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Simultaneous generation and correlation of complete sets of generalized pairwise complementary sequences

Matías N. Hadad[1,*,†], Marcos A. Funes[1], Patricio G. Donato[1], Enrique García[2], Jesús Ureña[2] and Daniel Carrica[1]

[1]*Laboratory of Instrumentation and Control, Department of Electronics, National University of Mar del Plata, CONICET, Argentina*
[2]*Department of Electronics, University of Alcalá, Madrid, Spain*

SUMMARY

New coding schemes are increasingly being developed to adapt multisensor and communication systems to the requirements of the current technological environment. In some cases, the practical implementation of these schemes involves a great amount of signal processing operations. In order to achieve and make suitable their use in different platforms, efficient algorithms are required to process coding. In the last years, a new kind of complementary sequences, known as Generalized Pairwise Complementary sequences, has been proposed. These sequences provide high noise immunity when they are applied to multiuser systems, as well as a low cross-correlation, which is convenient in those applications. The aim of this paper is to propose optimized generation and correlation architectures for Generalized Pairwise Complementary sequences that would allow to process a complete set using a reduced amount of operations. An analytical demonstration is provided, with a final comparison to evaluate the reduction with regard to other algorithms.

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KEY WORDS: GPC sequences; generation; correlation; signal detection

1. INTRODUCTION

In recent years, more research efforts have been devoted to the theory and application of Complementary Sets of Sequences (CSS) and other derived sequences. A growing number of publications have addressed this topic and dealt with theoretical and practical aspects. The applications involving the use of these sequences range from communications [1] to ultrasound image processing [2], including sensors [3] and local positioning systems [4]. The possibility of detecting signals immersed in noise, their multiuser properties and/or the possibility of performing channel identification are some of the key aspects of these sequences.

Furthermore, theoretical research has focused on generation and correlation algorithms, computational efficiency, and other implementation-related issues [5–9]. One of them is the synthesis of new types of sequences derived from the traditional CSS in order to obtain or improve some of their characteristics [10, 11]. As a result, in the last years, different types of sequences have been developed, such as the ZCZ (*Zero Correlation Zone* [12]), LS (*Loosely Synchronous* [13]), and GPC (*Generalized Pairwise Complementary* [14]), which are characterized by an Interference Free Window (IFW) in their Autocorrelation Function (ACF) and Cross Correlation Function (CCF). In

*Correspondence to: M. N. Hadad, Laboratory of Instrumentation and Control, Department of Electronics, National University of Mar del Plata, CONICET, Argentina.
†E-mail: mhadad@fi.mdp.edu.ar

particular, GPC sequences have attracted an increasing attention because of their particular mathematical properties, namely, greater energy efficiency and larger IFW for a given length, as compared with other sequences. In addition, they have controlled correlation side lobes outside the IFW.

The GPC sequences are pairs of binary sequences initially introduced by Chen *et al.* [14]. They can be generated and correlated using schemes derived from CSS, as proposed by García *et al.* [15]. However, the aforementioned schemes entail a large amount of operations, which is not a minor issue in the applications of limited resources and/or clock frequency.

Earlier works [6,16, 17] have introduced new approaches for CSS generation and correlation, which result in a significant reduction in the number of required operations to process CSS. Accordingly, similar approaches can be applied to GPC sequences in order to reduce the amount of operations required by the generation and correlation processes. This work presents optimized algorithms for the generation and correlation of GPC sequences that allow to process a complete set of GPC sequences in a simultaneous basis.

The paper has been structured as follows: section 2 deals with a summary of the fundamentals on GPC sequences and previous generation and correlation architectures [15]. A new proposal is developed in section 3, analyzing each step of the generation and correlation processes. A comparison between the new proposal and previous ones is provided in section 4 and, finally, section 5 draws some conclusions.

2. GENERALIZED PAIRWISE COMPLEMENTARY SEQUENCES

Generalized Pairwise Complementary sequences were introduced by Chen *et al.* [14] as a set of $2G$ ($G=2^a, a\in\mathbb{N}-\{0\}$) pairs of binary sequences $\left(u^j_{g,0}[k], u^j_{g,1}[k]\right)$ ($0\leq g\leq G-1$) of length $L=4GL_0$ derived from generalized even shift orthogonal sequences of length $4L_0$ [14]. These sequences are divided into two subgroups (U^0, U^1) of G pairs of sequences, expressed in the rest of the paper by the super index $j\in\{0,1\}$, which offer a uniform IFW. The main properties of GPC sequences are:

- The sum of the ACF of each pair is always zero, except at the time shifts equal to $8nL_0$, with $n\in\mathbb{N}-\{0\}$.

