractiveness – weighted function of: total routing time; # connected components; workload time imbalance	[71]
	Max(attraction) Min(cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R = \emptyset$	Balance taking into account the # inhabitants Define: tasks "repulsion" and "attraction" Sectors' evaluation: balance; compactness; connectedness 1 sector – 1 vehicle; IFs with limited capacity	[169]
	Min(length of the longest route)	U/W	$A_R = \emptyset$	$E_R \subset E$	$N_R = \emptyset$	Compactness, contiguity and geographic separation	[135]

A fleet of K homogeneous vehicles with fixed capacity is assumed.
U/D/M/W – Undirected/Directed/Mixed/Windy.

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compare the proposed methodology with some alternative methods published before 2010.

Zhang et al. [210] follow the same strategy and establish a hierarchical order for the same two objectives: to minimize first the number of vehicles and then the cost. The memetic algorithm in this work is improved by integrating a route decomposition step. The crossover operator and the local search procedure are applied. The population initialization phase is also improved to include both heuristic and random solutions. Instances of [144] were tried, and the authors conclude that this new procedure outperforms the previous ones and shows great effectiveness in solving large-sized instances.

In their work on the PARP, Monroy et al. [148] add irregular services, which they claim are well suited to applications such as road maintenance and street security surveillance. The major difference between their scenario and the basic PARP is that roads are serviced a given number of times in sub-periods along the time horizon and according to a hierarchy of link classes. A mathematical model, where binary variables represent the assignment of each street segment to a route, and a cluster-first, route-second heuristic are presented. A second phase includes the resolution of a single vehicle ARP, and the related model is also described. New instances were adapted from CARP benchmark instances (*gdb*, with 27/110/187 and *val*, with 50/194/329) and may be downloaded (see MAL_PARP in Table 9). Three new sets of instances were generated: *inso*, with 10/19/36; *insp*, with 19/44/111; and *ins*, with 50/98/243. Concerning the results, it is observed that: the largest gap from upper bound is 24% with running times of up to 32 minutes; 82% of the instances verify a gap lower than 15% and in 85% of them the CPU time is under 1 minute; and finally, the heuristic was able to find optimal solutions for four out of the eight smaller instances, the gap being inferior to 11% for the remaining instances.

Riquelme-Rodriguez et al. [165, 166] focus on the PARP with inventory constraints. Arcs in the network act as clients and consume a substance (e.g., water) over time until the vehicle arrives and replenishes the substance, thus justifying the concept of an inventory level. Each time the vehicle services the arc, the inventory level is reset, meaning that routing and inventory decisions need to be made simultaneously. As examples of applications we can find: (i) dust suppression from roads in open-pit mines or forest roads; and (ii) plant irrigation along sidewalks, where the roads of the network are periodically watered by a vehicle until a certain humidity level is reached. Water is then consumed (or evaporated) over time, while the vehicle does not again deliver water. The quantity of delivered water may either be fixed or variable, and the frequency of service is also considered within a given time horizon. A fleet of homogeneous sprayer trucks, with fixed capacity, is available at a depot. In the first paper, Riquelme-Rodriguez et al. [165] study models for both scenarios and validate them through computational results obtained with CPLEX. The first model considers the routing and the inventory problems combined. It identifies the edges to service as well as the quantity of water to be delivered at each edge. The second model suggests the routes to follow to distribute a fixed quantity of water. The authors created 10 instances specifically for the watering problem in open-pit mines, and 23 more were adapted from a set of CARP *gdb* benchmark instances.

In their ensuing work, Riquelme-Rodriguez et al. [166] investigate heuristics for the same problem. A new model is proposed considering a combination of two minimization objectives: an inventory one including a penalty for the lack of humidity, and a routing one with watering and travel costs. The heuristic, to tackle larger instances, is an adaptive large neighborhood search that dynamically uses several destroys and repair operators. Initial solutions are obtained with a cluster-first, route-second procedure. The model may solve smaller instances (with up to five trucks and 30 time periods), and the heuristic provides solutions for larger instances. For the reality-based instances, however, the heuristic generates solutions very similar to the initial one, with improvements of up to 10%. These results were obtained over adapted CARP *gdb* instances available online (see B_CARP in Table 9) and over new instances based on real open-pit mine networks with three or five trucks, 21–51 nodes and 22–60 edges.

Huang and Lin [112] apply ant colony optimization to a PK-ARP with Refill Points (PK-ARP-RP), with the inclusion of refill points generalizing the previous problems. Refill points are nodes where vehicles make a pit stop to recover their capacity. Huang and Lin [112] formulate the problem and transform it into an equivalent vehicle routing problem, making it possible to apply a known ant colony algorithm incorporating a local search procedure. The known *gdb*, *val* CARP benchmark instances (B_CARP in Table 9) are adapted, and larger ones generated from a real case study in the city of Kaohsiung, Taiwan (with 212 nodes and 321 arcs) are also applied. The authors point out that promising results were obtained in competitive computational times.

3.7.3. Sectoring Problems. Sectoring (or districting) problems are routing problems encompassing the subdivision of the networks, resulting in smaller and more easily tractable problems. Usually the road maps associated with real case studies (snow plowing, waste collection, street operations, etc.) are so huge that they cannot be used directly, and they become the base networks of such NP-hard problems. Sectoring emerges as a natural way to deal with such a difficulty. Mixing sectoring with arc routing is an extra challenge, as two NP-hard problems are involved. To try to avoid sub-optimization, it makes sense to simultaneously embrace both problems. Moreover, even when sequentially considered, sectoring must take some characteristics into account to avoid the development of routes may never be applied to a real situation at all. In fact, solutions need to be visually attractive for practitioners. Although attractiveness is a difficult concept to ascertain, we may consider the following characteristics as a measure of a solution's attractiveness: balance, compactness, connectivity or connectedness, and non-overlapping, each and all promoting fairness in assigning

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jobs to different crews. Balance usually refers to similar times in serving different sectors, or similar workloads. Compactness represents a small diameter square/round shape, which reduces unproductive travel time. Connectedness refers to the avoidance of deadheading and vehicle crossings, while non-overlapping has to do with the avoidance of crossed paths among the vehicles.

Generally, a Sectoring ARP (SARP) aims to distribute all the required network links into sectors (smaller zones). Each link is assigned to only one sector, to be serviced by one vehicle only. A fleet of homogeneous vehicles with a fixed capacity is available at a depot. Sectors must satisfy some predefined characteristics, and the cost of vehicle routes in all sectors is also considered.

SARP methodologies often resort to route cost estimates to assign the network links to the different sectors. Good ways to estimate these values are therefore highly valuable. Bostel et al. [45] develop a simple way to compute approximate values for the length of an optimum CPP on undirected or directed, strongly connected planar graphs. The approximation formulae were obtained from a multi-linear regression analysis. The authors note that even though some problems are known to be easily solvable, being able to predict their optimum value is of significant importance for large instances. Moreover, these computations are of major relevance, for example, in sectoring problems, since good estimates for these values are needed in order to balance the different sectors without first identifying the optimal solutions.

Butsch et al. [48] look for connected sectors including all required links and consider as soft criteria: workload balance related to the routing distance; small deadheading; and local and global compactness. These latter are tackled through a weighted objective. A model for the sectoring problem is developed, and a two-phase algorithm for the arc districting over an undirected network is also proposed. Firstly, an initial solution is achieved, then it is improved within a two-stage iterative procedure combining a tabu search and an adaptive randomized neighborhood search. Different strategies are applied to improve balance, deadheading time, and local and global compactness. The methodology was tested on instances generated from German road networks, which differ in dimension, the aspect ratio of the selected area, and whether they result from urban or rural areas. Instances with up to 400 edges have 4–8 districts; those with 400–600 edges have 5, 7, 8, or 10 districts; and for those with more than 600 edges, 6, 8, 10, or 12 districts are considered. The number of zones increases with the number of edges. Butsch et al. [48] conclude that results confirm the efficiency of their methodology.

García-Ayala et al. [89], also motivated by arc territory design, proposed a new formulation where new parity constraints on nodes are derived to favor the identification of Eulerian districts, that is, districts as close to Eulerian graphs as possible. Arc partitions inducing odd degree nodes are therefore penalized. The graph is considered planar and undirected, and constraints are imposed to obtain balanced districts in respect to service demand within an allowed tolerance. A branch-and-cut algorithm is developed and tested on 80 instances (with up to 401 nodes, 834 links and six depots), adapted from *lpr* benchmark instances (B_MCARP in Table 9), over which the effect of: (i) parity constraints; (ii) objective function weights; and (iii) number and location of depots are analyzed. The authors claim the model to be useful at a tactical level, as the characteristics of the obtained sectors are of interest.

Rodrigues and Ferreira [169], motivated by a waste collection application, tackled the SARP, where the balance criterion is met by the number of inhabitants. Connectedness and compactness of the sectors are also among the objectives. A two-phase approach is devised, including sectoring followed by an arc routing phase. Each sector is serviced by only one route per vehicle. Intermediate facilities are available but have a limited capacity. A new sectoring methodology is proposed, inspired by electromagnetism: required links will be assigned to a sector if there is some "attraction" between them; a "repulsion" measure is also defined for services to be assigned to different sectors. Borrowed from electromagnetism, Coulomb's law is adapted to measure "attraction" and "repulsion" when constructing sectors, which are then evaluated in terms of the considered objectives: balance, compactness, and connectedness. More sectors are designed as needed. For the routing phase, a model for the mixed CARP with limited multiple landfills (referred to in Section 3.4) is considered. Results were obtained from adapted CARP *gdb* and *mval* benchmark instances and from generated data for real base networks at Monção, Portugal (see RF_Waste in Table 9). Monção instances have 40–200 nodes, 31–165 required links, 31–124 deadheading links, and two disposal facilities. In addition to the effectiveness of its results, this method also has the advantage of allowing decision makers to define levels of "attraction" and "repulsion," thus meeting their expectation of having some control over the type of outcome.