$$C_{u^j_{g,0},u^j_{g,0}}[k] + C_{u^j_{g,1},u^j_{g,1}}[k] = \begin{cases} 8GL_0 & k=0 \\ 0 & k\neq 8nL_0, \end{cases} \tag{1}$$

where $C_{u^j_{g,0},u^j_{g,0}}[k]$ is the ACF of $u^j_{g,0}$

$$C_{u^j_{g,0},u^j_{g,0}}[k] = \sum_{i=0}^{L-1} u^j_{g,0}[i]\cdot u^j_{g,0}[i+k]. \tag{2}$$

- Their intragroup sum of CCFs (between sequences from either U^0 or U^1) is always zero, except at the time shifts equal to $8nL_0$.

$$C_{u^j_{g,0},u^j_{g',0}}[k] + C_{u^j_{g,1},u^j_{g',1}}[k] = \begin{cases} 0 & k=0 \\ 0 & k\neq 8nL_0 \end{cases} \tag{3}$$

with $g\neq g'$, where $C_{u^j_{g,0},u^j_{g',0}}[k]$ is the CCF of $u^j_{g,0}$ and $u^j_{g',0}$

$$C_{u^j_{g,0},u^j_{g',0}}[k] = \sum_{i=0}^{L-1} u^j_{g,0}[i]\cdot u^j_{g',0}[i+k]. \tag{4}$$

- Their intergroup sum of CCFs (between sequences of U^0 and U^1) is always zero for any time shift

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$$C_{u^{j}_{g,0},u^{j'}_{g',0}}[k]+C_{u^{j}_{g,1},u^{j'}_{g',1}}[k]=0 \tag{5}$$

with $j \neq j'$.

A. *Generalized Pairwise Complementary generation and correlation*

García *et al.* [15] formulated efficient generation and correlation schemes for pairs of GPC sequences of length $L=2GL_0$ derived from even shift orthogonal sequences (E-sequences [18]) of length $2L_0$, obtained by interleaving Golay binary pairs of sequences [19].

Golay binary pairs of sequences (also known as complementary pairs of sequences) are defined as a pair of sequences $(a[k], b[k])$ of the same length L, composed of two binary elements (± 1). The main property of these sequences is that the addition of their autocorrelation functions is zero for every time displacement difference of zero and $2L$ at zeros displacement (which is known as a Kronocker delta):

$$C_{a,a}[k]+C_{b,b}[k]=2L\delta[k]. \tag{6}$$

Other important property of these sequences is the existence of an orthogonal pair for every pair of sequences. Two pairs of sequences $((a[k], b[k]), (c[k], d[k]))$ are orthogonal if the addition of their CCFs equals to zero for every time displacement (Eq. 7). The limitation of this property is that for a given pair there is only one orthogonal pair to it [16].

$$C_{a,c}[k]+C_{b,d}[k]=0 \tag{7}$$

1. *Generation algorithm*: The generation algorithm can be summarized as a concatenation process with a delay cascade and a Hadamard matrix row as follows:

$$U^{j}_{g,m}(z)=\left[E_j(z)\sum_{i=0}^{G-1}h_{g,i}z^{-i\cdot 2L_0}\right]V_m(z), \tag{8}$$

where:

- $U^{j}_{g,m}(z)$ with $m,j \in \{0,1\}$ and $0 \leq g \leq G-1$ refers to the sequence m of the pair g belonging to the subgroup j (Figure 1) in the z domain.
- $h_{g,i}$ is the element of row g and column i of the Hadamard matrix of order G.
- L_0 is the length of the Golay pairs of sequences.
- $E_j(z)$ is an E-sequence obtained by interleaving the Golay pair j.

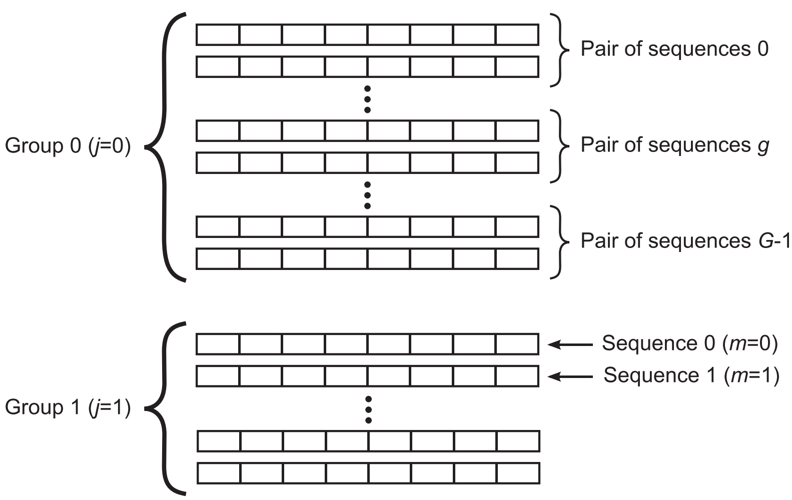

Figure 1. Both groups of a Generalized Pairwise Complementary sequences set.