Keeping the same motivation in a slightly different context, Cortinhal et al. [71] built two Local Search (LS) metaheuristics for the SARP. One follows a hill climbing base methodology, while the other stems from a tabu search. Both heuristics are tailored to improve the attractiveness of the solutions while maintaining the good features of the initial solution, generated with a constructive algorithm. Attractiveness is measured through a function combining three characteristics: total routing time, the number of connected components and workload time imbalance. Parameters for the tabu search were tuned via IRACE software [133]. Adapted CARP *lpr* benchmark instances (B_MCARP in Table 9), as well as instances generated from a real network in Seixal, Portugal, were used to assess performance. Seixal instances have 106 nodes, 214 links including 84 required edges and 52 required arcs, and 2–4 sectors. High imbalance values observed for the initial solutions are repaired with the LS procedures, and the tabu search seems to be a better option for providing "nicer" sectors for a real application.

Also with regard to the "nice" shape of tours, we refer to Constantino et al. [59], already mentioned in the CARP

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section (Section 3.3), who develop an approach including a set of measures for shape evaluation.

Another concern in daily maintenance operations on road networks is sector design, as pointed out by Chen et al. [52]. The decision problem involves not only the assignment of road segments to sectors, but also the location of depots. Moreover, routing operations are taken into account so as to evaluate both deadheading and the required number of vehicles. The problem, which also includes location arc routing, as mentioned in the LARP section (Section 3.7.1), is modeled and solved with a branch-and-cut algorithm. Additional constraints are set to impose a service level. This service level is measured as the maximum percentage of services allowed to exceed a given maximum distance from their assigned depot. Medium- and large-sized instances are solved with a three-phase algorithm: (i) the clustering phase, in which arcs to be serviced by a single vehicle are clustered; (ii) clusters of arcs are assigned to potential depot sites while meeting the service level imposition; (iii) localization of depots, in which the final localization of depots needed to guarantee the overall service level is decided upon. Moreover, solutions are evaluated not only regarding service cost, but also based on imbalance, overlapping and compactness. Random instances (with 12–20 nodes and 23–61 arcs) and instances based on the city of Shanghai, China (with 31 nodes and 88 arcs), were generated, and some sensitivity analysis was performed by varying the service level and the percentage of two-way roads on instances with 25–42 nodes and 50–157 arcs. The authors conclude that the heuristic provides nicer solutions than the model with regard to performance measures and in the shortest computational times. They also conclude the heuristic is able to provide high-quality solutions.

Very recently, Lum et al. [135] proposed a new way to find an arc partition of a network promoting compact and separate areas. The undirected min-max K windy RPP is considered to identify K vehicle tours, minimizing the length of the longest route. The three criteria taken into account to visually identify appealing routes are: compactness, contiguity and geographic separation (also called non-overlapping). A new metric is defined, taking all criteria into account. A cluster-first, route-second heuristic is applied to produce compact and separated routes. Results are reported for two sets of instances: one set based on real-world street networks (San Francisco, CA, USA, with 703 nodes and 840 edges; Washington, DC, USA, with 593 nodes and 663 edges; London, UK, with 855 nodes and 1,004 edges; Istanbul, Turkey, with 693 nodes and 865 edges; Perth, Australia, with 532 nodes and 592 edges; Auckland, New Zealand, with 1,209 nodes and 1,297 edges; Helsinki, Finland, with 1,310 nodes and 1,540 edges; Vienna, Austria, with 506 nodes and 586 edges; and considering 3, 5, or 10 vehicles with different depot locations) and a second set of artificially created rectangular networks (with 225–576 nodes and 420–1,104 edges; 3, 5 or 10 vehicles with different depot locations). The attractiveness of the solutions is measured with the newly devised metric and with the metrics in [59]. The constructive heuristic developed is compared with metaheuristics from [35] and was able to find routes with appealing visual qualities, with only a small increase in the min-max objective function value. Lum et al. [135] will make part of the code available in an open-source arc routing library (OAR Lib) (see LGCS_AR in Table 9).

3.8. Multiple Vehicle General Routing Problems (K-GRPs)

This section reports recent developments in GRPs with multiple vehicles (K-GRPs) that combine arc routing and node routing. A central problem of this type is the Mixed Capacitated GRP, MCGRP, sometimes also called the Node, Edge and ARP. The base network is a mixed one, and differs from the Mixed CARP in that not only do a subset of arcs and edges require service, but a subset of nodes requires service as well. Required nodes thus have an associated service cost and demand. The motivation to study this problem derives from its flexibility in capturing details from real-world routing applications.

The problems' characteristics as well as corresponding papers are summarized in Table 6.

In [24] a lower bounding procedure is developed for the MCGRP and represents a further development of the multiple cuts node duplication lower bound algorithm known for the CARP. To assess the quality of the obtained bounds, two new sets of benchmark instances were proposed, namely: one derived from some CARP instances with 11–140 nodes (4–50 required), up to 51 edges (51 required), 22–380 arcs (7–380 required); and another drawn from six real-life cases involving the design of carrier routes for home delivery of subscription newspapers and other media products in Nordic countries, with 563–1,120 nodes (120–347 required) and 815–1,450 edges (120–486 required). A third set of MGRP benchmark instances was also used, with 11–115 nodes and 29–311 links, 3–93 required nodes, up to 94 required edges and 149 arcs. All data sets are available online (see SINTEF_CGRP in Table 9) and detailed in [108], where upper bounds obtained with spider software (SINTEF_CGRP in Table 9) can also be found. These upper bounds were used to compute the gaps. Regarding computational test results, authors believe the large average gap value (25.1%) to come, in fact, from the upper bound side.

To close this gap from the upper bound perspective, Dell'Amico et al. [75] propose an adaptive iterated local search, and Bosco et al. [43] develop the first matheuristic for the MCGRP. The hybrid metaheuristic of [75] is a new iterated local search including an adaptive large neighborhood search combined with further intensification. Computational experiments show that the proposed metaheuristic is highly effective on five published benchmark sets for the MCGRP, the sets used by Bach et al. [24] and also some instances from two sets derived by Bosco et al. [44]. Good behavior of the metaheuristic is reported also on isolated CVRP and CARP instances (*gdb*, *val*, *egl*, *bmcv*). The matheuristic of [43] involves a large number of neighborhood structures and draws upon the branch-and-cut algorithm developed

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TABLE 6. K-GRPs' characteristics and related published papers.

Problem	Objective	Network $G=(N, A\cup E)$				Other characteristics	Papers
		U/D/M	A_R	E_R	N_R		
K-GRP	Min(workload balance; total time)	U/W	$A_R=\emptyset$	$E_R\subset E$	$N_R\subset N$	Turn penalties; Forbidden turns	[146]
		U	$A_R=\emptyset$	$E_R\subset E$	$N_R\subset N$	City-courier routing and scheduling; Time windows	[51]
MCGRP	Min(cost)	M	$A_R\subset A$	$E_R\subset E$	$N_R\subset N$		[24, 25, 43, 44, 75, 115]
						Turn penalties; Forbidden turns	[47]
	Min(cost; makespan)					makespan= (most – least) costly route	[137]
	Min(cost; imbalance)					4 alternatives for the imbalance	[103]
MCGRPSD	Min(cost)	M	$A_R\subset A$	$E_R\subset E$	$N_R\subset N$	Stochastic demands	[36]

A fleet of K homogeneous vehicles with fixed capacity is assumed. U/D/M/W – Undirected/Directed/Mixed/Windy.

by Bosco et al. [44] for improving the substructures of a solution obtained by considering two routes at a time. The effectiveness of the heuristic is demonstrated through an extensive computational study. Two sets of instances, with up to 10 vehicles, were generated by Bosco et al. [44]: one with 24–50 nodes (7–37 required), 25–106 arcs (13–97 required) and 12–44 edges (6–33 required); and a second with 7–22 nodes (3–15 required), 16–68 arcs (4–25 required) and 2–11 edges (1–8 required). A third set of MGRP benchmark instances was also used, with 11–140 nodes (4–50 required), 0–51 edges (0–51 required) and 22–380 arcs (7–380 required).

When discussing exact methods for MCGRP, we must mention the works of Bosco et al. [44], Irnich et al. [115] and Bach et al. [25]. Bosco et al. [44] present a three-index integer programming model, derived from the CARP, and capacity and odd cut set constraints, to be incorporated in a branch-and-cut algorithm. Computational experiments are shown on the two sets of instances here proposed for the MGRP, achieving 56 optimums out of 114 instances in one set and 45 out of 150 in the other set. A group of 34 MCARP instances was also used, and optimality was proved for 29 of them. Irnich et al. [115] develop a new mathematical model based on two-index variables and a two-phase branch-and-cut algorithm, which uses an aggregate formulation to develop an effective lower bounding procedure. This procedure also provides strong valid inequalities for the two-index model. Extensive computational experiments over benchmark instances are included. Also using this formulation, Bach et al. [25] propose a branch-and-cut-and-price algorithm. The method is compared, through extensive computational experiments, with the other two above-mentioned algorithms. Nearly all the previously mentioned MGRP benchmark instances were used. Usually, the algorithm produces strong lower bounds and, for instances with many vehicles, performs better than the algorithm of [44], which is more sensitive to the number of vehicles because it is based on a three-index model. When working with MCGRP instances derived from *mval*, the algorithm of [115] seems to be right choice.