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$V_m(z)$ is a polynomial that equals:

$$V_m(z) = \sum_{i=0}^{L-1} (-1)^{i \cdot m} \cdot z^{-i}. \tag{9}$$

The aim of this polynomial is to eliminate the interference in the odd shifts of the sum of aperiodic correlation functions.

2. *Generation Architecture*: From the algorithm previously introduced, an efficient generator for a single pair of GPC sequences (EGPCG) can be obtained, based on 8, following the steps indicated in the following (Figure 2).

- *Step 1*: To generate the E-sequence, it is necessary to interleave the Golay pair of sequences. To accomplish this, three sub-steps are required. First, the Golay pairs A^j and B^j are generated by using an efficient generator (EGG) [20, 21]. Then the Golay pairs must be zero-padded by inserting one zero between every two bits of both sequences. Finally, the zero-padded sequence A^j must be delayed for one bit and then added with the other zero-padded sequence to obtain the E-sequence $E_j(z)$. In order to generate the zero-padded sequences, the delays in the efficient generator must be multiplied by 2.
- *Step 2*: A Walsh–Hadamard expansion is made to increase the length of the set. According to 10, a cascade of z^{-2L_0} delay blocks is used, weighted by the h_g elements.

$$U^j_{g,0}(z) = E_j(z) \sum_{i=0}^{G-1} h_{g,i} z^{-i \cdot 2L_0} \tag{10}$$

- *Step 3*: Finally, to generate the sequence $u^j_{g,1}$, the sign of the even bits of $u^j_{g,0}$ must be changed (Eq. 9). This is implemented by using a multiplexer driven by a signal of half the clock frequency (*clk*/2) synchronized with the clock edge used in the previous stages.

3. Correlation Architecture: An efficient correlation architecture (EGPCC) can be obtained as a reversed version of the generator. To do so, the generation algorithm should be modified as shown in the following steps (refer to Figure 3):

- *Step 1*: The inverse Walsh–Hadamard expansion of the input signal $R(z)$ must be performed by reversing the delay orders. This process can be summarized through the following algorithm:

$$R_e(z) = R(z) \sum_{i=0}^{G-1} h_{g,i} z^{-(G-1-i)2L_0}. \tag{11}$$

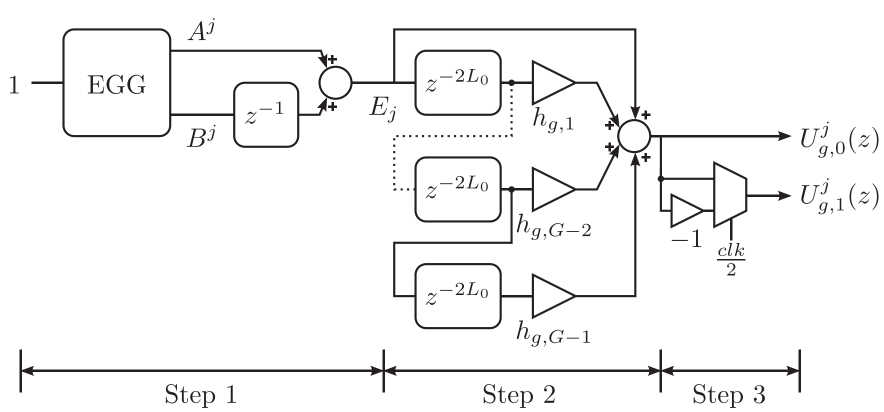

Figure 2. Efficient generation architecture for Generalized Pairwise Complementary sequences (EGPCG [15]). EGG, efficient Golay generator.

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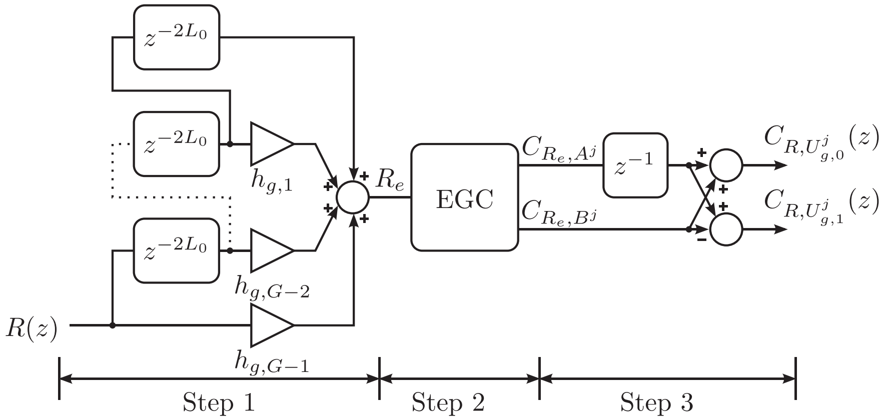

Figure 3. Efficient Generalized Pairwise Complementary sequences correlator (EGPCC). EGC, Efficient Golay Correlator.

- *Step 2*: The resulting sequence must be correlated with the zero-padded sequences by replacing the Golay generator with an Efficient Golay Correlator (EGC) with the delays multiplied by 2.
- *Step 3*: The delay order at the output of the EGC must be reversed to obtain the aperiodic correlation between the input signal, $R(z)$, and the sequence $U^j_{g,0}(z)$, as follows:

$$C_{R,U^j_{g,0}}(z) = C_{R_e,A^j}(z)z^{-1} + C_{R_e,B^j}(z), \tag{12}$$

and the aperiodic correlation between the signal $R(z)$ and the sequence $U^j_{g,1}(z)$ is:

$$C_{R,U^j_{g,1}}(z) = C_{R_e,A^j}(z)z^{-1} - C_{R_e,B^j}(z). \tag{13}$$

3. SIMULTANEOUS GENERATION AND CORRELATION ARCHITECTURES

The previous section dealt with the generation and correlation architectures for GPC sequences. By using such generation architecture, it is possible to generate a single pair of GPC sequences. On the other hand, the correlation architecture computes the correlation of a single input, $R(z)$, with both sequences of a GPC pair. To fully benefit from the properties of GPC sequences in a practical application, the generation or correlation of the complete set of $2G$ GPC sequences should be carried out on a simultaneous basis. In these cases, it is necessary to use $2G$ generators or $4G$ correlators, with the resulting increment in the number of operations. In this section, the architectures that enable to simultaneously generate or correlate a complete set of GPC sequences with a reduced amount of operations are analytically developed. Because the correlation of a complete set of GPC sequences is the most demanding operation, the proposal starts with the development of an optimized correlation architecture with the goal of a minimum operation amount. Later, some of the used strategies are applied to the generation algorithm in order to obtain a similar reduction.

A. *Simultaneous Generalized Pairwise Complementary Correlation Architecture*

Starting with the architecture of Figure 3, the correlation of the input signal with both sequences of a GPC pair can be expressed in z-domain as follows:

$$\begin{aligned} C_{R,U^j_{g,0}}(z) &= \left(A^j(1/z)z^{-1} + B^j(1/z)\right)\alpha(z)R(z) \\ C_{R,U^j_{g,1}}(z) &= \left(A^j(1/z)z^{-1} - B^j(1/z)\right)\alpha(z)R(z), \end{aligned} \tag{14}$$

where $A^j(1/z)$ and $B^j(1/z)$ are the terms in the z-domain to compute the correlation with the Golay pair

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$A^j(z)$ and $B^j(z)$. $\alpha(z)$ is the inverse Walsh–Hadamard expansion at the correlator input:

$$\alpha(z) = \sum_{i=0}^{G-1} h_{g,i} z^{-(G-1-i)2L_0}. \tag{15}$$

In order to compute the sum of correlations between two input signals and a pair of GPC sequences, two correlator architectures must be used (Figure 4):

$$\begin{aligned} Y_g^j(z) = C_{R_1,U_{g,0}^0}(z) + C_{R_2,U_{g,1}^0}(z) &= \left(A^j(1/z)z^{-1} + B^j(1/z)\right)\alpha(z)R_1(z) \\ &+ \left(A^j(1/z)z^{-1} - B^j(1/z)\right)\alpha(z)R_2(z). \end{aligned} \tag{16}$$

The use of the associative property with α in 16

$$Y_g^j(z) = \alpha(z)\left[\left(A^j(1/z)z^{-1} + B^j(1/z)\right)R_1(z) + \left(A^j(1/z)z^{-1} - B^j(1/z)\right)R_2(z)\right] \tag{17}$$

yields the block diagram in Figure 5, which leads to a significant reduction in the calculations' requirements.

Upon analyzing 17, a further reduction can be obtained by splitting the correlation with A^j and B^j (Figure 6):

$$Y_g^j(z) = \alpha(z)\left[A^j(1/z)z^{-1}(R_1(z) + R_2(z))B^j(1/z)(R_1(z) - R_2(z))\right]. \tag{18}$$

The architecture in Figure 6 uses two EGCs to perform the correlation with the complementary sequences. However, this implementation performs two correlations that are not used (C_{R_1',B^j} and C_{R_2',A^j}, respectively). The sum of correlations with respect to A^j and B^j can be implemented by using the algorithm proposed in [22] (Optimized Golay Correlator, OGC), further reducing the total number of required calculations (Figure 7).