Other K-GRPs. Bräysy et al. [47] and Micó and Soler [146] address the capacitated GRP (CGRP) with turn penalties and forbidden turns. The first authors do it on a mixed graph while the second authors consider a windy version. The work of [146] proposes a transformation of the windy CGRP with turn penalties into an asymmetric capacitated vehicle routing problem. Likewise, Bräysy et al. [47] present a transformation of the MCGRP into an asymmetric capacitated vehicle routing problem and also use it to obtain feasible solutions with a memetic algorithm for node routing. Computational results were obtained on generated sets of MCGRP with turn penalty instances, with up to 700 arcs, 160 edges and 160 nodes, or 225 required arcs, and 28 required nodes. The algorithm was able to produce feasible solutions for all 336 instances within reasonable computing times. Concerning the quality of the achieved solutions, extensive experiments were performed on node routing benchmark instances with quite impressive results.

Salazar-Aguilar et al. [173] introduce the Synchronized CGRP (SyCGRP), motivated by a real application arising in road marking operations (Section 4.4.4). Two types of vehicles are involved, those performing the service and those refilling them. Synchronized routes must therefore be found for all vehicles. An adaptive large neighborhood metaheuristic combining seven destroy/repair operators is developed and evaluated over a large set of instances (up to 400 nodes and 1,500 arcs), comparing three different replenishment policies.

In the MCGRP, the only objective usually pursued is the minimization of the total routing cost. However, in real applications, other intents are also relevant. One such objective is balanced routes, which may be achieved through minimizing the difference between the most and the least costly routes, also known as makespan minimization. Mandal et al. [137] study a bi-objective MCGRP considering the two previously mentioned objectives. They present a mathematical model and a multi-objective evolutionary algorithm designed to generate the optimal/near optimal Pareto frontier with a good spread of non-dominated solutions in a single optimization

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TABLE 7. Terminology for K vehicles' arc routing problems with profits.

	Problem	Objective				Constraints		
Initials	Name proposed	Max (profit)	Max (profit-cost)	Min (cost)	Multiple visits	Travel time limit	Min. bound collected profit	Capacity
CTOARP	Capacitated TOARP	✓				✓		✓
KP-ARP	Multiple Profitable ARP		✓			sometimes		
KP-ARPM	KP-ARP with Multiple visits		✓		✓	✓		
K-MBPARP	Multiple Minimum Bound collected Profit ARP			✓			✓	
TOARP	Team Orienteering ARP	✓				✓		

Undirected, Directed, Mixed or Windy graphs may be emphasized with the respective initial (U, D, M, W) before the name (for instance, the mixed KP-ARP is represented by MKP-ARP).

run. The proposed algorithm was tested on twenty-three standard MCGRP benchmark instances, with 11–115 nodes and 29–311 links, 3–93 required nodes, up to 94 required edges and 149 required arcs. The obtained results point to the effectiveness of the methodology.

Later on, Halvorsen-Wearea and Savelsbergh [103] stress that several concepts may be applied to the definition of balanced routes. They thus propose alternative definitions and investigate their impact. Four alternatives are tried: (i) minimize the difference between the longest and the shortest route travel distances; (ii) minimize the travel distance of the longest route; (iii) minimize the sum of the differences between the travel distances and a desired target route travel distance; and (iv) minimize the sum of the differences between the travel distances and the average travel distance. Following tests on MCGRP benchmark instances (with 8–12 nodes (3–8 required), 18–34 arcs (4–11 required), 2–5 edges (1–3 required) and 3–5 vehicles), Halvorsen-Wearea and Savelsbergh [103] conclude that the choice among these alternatives has significant effects on the Pareto front and that (i) seems to be the most robust alternative.

The MCGRP with stochastic demands is studied in [36]. A chance-constrained integer programming formulation of the problem is presented, that is, an integer formulation where instead of a constraint, which sets a vehicle capacity that cannot be exceeded, a threshold for the probability of respecting capacities is imposed. An equivalent deterministic formulation is also derived. A branch-and-cut and a heuristic are developed, the first providing the optimal solution to small-sized instances. The effectiveness of the methods is assessed on a probabilistically constrained version of the benchmark instances for the MCGRP, with 7–27 nodes (3–11 required), 32–84 arcs (6–29 required), 4–13 edges (1–9 required) and 4–17 vehicles.

3.9. Multiple Vehicle Arc Routing Problems with Profits

As with the single vehicle case, we devote a section to recent studies on multiple vehicle ARPs with profits. In so doing, we follow the suggestion of [17] in their survey of single and multiple ARPs with profits, since current terminology is far from consensual and often the same problem is referred to by different names. We present the terminology in Table 7. On one hand, problems that maximize gross profit are defined as Team Orienteering ARPs (TOARP). On the other hand, we call Multiple Profitable ARPs (KP-ARPs) problems maximizing net profit, that is, the difference between the collected profit and the travel cost, and sometimes also including a time limit constraint for route duration. Lastly, Multiple Minimum Bound collected Profit ARPs (K-MBPARP) are problems that minimize cost with a requirement for a minimum profit threshold.

The works' characteristics, as well as corresponding papers, are summarized in Table 8.

3.9.1. Team Orienteering Arc Routing (TOARP). The recent increase in the number of papers on orienteering ARPs can be noticed just by comparing the references in the survey by Vansteenwegen et al. [196] with those in a survey 5 years later [99].

Archetti et al. [13] define a team orienteering ARP on an undirected graph under the designation of undirected CARP with profits. A profit and a demand are associated with each profitable edge, and a travel time is assigned to each edge on the graph. A fleet of capacitated vehicles is available to service the profitable edges, and a maximum duration for a route is also imposed. The profit from an edge is collected only once and by the same vehicle that services its demand. The objective of this problem is to find a set of routes, starting and ending at the depot, that satisfy the constraints on route duration and vehicle capacity, further maximizing the collected profit. In terms of exact algorithms, a branch-and-price algorithm is presented in [13]. The method was tested on two groups of 102 instances, each derived from *val*. Both groups have 24–50 nodes, 34–97 profitable edges, and 2–4 vehicles, but differ in their capacity and maximum duration of a tour. For the group of instances that do not allow long routes (that is, with low-capacity vehicles and shorter time limit), instances with up to 97 profitable edges can be optimally solved. A number of metaheuristics can be chosen to find feasible solutions to the problem: Archetti et al. [13] develop a variable neighborhood search and two tabu search methods (one enlarges the solution space, allowing infeasible solutions), and Cura [72] proposes a bee colony approach. These

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TABLE 8. Multiple vehicle problems with profits: characteristics and related published papers.

Problem	Objective	Network $G=(N, A \cup E)$				Other characteristics	Papers
		U/D/M/W	A_R	E_R	N_R		
KP-ARP	Max(profit-cost)	U	$A_R=\emptyset$	$E_R \subset E$	$N_R \subset N$	Time limit constraint; Clients on nodes and links	[10]
	Max(profit); Min(cost)	U	$A_R=\emptyset$	$E_R \subset E$	$N_R=\emptyset$		[209]
	Max(profit-cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R=\emptyset$	Time limit constraint	[32]
	Max(profit from coalition)	D	$A_R \subset A$	$E_R=\emptyset$	$N_R=\emptyset$	Collaboration ARP; Mandatory tasks and tasks to be dealt with in collaboration to improve profit	[86]
KP-ARPM	Max(profit-cost)	D	$A_R \subset A$	$E_R=\emptyset$	$N_R=\emptyset$	Profit collected more than once but upper limited	[84]
	Max(weight)	D	$A_R=A$	$E_R=\emptyset$	$N_R=\emptyset$	Fixed or unfixed depots; Lower and upper limits for traversing links; Time limit per tour	[176]
K-MBPARP	Min(cost)	M	$A_R \subset A$	$E_R \subset E$	$N_R=\emptyset$	Profit lower bound	[32]
TOARP	Max(profit)	U	$A_R=\emptyset$	$E_R \subset E$	$N_R=\emptyset$	Profit collected only once; Mandatory and optional tasks; Time limit per tour	[13, 72]
		D	$A_R \subset A$	$E_R=\emptyset$	$N_R=\emptyset$		[12, 18, 164]
		M	$A_R \subset A$	$E_R \subset E$	$N_R=\emptyset$		[32]
		U/W	$A_R=\emptyset$	$E_R \subset E$	$N_R \subset N$	Clients on links and nodes; profit collected once	[91]

A fleet of K homogeneous vehicles with fixed capacity is assumed.
U/D/M/W – Undirected/Directed/Mixed/Windy.

algorithms were tested on the above-mentioned instances. The VNS proved to be more effective when compared with the tabu search (with an average error below 1% compared to the best solution values). In [72], results show solutions of equal quality with short CPU times.

A TOARP is also defined in [18], this time on a directed graph and considering that a profitable arc may be either required or optional, that is, service to some arcs is mandatory, while other arcs are only serviced if they are worth it. Archetti et al. [18] propose a formulation and study a relaxation of its associated polyhedron. A branch-and-cut is presented, embedding families of valid inequalities and facet-inducing inequalities. Riera-Ledesma and Salazar-González [164] offer exact algorithms to solve the problem, after transforming it into a node routing problem, namely: two column generation approaches, a branch-and-price and a branch-and-price-and-cut. A matheuristic can be found in [12], where the algorithm successfully combines the solution of integer models with a tabu search and a diversification phase allowing deep exploration of the solution space. All three of these papers, [18], [164] and [12], offer computational experiments on the same set of instances derived from undirected RPP benchmark instances (with 11–100 nodes, 42–846 arcs (0–64 mandatory, 1–121 optional), and 2–4 vehicles) in [18]. Riera-Ledesma and Salazar-González [164] report a certain complementarity between branch-and-cut and column generation approaches, and point out that in instances with a tight time limit, constraint column generation works better.