In the previous section, it was stated that the intra-group GPC sequences differed in the row of the Hadamard matrix used in the Walsh–Hadamard expansion. Rewriting 15 in a matricial form and replacing in 18:

$$Y_g^j(z) = \boldsymbol{Hg}\cdot\boldsymbol{D}'(z)\cdot\boldsymbol{C}^j, \tag{19}$$

where:

- $\boldsymbol{C}^j$ is a column vector with the output of the OGC (Figure 7)

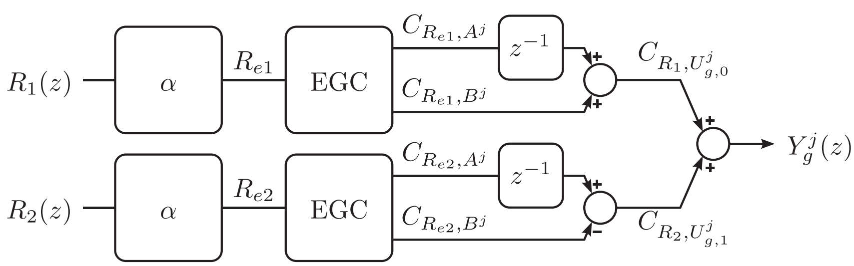

Figure 4. Block diagram of the sum of Generalized Pairwise Complementary correlations. EGC, Efficient Golay Correlator.

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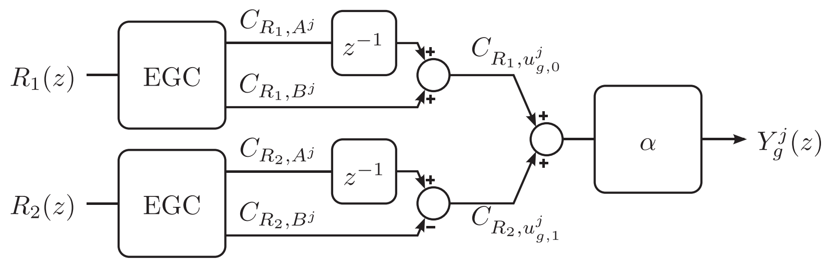

Figure 5. Generalized Pairwise Complementary correlator with the $\boldsymbol{\alpha}$ block located at the output. EGC, Efficient Golay Correlator.

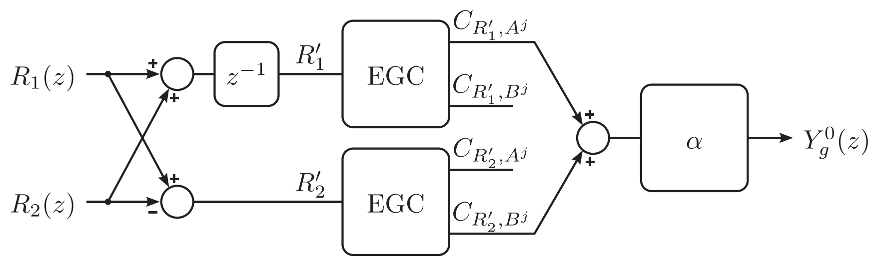

Figure 6. Generalized Pairwise Complementary correlator based on equation 18. EGC, Efficient Golay Correlator.

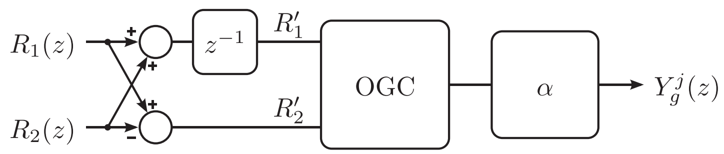

Figure 7. Generalized Pairwise Complementary correlator based on the Optimized Golay Correlator architecture and equation 18.

$$\boldsymbol{D}'(z) = \begin{bmatrix} z^{-(G-1)2L_0} & 0 & \cdots & 0 & 0 \\ 0 & z^{-(G-2)2L_0} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & z^{-2L_0} & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \tag{20}$$

$$\boldsymbol{Hg} = [\, h_{g,0} \quad h_{g,1} \quad \cdots \quad h_{g,G-2} \quad h_{g,G-1} \,]. \tag{21}$$

By applying a similar reasoning to the one shown in [17], it may be assumed that the correlation of a subgroup of GPC sequences can be obtained by replacing the single row of the Hadamard matrix ($\boldsymbol{Hg}$) with the full Hadamard matrix, as it is shown in 22 and Figure 8.

$$\boldsymbol{Y}^j(z) = \begin{bmatrix} Y^j_0(z) \\ Y^j_1(z) \\ \vdots \\ Y^j_{G-1}(z) \end{bmatrix} = \begin{bmatrix} h_{0,0} & h_{0,1} & \dots & h_{0,G-1} \\ h_{1,0} & h_{1,1} & \dots & h_{1,G-1} \\ \vdots & \vdots & \vdots & h_{G-1,0} \\ h_{G-1,0} & h_{G-1,1} & \dots & h_{G-1,G-1} \end{bmatrix} \cdot \boldsymbol{D}'(z) \cdot \boldsymbol{C}^j. \tag{22}$$

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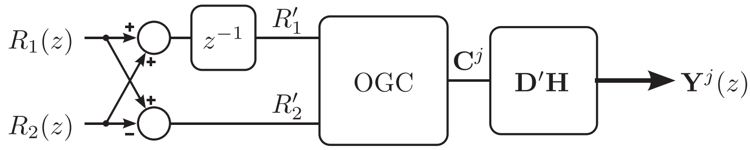

Figure 8. Architecture to perform the correlation of all the sequences of a Generalized Pairwise Complementary group. OGC, Optimized Golay Correlator.

Figure 9 provides an example of the scheme that is used to implement the product $\boldsymbol{H}_4{\cdot}\boldsymbol{D}'_4(z){\cdot}\boldsymbol{C}^j$ by using a reduced architecture.

In order to obtain the correlation with respect to the complete set of GPC sequences, the correlation with the sequences of the second subgroup should be obtained as well. These sequences are generated by using an orthogonal pair of complementary sequences. Then, the correlation with both pairs of complementary sequences has to be computed. This operation can be performed by using two OGCs, each one with a different seed (Figure 10).

$$\begin{aligned} Y^0(z) &= \boldsymbol{H}{\cdot}\boldsymbol{D}'(z)\cdot C^0 \\ Y^1(z) &= \boldsymbol{H}{\cdot}\boldsymbol{D}'(z)\cdot C^1 \end{aligned} \tag{23}$$

Moreover, to further reduce the architecture, it is advisable to use the algorithm proposed in [16] (Optimized Orthogonal Golay Correlator, O^{2}GC), which simultaneously computes the correlation of both pairs of complementary sequences. The proposed architecture is presented in Figure 11.

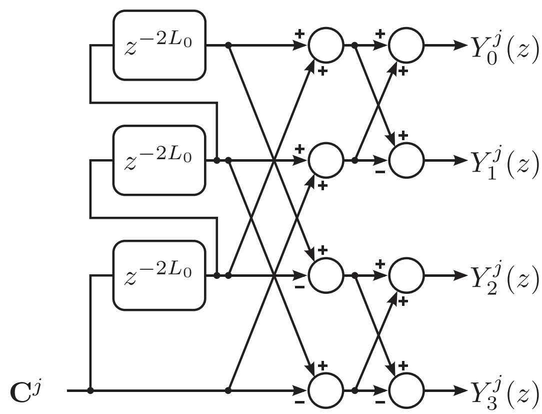

Figure 9. Architecture for computing the product $\boldsymbol{HD}'$ for a Hadamard matrix of order 4.

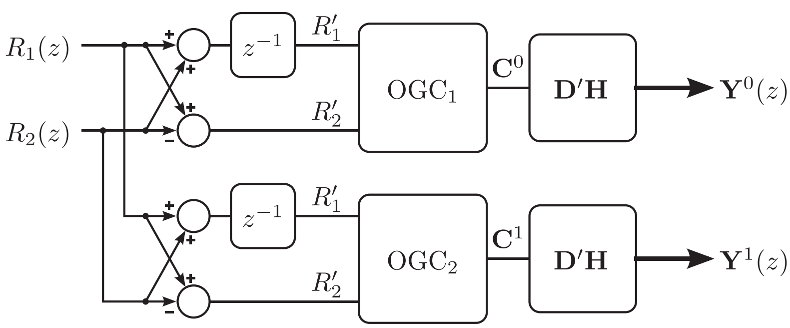

Figure 10. Architecture to perform the correlation of all the sequences of both Generalized Pairwise Complementary groups. OGC, Optimized Golay Correlator.

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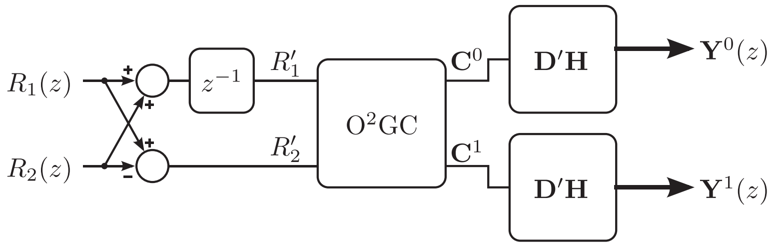

Figure 11. Optimized full Generalized Pairwise Complementary correlator.