Where mixed graphs are concerned, a valid and an aggregated model for TOARP can be found in [32]. Both models are flow-based compact ones and were tested on the two sets of above-mentioned instances adapted by [13] and [18]. On average, the gap values of [13] and [18], although very similar, are slightly better.

A multiple vehicle GRP with profits and TW is presented by Gavalas et al. [91]. The problem is defined on a windy multigraph. It was motivated by route planning to find multiple-day itineraries for tourists who are visiting a destination featuring several points of interest and scenic views. The objective is to plan a specific number of walks spanning a subset of nodes and edges of the graph so as to maximize the overall collected profit; some nodes represent points of interest and some edges are scenic street segments. Included nodes and edges should be visited within their respective TW, and the overall duration of each walk should be below a certain threshold. To tackle this problem, a formulation, a preprocessing procedure, an iterated local search and a simulated annealing are proposed. The methodology was tested on instances created by the authors based on data related to the city of Athens (with 249 nodes, 113 points of interest, 18 scenic routes and 100 hotels, which are the starting and ending locations of daily tourist walks). The iterated local search method outperforms the simulated annealing in the quality of solutions produced, although it is usually more time-consuming. Even when CPU time is equal for both algorithms, the iterated local search still provides a better solution value.

3.9.2. Multiple Profitable Arc Routing (KP-ARP). The Multiple Profitable ARP (KP-ARP) is the generalization of the P-ARP (Section 2.4.2) for multiple vehicles that each perform a somehow limited route. The objective is to identify K vehicle tours within the crews' time limit, starting and ending at the depot and maximizing net profit.

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Considering also that the profitable links may be either mandatory or optional, Benavent et al. [32] present compact flow-based models for KP-ARP on a mixed graph. The models were tested on a set of 204 instances (with 24–50 nodes, 25–106 arcs, 12–44 edges and 4–12 vehicles) that includes different proportions of mandatory/optional links and capacities. A valid model achieves optima a few times (24); however, an aggregated model provides good lower bounds, and linear relaxation values show an average gap of 5%.

Recently, Archetti et al. [10] study a profitable GRP. In addition to the set of profitable edges, there is also a set of profitable nodes with a profit that can be collected only once and by its serving vehicle. The objective function is the maximization of the net profit, that is, the difference between the total collected profit and the total travel cost. A mathematical formulation and valid inequalities to strengthen the linear relaxation are provided. An aggregated formulation is also proposed, allowing for the introduction of further inequalities. A two-phase exact algorithm is then presented. This consists of, firstly, a branch-and-cut involving the aggregated formulation and the identification of valid inequalities; secondly, the use of these inequalities on a branch-and-cut that hopefully ends with an optimal solution. The algorithm's good performance is shown by computational results on two groups of instances adapted from *gdb* and *val*: one with 7–22 nodes (3–15 required), 10–48 edges (7–36 required) and 5–50 vehicles; and the other with 24–50 nodes (13–33 required), 34–92 edges (27–73 required) and 2 or 3 vehicles.

Zachariadis and Kiranoudis [209] present a local search metaheuristic for a bi-objective problem. Instead of considering net profit, they maximize gross profit and minimize travel time. Instances generated in [13] were used to validate the methodology.

3.9.3. Multiple Minimum Bound Collected Profit (K-MBPARP).

The Multiple Minimum Bound collected Profit ARP (K-MBPARP) on a mixed graph, under the designation of multi-vehicle prize-collecting mixed capacitated ARP, is presented in [32]. The aim is to find a minimum cost set of routes, beginning and ending at the depot, collecting all the required links, and choosing optional links so that the total collected profit meets an imposed minimum limit, while satisfying the capacity of the vehicles. One valid and one aggregated compact flow-based formulation were presented and tested in a large set of 1,942 instances, adapted from MGRP instances (with 2–4 vehicles, 116–428 nodes, 13–789 arcs and up to 285 edges). For the instances with two vehicles, the valid model obtained 612 optimums out of 972. As expected, as the number of vehicles increases, the results deteriorate, particularly for instances with a larger imposed profit.

3.9.4. Multiple P-ARP with Multiple Services (KP-ARPM).

In this subsection we address profitable problems with multiple vehicles where profit associated with a link may be collected more than once.

Euchi and Chabchoub [84] study the problem on a complete directed graph, where there is a maximum number of times the profit of each arc can be collected. They present two metaheuristics, both based on an adaptive memory procedure but combined with two different methods: a tabu search and a variable neighborhood search. A 2-opt procedure is also implemented. Computational tests on instances with 5–140 nodes show that the best option consists of using the adaptive memory procedure combined with the variable neighborhood search and the 2-opt procedure. They report small gaps, of up to 1%, over the best known solution values.

Also assuming a task can be serviced more than once, Shafahi and Haghani [176] define a problem they refer to as the Generalized Maximum Benefit Multiple CPP. This work studies alternatives to the requirement that all routes must begin and end at the depot. A mixed integer LP model is proposed, and a set of constraints is changed to accommodate each alternative. The problem has its application in security patrolling, and an illustrative example of the University of Maryland College Park campus network (with 12 nodes and 18 links) was used to compare the alternatives.

3.9.5. Other Multiple ARPs with Profits.

Fernández et al. [86] introduce the collaboration uncapacitated ARP. A set of carriers is considered, each associated with a depot and a tour. There is also a set of arc tasks, each assigned to a carrier. Some tasks are mandatory, while others may be dealt with by a collaboration of carriers to achieve the objective of maximizing the coalition of carriers' profit. An integer programming and a branch-and-cut are presented. The authors conclude that the total gain is always greater if cooperation is allowed; however, carriers may significantly decrease their individual profit. Another version is therefore proposed in which carriers may set thresholds on profit in the collaborative scheme so that the overall profit is allocated in a more balanced manner. A set of benchmark instances were generated (with 16–102 nodes), and results of extensive computational experiments are presented and analyzed. Authors also analyze the problem from a game theory perspective. Other works combining ARPs and game theory can be found in Hochbaum et al. [109], who address security routing games with a multivehicle CPP, in [98] with the *k*-centrum Chinese postman delivery problem and a related cost allocation game, or in [156] on games arising from multi-depot CPPs.

4. ARC ROUTING APPLICATIONS

Arc routing applications embrace several complicated real-world systems in which routing activities are to be performed along the links of a network. This section is devoted to the study of such applications from 2010 onwards, surveying papers on distribution and collection operations, outdoor activities, post-disaster operations and street cleaning and marking, among others. Each subsection begins by outlining the specific applications, while the last subsection groups together a few other, less researched applications.

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4.1. *Distribution and Collection Routes*

Routing for delivery or collection activities is one of the most studied applications of ARPs. Very complex logistics are usually associated with these systems, with the routing phase being very important not only because of its high cost, but also because of the substantial environmental impact. This section surveys some real applications, namely: money collection, postal and newspaper delivery, and waste collection.

4.1.1. Money Collection. Money collection gives rise to some challenging new issues, since in order to prevent robberies, planned routes should be as dissimilar as possible while still minimizing the travel cost. To address this problem, Constantino et al. [60] define the Dissimilar ARP (D-ARP), where for one vehicle and during a time horizon, the objective is to collect all the tasks of a network every day, making its tours on two different days as dissimilar as possible while minimizing the total cost for the time horizon. The dissimilarity between two routes is defined. Authors propose a mixed integer LP model and a matheuristic crafted for the resolution of compact models. The strategy was tested over proposed instances for the problem.

4.1.2. Postal and Newspaper Delivery. Although we may argue that delivery of print media is becoming less relevant nowadays, there is still some space for research in this area, mainly in the field of postal delivery. In fact, everyday postal distribution is still a necessary activity that is closely linked to ARP applications. A study by Chang and Yen [51] appears to be one of the few post-2010 papers invoking a city courier routing and scheduling problem to reduce operating costs and improve service levels. The network is divided into two levels based on whether customers are inside or outside the central business districts (CBDs), with customers represented by links or by nodes, respectively. The problem is thus considered as a generalization of a capacitated GRP, since clients' time-windows need to be met and two objectives are established: balancing workload among different couriers and minimization of total tour time. The problem is transformed into a multi-objective multiple traveling salesman problem with strict TW, and a scatter search procedure is developed. Real data from Maple Logistics, a city courier company in Taipei, Taiwan, is used. As a conclusion, the authors write: "the owner of Maple Logistics appreciates the results with, however, one concern. That is, the daily route plan could become quite different and may thus reduce his couriers' working efficiency due to unfamiliar surroundings."

Corberán et al. [62] suggest mail delivery in certain countries (Germany, for instance) as a potential application for the windy clustered prize-collecting ARP (2.4.4). As privatization of postal services is a growing trend, companies may potentially expand services to different districts.

The recent book chapter by Hasle [107] links ARPs to newspaper delivery, detailing the inherent logistics, challenges, and the historical evolution of this field. It is noted that the mixed capacitated GRP, including both required links and nodes, is usually the most adequate model for such problems.