The architecture obtained performs the sum of correlations of two inputs with respect to all the sequences of both subgroups of a GPC sequence set. This approach certainly represents a significant improvement on the correlation architectures of GPC sequences, avoiding the replication of architectures and/or the employ of useless operations (e.g., unused correlation outputs). The next section analyzes the calculation requirements by means of a detailed comparison with other algorithms.

B. *Simultaneous Generalized Pairwise Complementary Generation Architecture*

The GPC generator can be redesigned in order to generate all the sequences of a GPC set. Starting with the generation architecture of [15] (Figure 2), the product with a Hadamard matrix row can be replaced with an architecture similar to Figure 9, which computes the product with the whole Hadamard matrix to generate all the G sequences $U^j_{g,0}(z)$ of the subgroup j (Figure 12).

The last stage of the GPC generator must be replicated for each of the G generated sequences to generate all the pairs $\left(U^j_{g,0}(z), U^j_{g,1}(z)\right)$. Figure 13 shows the simultaneous generator architecture obtained for a subgroup of GPC sequences.

To generate a complete set of GPC sequences, two of the previous architectures should be used, each one with the corresponding complementary pair to obtain E_0 and E_1. However, in [23], it was demonstrated that two orthogonal pairs of sequences can be obtained by generating one pair and making alternate sign changes on both sequences to obtain the orthogonal pair:

$$\begin{aligned} A^1 &= A^0 \circ [+1\ \ -1\ \ +1\ \ -1\ \cdots] \\ B^1 &= B^0 \circ [+1\ \ -1\ \ +1\ \ -1\ \cdots], \end{aligned} \tag{24}$$

where $\circ$ is the Hadamard product of the vectors.

The generation algorithm is based on the concatenation, with sign changes, of the E-sequences, generated by interleaving the bits of a Golay pair. Thus, the pair of E-sequences can be obtained using a single generator architecture:

$$E_1 = E_0 \circ [+1\ \ +1\ \ -1\ \ -1\ \ +1\ \ +1\ \ -1\ \ -1 \cdots]. \tag{25}$$

By applying this sign pattern to every output sequence of the generator in Figure 13, a complete set of GPC sequences can be generated. The pattern is the inversion of two bits every two bits, and it is implemented by using a multiplexer driven with a signal four times slower than the clock frequency (*clk*/4), synchronous to the *clk*/2 and *clk* signals. Figure 14 shows the proposed architecture of the simultaneous generator.

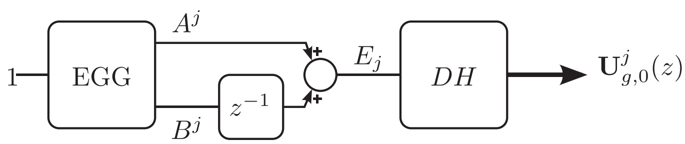

Figure 12. Generator architecture for all the first sequences of a Generalized Pairwise Complementary subgroup. EGG, efficient Golay generator.

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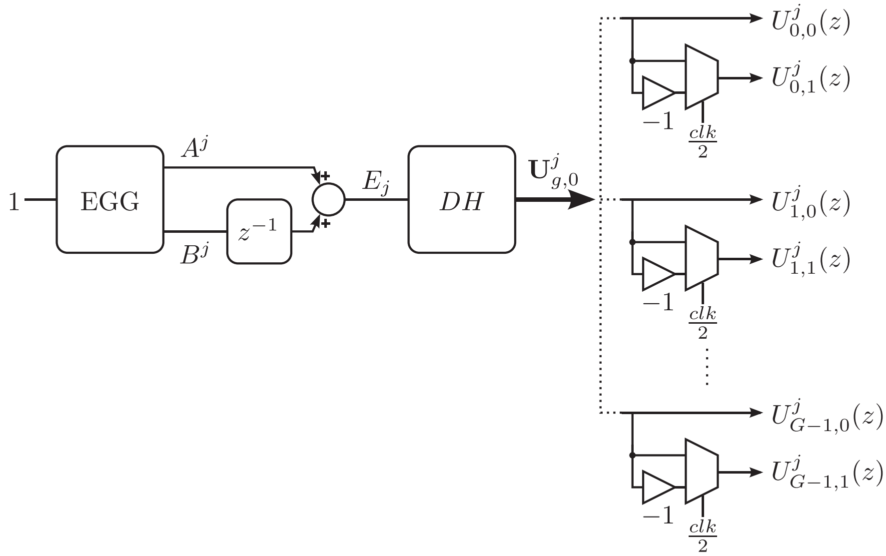

Figure 13. Generalized Pairwise Complementary generator of all the pairs of a subgroup. EGG, efficient Golay generator.