4.1.3. Waste Collection. Waste collection routes on a door-to-door system are usually better handled through an ARP. Surveys on waste systems (WS), including routing collection, are conducted by Beliën et al. [30], Ghiani et al. [94] and Han and Ponce-Cueto [106], who also distinguish between collection systems based on whether they benefit from a node or an arc routing approach. Several specific cases of real-world WS demand generalizations concerning modeling and solution methodologies. In developing these generalizations, researchers identify factors that affect routing within these complicated systems, including vehicle types and number; location, number and types of disposal facilities; number and types of waste containers; network type; etc.

Martins et al. [141] and Ghiani et al. [94] study mixed CARP compact models and matheuristics for a waste collection problem in Seixal, a municipality near Lisbon, Portugal. Capacity constraints are replaced by time limit constraints. Martins et al. [141] pursue a joint objective of minimizing total collection time (including service and deadheading times) and minimizing the largest time difference among the total collection times of any two tours (with one tour per vehicle). This time difference is used to better balance tours in terms of total collection times. The proposed matheuristic begins with the resolution of an aggregated relaxed model to identify the required links to be used as seeds within a heuristic where some services are fixed for different vehicle tours. Some required links are then assigned to a vehicle service and linked with the vehicles' seed. Once the problem has been reduced, the valid model is applied to identify feasible solutions. Real-world-based instances are generated from the Seixal network, with dimensions varying from 106 to 257 nodes and 143 to 439 links. Although tested on a relatively small set of instances, results point to good solutions for identifying a set of balanced tours. Collection times increase as imbalance decreases, as expected. Ghiani et al. [94] model imbalance between tours by looking into the total collected demand. The objective function includes only the minimization of total deadheading. The matheuristic considered here, although similar to that of [141], differs in its selection process, where service links are fixed to vehicle tours. It also begins with the resolution of a slightly different aggregated model and ends with the resolution of the new valid model for a smaller instance, as some services have become fixed in the meantime. Larger reality-based instances were successfully tested in terms of their solution characteristics and specifically in terms of imbalance.

A waste collection study in Monção, a region in the north of Portugal, was suggested by Rodrigues and Ferreira [170] (and previously mentioned in Sections 3.4 and 3.7.3). This scenario involves a heterogeneous vehicle fleet, multiple landfills, different types of waste, and a mix of rural and urban areas. Three formulations are proposed based on the mixed CARP (MCARP) of [96]. The first one generalizes the MCARP model to include a heterogeneous fleet; the

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second considers multiple landfills; and the third joins the first two, that is, extends the MCARP model to include both the heterogeneous fleet and multiple landfills. A new districting methodology is first applied to get smaller instances (Section 3.7.3). Formulations were solved with CPLEX to obtain solutions over three sets of instances available online (see RF_Waste in Table 9): the first two were adapted from some benchmark instances; the third represents real data from the Monção case study. This region is about 220km^2 and has 20,000 inhabitants. Judging from gap values obtained with CPLEX, the models produce good solutions, always providing the optimum in scenarios with up to five tours, which, according to the authors, fits the real situation.

The recyclable waste collection system in the municipality of Morón, a suburb of Buenos Aires, Argentina, with about 320,000 inhabitants in an area of 55.6km^2, was studied by Braier et al. [46] as a generalized directed RPP. Traffic regulations, for example, forbidden left and U-turns at signal-controlled intersections, are also considered. The municipality comprises five administrative districts, each divided into seven sectors, one per vehicle. Refuse is collected once a week in all districts and throughout a district on the same day. By considering the minimization of the travel distance as the objective, the authors claim they also obtain "lower fuel consumption, reduced emissions, reduced collecting times and less traffic jams," which seem to be credible side effects. As previously mentioned in our discussion of the generalized problem (Section 2.2.4), at least one link from each pre-specified group of links needs to be serviced. An expanded graph is applied to represent traffic regulations, and open tours are considered. An integer programming model is provided and feasible solutions identified through a simple approach, using a model relaxation with no subtour elimination constraints. A procedure to generate feasible tours in short computational times is then applied to real data networks. Although this procedure is fit for the real-case study, it can easily be applied to solve general instances. The generated routes, which are slightly better than the previous routes manually defined by drivers, cover more streets.

Whenever private companies are involved, garbage collection can be viewed from a profitability perspective. Corberán et al. [62] refer to garbage collection as a potential application for the windy clustered prize-collecting arc-routing problem (Section 2.4.4). Their paper gives the example of "cities in Minnesota (see the Web page of the Minnesota House of Representatives) or in Buenos Aires, where garbage collection companies put in bids to municipalities for parts of cities so that service is provided by different companies depending on the areas."

Very recently, and as the outcome of a waste collection project, Kiilerich and Wøhlk [118] suggest new CARP variants, propose formulations and list several related real-world instances. The new CARP variants, which authors claim to better fit reality, include: three cases of multi-compartment CARPs, one of which also considers a time horizon (No-split multi-compartment CARP, Commodity-split multi-compartment CARP, Multi-Day Commodity-split multi-compartment CARP); vehicle coordination (coordinated CARP), that is, the coordination of different vehicles to collect different types of waste in a neighborhood on the same set of days; and different service frequencies dependent on need, that is, customers in each link have different frequency needs (Semi-Periodic CARP). New large-scale reality-based instances with thousands of nodes are proposed (see W_CARPs in Table 9).

4.2. *Outdoor Activities*

Cycling is very popular in countries like Belgium or the Netherlands, and it has become an "important source of income for local economy of East Flanders" [185]. Traversing scenic routes offers cyclists a mixture of sport and leisure. Nowadays, these regions provide online applications for cyclists to design their own trips, using the so-called cycling networks. As state by Souffriau et al. [185], "The East Flanders' network is a concatenation of five regions and is composed of 989 nodes and 2,963 arcs, with a total of 3,585km." This software, developed by Souffriau et al. [185], generates cycling tours on demand, with parameters such as length, difficulty, area, budget limit, etc. The problem is tackled as an arc orienteering problem (OARP) on a directed graph, as the objective is to maximize the total collected score (each arc has an associated score) within the defined budget or length constraint (see Section 2.4.1). The length constraint does not represent a hard imposition, given that cyclists usually do not mind pedaling a bit more. A greedy randomized adaptive search procedure (GRASP) is developed to generate the route, and a procedure is applied to try to escape from local optima. Solutions provided by the GRASP heuristic are compared with those produced by CPLEX for a proposed model, over real-world data from the East Flanders cycle network. Results suggest good performance for the GRASP methodology, which was able to provide good solutions in one second CPU time. As the authors state, "The approach was implemented in two applications: an on-line cycle route planning application offering personalized cycle routes based on user preferences and an SMS service providing cyclists 'in the field' with routes on demand."

In a more recent paper, Verbeeck et al. [198] propose a new mathematical model and two methods to generate solutions for this same route planning problem. In this paper, a cyclist may start the tour at a point selected from a set of possible starting points, while in [185] the starting point is first chosen and fixed. There are several other differences in [198]: a node can be visited more than once, a score is associated with each arc regardless of its length, and a lower bound is imposed on the tour's length. One of the proposed methods is a branch-and-cut approach able to solve small-sized instances only. For larger instances, a metaheuristic is devised and is capable of producing good-quality solutions in a few seconds. Results were assessed using instances from [185], including the reality-based ones, and some OARP instances were adapted. New instances were also generated for the cycle trip planning problem. This methodology may be incorporated in

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a web-based cycle route planner like that of [185]. The user can specify a maximum distance between her/his home and a starting point and select some route preferences, and the best starting point and the best route are automatically tailored in just a few seconds. The route may then be stored to a personal device and sent to the user by SMS.

The company Route You envisaged the design of an interactive web application to suggest tours for outdoor activities, such as mountain biking. The underlying problem, tackled by Maervoet et al. [136], is a GRP with profits, as both arcs and nodes have an associated attractiveness. Some nodes represent the so-called POIs (point of interest), and both the starting/ending point and tour length are defined by the user. The aim is to find a tour that maximizes attractiveness within a target path length and tolerance. The tour must match user preferences, including some typified POIs, and may be open or closed. A constructive heuristic approach is developed to suggest multiple spatially different tours in a short computational time.

The identification of scenic tours may also represent an ARP with profits application, as is suggested by, for instance, [91], and is better detailed in Section 3.9.

4.3. Post-Disaster Operations

Berktaş et al. [37] relate unsettling figures concerning disasters: between 1990 and 2012, as many as 340 disasters occurred per year, involving a yearly average of 240 million victims. Figures like this make the study of post-disaster operations even more relevant.

Post-disaster actions usually involve several operations research problems. Obviously, these problems require the shortest possible decision-making process, time being a crucial factor. Debris removal, in order to quickly restore connections and channel the distribution of humanitarian aid, lies within the scope of routing applications, and it is also part of the newest application areas for arc routing. Çelik [50] wrote a survey paper addressing several problems involving network restoration and humanitarian operations after catastrophic events, such as earthquakes, storms or other natural disasters or attacks with large affected areas. As the author states, "the costly and complicated nature of these activities has led to an increased level of interest concerning this field in the OR/MS literature over the recent years." In addition to summarizing extant work, the survey suggests "potential directions for future research by pointing to the gaps between the needs in the field and the existing body of literature."

Among the several objective functions that may be considered, we find: time to complete the repair operations; utility functions related to demand satisfaction or penalties paid for unsatisfied demands; measures of accessibility; and the flow of people or commodities that can be supported by the repaired network. Some equity-based objectives are often also considered in order for humanitarian aid to be evenly distributed. Problems are classified into different categories, including: (i) "road restoration and rehabilitation (...) involves either restoration of the transportation infrastructure into its pre-event state, or the improvement or strengthening of the network so that increased flow of people can be handled"; (ii) snow removal problems emerge "in the aftermath of large scale snowfall or avalanches" (here addressed in Section 4.4); (iii) debris clearance, which "consists of opening up debris-covered roads by pushing the debris to road sides"; and (iv) debris removal, where the aim is to "collect, transport, and dispose or recycle the debris in special temporary facilities." Çelik [50] summarizes these studies in tables, classifying them by problem types, solution methods, and objectives as well as involved decisions.

The Arc Routing for Connectivity Problem (ARCP) links ARPs with network design and appears in relation to post-disaster operations. Asaly and Salman [19] introduce this problem to work on the connectivity of road networks in the immediate response stage. First steps involve the evaluation of road conditions as well as the estimation of the time needed to clear/open them. Only those blocked roads that may be unblocked within a short period of time are considered, and the network is then constructed by adding other roads that were not affected by the disaster (i.e., that are initially unblocked) and that will enable connectivity. All blocked arcs must be cleared to obtain a strongly connected network. The ARCP is then defined on directed and strongly connected networks (including blocked and unblocked arcs). Since traversal speed depends on the road damage, arc costs are computed from estimated traversal times and are not proportional to distances. Each blocked arc has two associated deadheading costs, one before and one after it is unblocked. The unblocked (unblocking service) cost is also considered. An emergency response fleet may include several types of machines (e.g., drainage dump, satellite communication vehicles, lighting vehicles, etc.) moving together as a single vehicle, located at an emergency response facility. The ARCP aims to identify a minimum cost tour, including the depot, traversing all the blocked arcs at least once. Asaly and Salman [19] formulate the ARCP, prove it to be NP-hard, and evaluate the size of instances solvable by CPLEX in short computational times, over instances generated from the Istanbul (Turkey) road network. On the same base network, ten scenarios including different sets of blocked roads are generated, and two levels of damage lead to 20 instances. The authors conclude that more instances should be used to validate the methodology, and in the case of larger damage networks that demand several vehicles, a partitioning step may first be applied to assign each vehicle a zone unblock.

Debris clearance in an arc routing context is also addressed in references [3, 4, 37, 117, 153, 171, 199], as detailed below.

Kasaei and Salman [117] work on two ARPs. One coincides with the ARCP while the other pursues the maximization of the total benefit gained by reconnecting the network. This second problem is called the Prize Collecting Arc Routing for Connectivity Problem (PC-ARCP). We emphasize that this name is coherent with the terminology adopted in Table 3, because although the problem is similar to an OARP with no time limit constraint, it differs in the connectivity restoration aspect. Here, the time

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limit is the minimum time needed to restore network connectivity. Two mixed integer programming models and two versions of a variable neighborhood search matheuristic are presented and are capable of providing feasible solutions in short computational times. Computational experiments were conducted through adapted RPP instances (with 41–465 nodes and 98–1,055 edges) and real-life data based on the Istanbul city road network (with 349 nodes and 647 links) within several disaster scenarios. The matheuristic values are compared against those obtained by solving the models with CPLEX solver within a fixed time limit. Small gap values and CPU times allow Kasaei and Salman [117] to conclude that the heuristics produce high-quality solutions.

Akbari and Salman [4] generalize the second problem of [117], which is the maximization of the prize in reconnecting the damaged network (PC-ARCP), by including a time limit constraint. Here, the prize gained from reconnecting a component is derived from the number of people it benefits. That prize may also be weighted to prioritize the components, that is, higher weights assigned to airports, schools, hospitals, etc. In this problem, referred to as KPC-ARCP, several vehicles are considered, and the authors include unblocking link decisions originating from their preliminary work in [5]. A MIP model and matheuristics are proposed. The matheuristics involve a relaxed model, a Lagrangian relaxation and the resolution of single vehicle problems in order to obtain lower and upper bounds. The instances used align with those in [3], and very good results are reported.

Akbari and Salman [3] focus on debris removal and on the need to identify a synchronized work schedule for road clearing teams when reconnecting the damaged network. Each team is assigned to a different vehicle, giving rise to the multiple ARCP (K-ARCP). Multiple depots are also considered, since vehicles may be located at different points. No blocked link may be traversed before having been cleaned, and thus node arrival times are also an important part of the solution. A MIP model and a matheuristic, mixing solutions from a relaxed formulation with feasibility and local search procedures, are developed. Three sets of instances were generated from the Istanbul network (with up to 349 nodes, 689 edges, 6 connected components and 4 vehicles), and randomly generated instances with Euclidean distances were also used to validate the heuristic's performance. Very good results are reported and commented on.

Sahin et al. [171] represent the first areas to be serviced, such as schools, hospitals, etc., as critical nodes, and they consider blocked roads connected to those areas as links needing service, thereby adapting a GRP. The objective is the minimization of the total effort to visit all critical nodes, measured by both travel and debris removal times. The proposed model identifies an order in which to visit the critical nodes, the route including these nodes, and the blocked roads to be cleaned. As the model fails to find solutions quickly, a heuristic method beginning with a constructive procedure and followed by an improvement procedure was also derived. The methodology was tested on instances generated from Istanbul's Kartal municipality (with 45 nodes, including 7 critical), with different sets and number of blocked roads. The heuristic gap values are considered to be quite good.

Berktaş et al. [37] addressed this same problem through two models, considering two different objectives. The first model aims to minimize the total time to reach all critical nodes, while the second minimizes the total weighted sum of visiting times, with weights being used to prioritize critical nodes. New models, with a smaller number of variables than those of [171], were provided and produced better results on the Kartal municipality instances. New data from the Bakirköy district in Istanbul (with 73 nodes, 15 of them critical) was also tested, and feasible solutions were obtained via a new heuristic procedure that includes the resolution of a simpler MIP model.

Ozdamar and Aksu [153] develop a constructive heuristic to identify the routes for a limited number of bulldozers to unblock roads within time-dependent travel times based on road conditions. A cumulative network accessibility is used to account for dynamic road conditions. Two objectives are considered: minimization of the unblocking times, that is, the maximization of accessibility; and minimization of the maximum unblocking time. Road networks based on the maps of two districts in Istanbul (Caddebostan and Fatih) were used to generate post-disaster scenarios. Several rules were applied to generate different feasible routes, even for large networks. The authors claim these methods "are robust under damage assessment uncertainty."

Walliser et al. [199] define an open hierarchical routing problem to deal with debris removal. The road network is divided into priority sets that must be serviced in a hierarchical order. Since the planning period usually extends throughout multiple days (all those needed to restore the road network), the aim is to identify minimum cost vehicle routes departing from a depot and servicing all required (blocked) links with no need to return to the depot. Personnel replacement and refueling is performed during the operation and leads to a time delay. Servicing a link consists of pushing debris to the roadsides without impact on vehicle capacity. Moreover, each link can be traversed more than once, serviced more than once, and serviced by different vehicles at the same time, if necessary to remove the accumulated debris. A blocked road may only be traversed (deadheaded) after being cleaned. The problem is modeled, and a constructive, rule-driven heuristic is proposed. Two beam search processes are also developed. These are modified branch-and-bounds, where the number of available nodes at each level of the branch tree is limited by a beam-width constraint that keeps elite nodes and discards others. A network with 808 nodes and 1,126 links was derived from Geographic Information System data for the Fort Stewart military installation in Georgia (USA), and different crew sizes were considered to compare the different heuristics proposed.

After debris is moved to the roadsides in the unblocking phase, the collection phase commences. Collection is a huge operation that is extremely expensive and may last for several months or even years. Pramudita et al. [159] and

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Pramudita and Taniguchi [158] study debris collection and transportation as an augmented new undirected CARP, where new constraints, representing accessibility between different areas, are added. Two types of required arcs may be considered: blocked roads, from which debris needs to be removed and transported; and cleared roads, from which the remaining debris still needs to be transported. The objective is to service all required arcs at minimum cost while including only feasible vehicle tours. As usual, blocked arcs can be traversed only after being serviced; therefore service sequence needs to be taken into account. Intermediate depots where vehicles may unload are also considered and vehicle capacity is imposed. In both studies, the CARP is transformed into a CVRP and feasible solutions are found with a tabu search for the CVRP. The two papers differ in the set of instances tested. Pramudita et al. [159] used small-sized instances to check the methodology. Pramudita and Taniguchi [158] report results on VRP benchmark instances and on data derived from real networks in the Tokyo metropolitan area. The authors conclude that their methodologies allow the resolution of large-scale instances.

4.4. *Road Cleaning and Marking*

Street cleaning and marking are common arc routing applications, as vehicles drive along streets to perform their services. Several additional and complicating constraints may be needed, as in snow plowing or debris removal operation, which demand hierarchical services. This section surveys post-2010 papers dealing with winter road cleaning (snow plowing or salt spreading operations), street cleaning (sweeping), and street maintenance operations.

4.4.1. Road Cleaning. Blazquez et al. [39] deal with a street sweeping routing problem by identifying an RPP in a special graph. Each street requiring cleaning needs to be serviced as many times as the number of its sides, since a vehicle can sweep only one side at a time. For instance, a street with a central divider or median needs four vehicle passages. Moreover, the vehicle tours must comply with traffic flow and turn constraints. The problem is transformed into an equivalent vehicle routing problem, and known methodology is applied to generate feasible solutions for the northeast area of the municipality of Santiago, Chile. The authors report savings of about 37% in the distance traveled.

4.4.2. Road Cleaning in Winter. In areas with severe winters where it often snows or where roadway icing often occurs, a common problem is the identification of the minimum cost (distance, time) tour(s) to be assigned to plow vehicle(s). As with snow plowing, we may also find several applications dealing with deicing vehicles that spread chemicals and abrasives on the streets. Deicing needs to be performed on a regular basis during bad weather and with special attention to highways, while plowing is essential when snow falls.

Perrier et al. [155], Campbell et al. [49], and Eglese et al. [83] survey papers considering the routing of vehicles for winter abrasive or chemical spreading operations. These surveys contribute to the literature by offering a classification scheme for the models developed and by providing methodologies including some practical operating constraints.

Xie et al. [206] and Gáspár and Bencze [90] study the routing of deicing vehicles to reduce road maintenance costs and environmental pollution. Xie et al. [206] consider different factors influencing routing decisions, namely: the characteristics of road networks, including the operating region boundary influences; the starting points; the type of the vehicles; location, type and number of stations to load the vehicles if needed; and tour constraints such as load balancing. A model is provided, as well as a genetic algorithm to generate feasible solutions for a numeric example.

Gáspár and Bencze [90] apply existing CARP methodologies to study salting route optimization in a Hungarian county. The authors consider several scenarios that include different salt depots representing intermediate facilities, and these are compared against the utilization of only one centralized depot, which the authors refer to as the standard variant. A cost-benefit analysis is provided.

Routing for snow plowing vehicles is the core work in Refs. 81, 82, 110, 126, and 172. As with the Hungarian salting route study discussed above, Liu et al. [126] also resort to existing CARP methodologies, in this case to minimize the total travel distance for snow plowing operations in the south part of the city of Edmonton in Alberta, Canada. The street map for the selected zone includes 91 roads and 55 nodes, and only one vehicle is used with a working time limit of four hours. The authors apply a memetic algorithm with extended neighborhood search and perform some sensitivity analysis concerning the depot location and the number of tours.

To deal with snow removal on secondary roads, such as cycle paths, sidewalks, or paths in pedestrian areas, Holmberg [110] develop heuristics for the undirected RPP. The problem differs from the one mentioned above in, for instance, the type of equipment used. Also, the primary roads, which are assumed to be already cleared, may be used to reach the secondary ones. The methods were tested over several groups of instances, not including real ones, and the authors conclude "that it is practically possible to solve instances of relevant sizes in real time."

Kramberger et al. [120] use GIS technology, together with undirected arc routing with priority nodes, to tackle a deicing problem. Penalties are associated with delays in reaching the priority nodes, and the aim is to find a tour that traverses each edge at least once and minimizes the total cost (including traversal and penalty costs). Results are reported for random and real data based on the street network of Celje town, Slovenia.

Salazar-Aguilar et al. [172] define the Synchronized ARP (SyARP) for snow plowing, based on the need to identify a set of tours that services all streets and in which streets with multiple lanes are plowed at the same time by several synchronized vehicles. Vehicles start and end at a depot, and by following one another they may push snow to the sides of large streets and avoid forming snow mounds in the middle.

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The objective is to minimize the longest route in order to service all street segments in the least possible time and thereby obtain a set of balanced tours. Salazar-Aguilar et al. [172] formulate a mixed integer programming model, prove it to be NP-hard, and develop an adaptive large neighborhood search matheuristic, including a construction and an improvement phase. Computational results are reported over random generated instances as well as data from a real network. This work focuses on the "city of Dieppe, a suburb of Moncton in New Brunswick, Canada, with a population of about 20,000." The associated network has 430 nodes and 1,056 arcs, all of which have one or two lanes. Tests are used to assess the quality of the operators applied, as well as the effect of the improvement phase. Tours generated for the real case seem to be realistic.

Jang et al. [116] propose a formulation and a heuristic that integrates several design decisions for a winter road maintenance operation, namely: depot location, sector design, vehicle route design, vehicle scheduling and fleet configuration. The procedure was applied on the state highway road network in Boone County, Missouri, USA, which is maintained by the Missouri Department of Transportation.

Dussault et al. [81, 82] resort to windy networks to address plowing with precedence constraints, motivated by the fact that deadheading a cleaned link is faster than deadheading it when it is covered with snow or ice. Traversal costs are therefore dependent on the sequence of streets in a tour. It is also observed that some steep streets are difficult or impossible to plow when traveling uphill. The aim, then, is to identify minimum cost tours "that try to avoid plowing uphill on steep streets and take advantage of the faster traversal time on plowed streets." Some streets are considered crucial and must be plowed first. Dussault et al. [81] formulate the problem by considering that no completely unplowed street may be deadheaded before it is cleaned. They develop a method capable of generating near-optimal tours for instances with up to 200 nodes. This method starts by solving a problem to identify the number of times each link needs to be traversed. Then an initial tour is identified, and it is next improved through a heuristic that takes precedence relationships into account. The heuristic uses a local search and a re-initialization procedure and is tested over adapted WRPP benchmark instances.

In contrast, Dussault et al. [82] ignore the concept of imposed precedence. In this paper, the authors assume snow depth allows streets to be deadheaded before being serviced. The aim is to find a set of tours, one per vehicle, starting and ending at the depot, plowing each arc twice (once on each side of the street), and minimizing maximum tour length. The procedure previously proposed in [81] is adapted, and results are generated for the same set of instances. Authors point out that this methodology may easily be adapted to some related problems.

The routing of street cleaners and the design of street snow plowing or snow-salting tours may also be tackled via the maximum benefit CPP addressed in [67]. The advantage of this approach is that it relaxes the imposition that all links must be serviced, while allowing some to be serviced more than once if it is beneficial.

4.4.3. Road Maintenance. Chen et al. [53, 54] study the daily maintenance operation of a road network, which involves several challenging tasks, including: visually checking the operational status of each segment, reporting defects on the roads, and evaluating the function of auxiliary facilities. The problem becomes more difficult when considering stochastic service and travel times (normal distribution is assumed). The problem is addressed as a generalization of the CARP, minimizing both the total deadheading cost and the number of vehicles needed, and such that the probability of having a tour lasting longer than a maximum permitted value does not exceed a given threshold (referred to as a chance constraint). A model, a branch-and-cut algorithm and an adaptive large scale neighborhood are presented. According to the authors, results obtained over randomly generated instances derived from real data from a road maintenance agency in Shanghai, China, show the effectiveness of the methodology. Chen et al. [54] state that this "is an initial work demonstrating the potential benefits of including stochastic service and travel time in ARPs." In a later paper, Chen et al. [53] propose a robust optimization approach for the same problem, allowing them to consider the risk of extreme outcomes. For this second study, the authors changed the objective to the optimization of the worst-case value over all uncertainty data within a bounded set. The robust model is solved via a branch-and-cut algorithm, and sensitivity analysis is performed on the level of robustness and on the number of vehicles. Chen et al. [53] conclude that this model is a better option than their previous one, and they state that "robust optimization is an attractive alternative for solving routing problems under uncertainty."

Generally similar in structure to the road maintenance problem, safety inspection of railroad tracks is addressed in Lannez et al. [122, 123]. With the minimization of total deadheading distance as their objective, Lannez et al. [122] first study this problem as a rich mixed ARP. A daily time limit is considered, given that inspection vehicles cannot be on the road for more than six hours per day. Furthermore, the vehicles have a water tank with a limited capacity, which can be refilled at special stations but no more than once per shift, as it is a very time-consuming task. The study focuses on the inspection of primary tracks requiring a visit once or twice a year. Complicating constraints are related to the time limit per shift, water supply, track outages and heterogeneous fleet. The huge real network size presents an additional challenge, supporting the need for the study. The authors find that a column generation approach performs better than a greedy algorithm over a real data set. The network has 1,600 arcs, 500 edges, and 770 nodes, of which 90 are refill nodes. Three different scenarios are considered. For the same problem, and the same real case study, Lannez et al. [123] later present a new model, a matheuristic based on both Benders and Dantzig-Wolfe decompositions and a dynamic programming

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heuristic. The resulting methodology and generated solutions may easily be used and adapted by the operator, taking into account the real speed of vehicles and precise dates of outages, or including desired pathways.

A recurrent problem in Taiwan is the re-pavement of roads. Huang and Lin [111] consider the routing of construction machinery for road resurfacing activities performed by contracted companies simultaneously working on several road resurfacing jobs. Time window constraints are imposed to guarantee some precedence constraints between road treatments, as each road needs multiple treatments in a pre-determined sequential order. A multi-treatment CARP with TW is then defined. The aim is to identify minimum cost routes that will treat all the required roads in the established order. This problem is transformed into an equivalent traveling salesman problem, and an ant colony optimization algorithm is applied. Benchmark classical CARP instances were adapted and used to validate the methodology, which was then applied to a real-case study. This was adapted from a sub-network of Kaohsiung City in Taiwan, containing 202 nodes and 348 arcs, 101 of which require only one type of treatment, 130 requiring a different type, and the remaining ones requiring both treatments. The authors concluded their proposed methodology to be an efficient one.

As detailed in Section 3.7.1, Riquelme-Rodríguez et al. [166] and [167] study a new application for open-pit mines. The problem is addressed as a LARP, with the goal of identifying the best locations for water depots to be used by vehicles periodically spraying the mine hauling roads to eliminate dust.

4.4.4. Road Marking. Road marking is a natural arc routing application. A real case study in Quebec, Canada, leads Amaya et al. [6] to a CARP with refill points and multiple loads, as detailed in Section 3.4. The two types of vehicles employed are: the finite capacity Servicing Vehicle (SV), which marks the roads; and the Refilling Vehicle (RV) tank trucks, which meet up with and replenish the SVs. A RV may refill an SV more than once. The objective in this case is to identify minimum cost vehicle tours for both vehicles on a directed graph. A model and a related cutting plane algorithm are presented to optimally solve the smaller instances. Moreover, a route-first cluster-second heuristic is developed. Randomly generated instances were used to assess the performance of the proposed methods, and a real road network from a region in Canada was also used.

Working on the same problem, Salazar-Aguilar et al. [173] assume that routes for SV and RV vehicles must be synchronized (as discussed in Section 3.8). They also assume that certain links and nodes demand service, and they tackle the problem as a synchronized GRP. Three different replenishment policies are considered. An adaptive large neighborhood search heuristic, generalizing the one developed by Salazar-Aguilar et al. [172] for the snow plowing problem, is presented and successfully tested over a large set of randomly generated instances.

4.5. *Other Applications*

A very common application for ARPs, already discussed in relation to the close-enough problem, is routing for meter readers. Eglese et al. [83] present a very interesting survey of meter reading applications, including the historical evolution, not only of the reader operation itself from its early roots to the current automated reality, but of the methodological evolution in tandem with computer evolution.

4.5.1. Security Patrol Routing. Some housing developments or residential areas need to provide 24-hour security for residents, so constant patrolling of roads is essential in these situations. Willemse and Joubert [201] investigate this problem as an ARP, or more specifically, as a min-max K-ARP, and present a tabu search algorithm for patrol route design. The study focuses on a case in South Africa, where gated communities are a growing phenomenon, but in fact these types of neighborhoods are proliferating in many countries all over the world, wherever security is an increasing concern, and this study may be applied to several real-world situations. Willemse and Joubert [201] identify several complications in patrol route design: patrolling must be unpredictable, that is, cannot be repeated the same way every day; all roads and paths have to be patrolled; and routes should be evenly distributed among the guards. A real problem instance from an estate in Gauteng, South Africa, is used to compare tours generated by the algorithm with previously existing ones. The tabu search methodology was able to produce better tours, which were also better balanced. Benchmark instances from the literature were used to assess the performance of the methodology. From the computational results, Willemse and Joubert [201] conclude that "the algorithm is robust enough to generate quality patrol routes on different road networks. (...) Most notable are the improvement of unpredictable patrolling and the placement of checkpoints." To generate an unpredictable set of tours, several solutions were generated and routes were chosen at random for implementation. However, no check was conducted to truly assess dissimilarity between the chosen tours.

Also embracing the design of security patrol routes, the generalized maximum benefit multiple CPP is defined in [176]. As discussed in Section 3.9.4, the paper's main purpose is to study alternatives to the requirement that all routes must begin and end at the depot. To evaluate the alternatives, integer programming formulations were developed, and the campus of the University of Maryland College Park, USA, was used as an illustrative example.

4.5.2. Industrial Cutting Applications. When units of raw material are industrially cut, arc routing may be applied to optimize the cutting path. In studying one such case, Rodrigues and Ferreira [168] deal with a continuous process path-cutting, meaning that the cutting tool never leaves the cutting surface, with no restrictions on the total cutting of a given piece. The authors propose a memetic algorithm to solve an associated RPP and illustrate its application for

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path-cutting operations within the company. The objective is to reduce the distance/time traveled by the cutting tool. An RPP is defined, where required edges represent pieces to cut and deadheading represents the paths between them and along which the tool needs to move without cutting. An RPP tour minimizing the total time then constitutes an optimal cutting path. Four reality-based instances for the cutting problem were used, as well as several known RPP instances, and results are compared with known procedures from the literature. The authors conclude that the memetic algorithm "behaved very well in general, except for some specific instances, which had relatively more connected components."

4.5.3. Mobile Mapping Van Problem. Digital maps are often created by teams driving so-called mapping vans through streets. Vansteenwegen et al. [197] studied this problem for Tele Atlas, a Netherlands-based company, which uses a fleet of vehicles to take photos of streets and road signs. To minimize the number of days a vehicle needs to traverse all the streets in a given network, a CARP is defined, with a maximum travel time per day (Section 3.5). Soft time windows are established to avoid taking pictures against the sun, and a penalty is incurred whenever a time window is violated. The network is a mixed one, and the company uses a single vehicle that begins and ends service each day at a fixed node (a hotel). Authors pursue the minimization of the number of days by minimizing the total travel time per day, and they aim to identify the sequence in which links must be serviced. The problem is transformed into an equivalent vehicle routing problem, and a hybrid metaheuristic is then applied. This latter first includes a local search phase to diminish the number of days, then uses an iterated local search phase to minimize time window violations. The algorithm was first tested on VRP instances and then applied to solve real cases generated from a cycling network in East Flanders, Belgium.

5. FINAL REMARKS

This annotated bibliography has followed the study of arc routing through seven very productive and exciting years. The number of authors and publications has increased at an impressive rate as a result of previous pioneering works, and the current panorama forms the corollary for promising years to come. To identify some emerging patterns and trends in arc routing, we conclude with the following observations.

It seems consensual that although a transformation into a node routing problem is possible (and sometimes beneficial), the direct study of ARPs and their characteristic uses leads to better results. It then becomes possible to develop methods that are finely tuned for the specificities of each problem, and this is most certainly a relative advantage.

Sophistication in crafting and a continued refinement of the underlying methodologies have been broadening the range of possibilities for arc routing. As computer technology evolves, it becomes possible to "attack" more complicated problems that are more accurate representations of real systems. In fact, real-world applications have already benefitted from research in this area, and arc routing methodologies to support decision making are both growing in number and broadening in range.

Interestingly, the very necessary standardization of test data has led authors to create sets of instances and make them globally available to the scientific community. In fact, many instance sets are now available online, and online software libraries can be used for instance generation or solving subproblems. Any comparison of methodologies gains clarity and depth when results are obtained over the same sets of data, as authors can readily obtain a performance assessment against the best results published online.

We conclude with several observations on methodology. First, we note that (meta)matheuristics and branch-and-cut procedures seem to have taken their place among the more useful methods for solving ARPs, and the theoretical study of these problems will be substantially beneficial. Second, we see a growing interest in the classification of solutions not only in terms of their global cost, but also in terms of their "attractiveness" or aesthetic characteristics. In fact, the practical implementation of the generated solutions often benefits from a carefully shaped output that is appealing to practitioners. Finally, a new and exciting field of study lies in the natural stochasticity of real problems. This is an interesting and promising avenue to explore in future research.

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APPENDIX: LINKS TO SETS OF INSTANCES AND OPEN SOURCE SOFTWARE

TABLE 9. Instance links and open source software (consulted on 03/05/2017). *Code not yet available.

Name	Link	Problems	Maintained by or referred by
BCL_CARPDD	http://mis.ucd.ie/staff/pkeenan/TCARP	CARPDD, sparse graphs	Bartolini, et al. [27] MIS group at University College Dublin, Ireland
B_CARP	http://www.uv.es/belengue/carp.html	CARP	J. Belenguer, E. Benavent
B_MCARP	http://www.uv.es/belengue/mcarp/index.html	MCARP	J. Belenguer, E. Benavent
Brescia_Inst	http://or-brescia.unibs.it/instances	TOARP; ClOARP; P-RPP	OR-Brescia: Operational Research Group University of Brescia
CPS_ARP	http://www.uv.es/corberan/instancias.htm	CPP; GRP; G-DRPP; HRPP; MMK-WRPP; OARP; P-ARPM; RPP; SCP - For U/D/M/W graphs and single or multiple vehicles	Á Corberán, I. Plana, J.M. Sanchis
GAS_DRPP	http://www.unibs.it/sites/default/files/ricerca/allegati/13004DPRPP.zip	DPRPP	G. Guastaroba, C. Archetti, M.G. Speranza
I_CARP	http://logistik.bwl.uni-mainz.de/benchmarks.php	ARP; CARP	S. Irnich
LGCS_AR	https://github.com/Olibear/ArcRoutingLibrary	Open-source Arc Routing Library (OAR Lib) - forthcoming *	O.Lum, C. Cerrone, B. Golden, E. Wasil.
LPFS_LARP	http://lore.web.ua.pt/	LARP	R.B. Lopes, F. Plastria, C. Ferreira, B.S. Santos
LIU	http://www.mdolab.net/Ray/Research-Data/InstanceGenerator.rar	CARP Instance Generator Software	M.Liu, H.K. Singh, T. Ray
MAL_PARP	http://ftpprof.uniandes.edu.co/∼pylo/PCARP-IS/instances.htm	Periodic ARP with irregular services	M. Monroy, C. A. Amaya, A. Langevin
RvB_MCARP	Instances: https://gitlab.com/rvb/mwcarp-ob Source code available at: http://gitlab.com/rvb/mwcarp-approx	Benchmark MCARP instances including several components	R. van Bevern
RF_Waste	http://www.inescporto.pt/∼amr/Limited_Multi_Landfills	CARP-IF heterogeneous fleet	A. M. Rodrigues, J.A.S. Ferreira
SINTEF_CGRP	Instances: https://www.sintef.no/projectweb/top/nearp/; https://www.sintef.no/pdptw Software: https://www.sintef.no/en/software/spider/	Mixed Capacitated GRP	G. Hasle
W_CARPs	http://www.optimization.dk/	CARP; Coordinated CARP; Semi-periodic CARP; Multi-compartment CARP; Time-dependent P-ARP; Time-dependent multiple P-ARP	Sanne Wøhlk
WJ_CARPIF	https://data.mendeley.com/datasets/9x4vd92rcj/3	NEARP; CARP with time constraints and IF	E.J. Willemse, J.W. Joubert

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