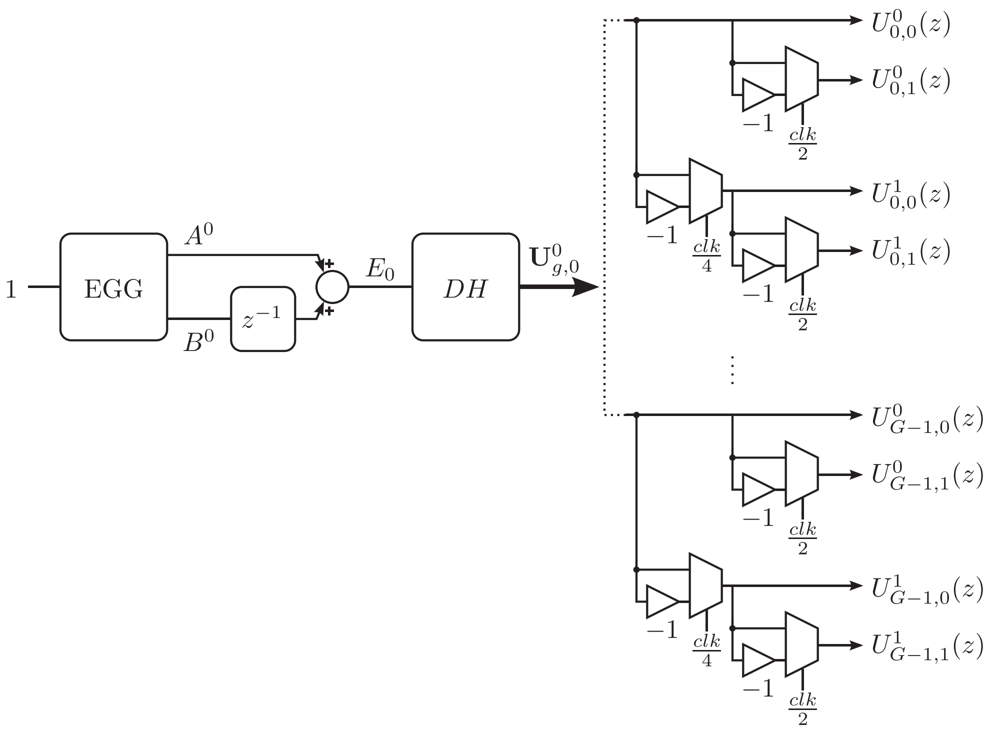

Figure 14. Simultaneous generator architecture for a complete set of Generalized Pairwise Complementary sequences. EGG, efficient Golay generator.

4. COMPARISONS

In order to formalize the efficiency of the new proposal, an analysis of the required operations to perform the generation and correlation of a complete set of GPC sequences was conducted. Table I lists the amount of operations required to obtain the correlation with respect to the complete set of GPC sequences, with L_0 being the length of the Golay pair of sequences used to generate the E-sequences

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Table I. Comparison of the operations required to perform a correlation with a complete set of Generalized Pairwise Complementary sequences.

	Proposed correlator		
	Products	Additions	Memory
First stage	0	2	1
O^{2}GC	$\log_2 L_0 + 1$	$2(\log_2 L_0 + 1)$	$2(L_0 - 1)$
2× $\boldsymbol{D'H}$	0	$2G \log_2 G$	$4(G-1)L_0$
Total	$\log_2 L_0 + 1$	$2(G\log_2 G + \log_2 L_0 + 2)$	$2L_0(2G-1)-1$
García *et al.* [15]			
Total	$2G(3\log_2 L_0 + G - 1)$	$2G(2\log_2 L_0 + G + 1)$	$2G(2GL_0 - 1)$
Straightforward correlation			
Total	$2G \cdot L_0$	$2(G \cdot L_0 - 1)$	$2(G \cdot L_0 - 1)$

O^{2}GC, Optimized Orthogonal Golay Correlator.

Table II. Operations required for the proposed generator and Efficient Generalized Pairwise Complementary Generator [15].

	Proposed generator			
	Products	Additions	Memory	Multiplexers
EGG	$\log_2 L_0$	$2\log_2 L_0$	$2(L_0 - 1)$	0
E-sequence generator	0	1	1	0
$\boldsymbol{DH}$	0	$G\log_2 G$	$2(G-1)L_0$	0
Final stage	$3G$	0	0	$3G$
Total	$\log_2 L_0 + 3G$	$2\log_2 L_0 + G\log_2 G + 1$	$2L_0G - 1$	$3G$
García *et al.* [15]				
$2G\times$	Products	Additions	Memory	Multiplexers
EGG	$2G\log_2 L_0$	$4G\log_2 L_0$	$4G(L_0 - 1)$	0
E-sequence generator	0	$2G$	$2G$	0
α	$2G(G-1)$	$2G(G-1)$	$4G(G-1)L_0$	0
Final stage	$2G$	0	0	$2G$
Total	$2G(G + \log_2 L_0)$	$2G(G + 2\log_2 L_0)$	$2G(2GL_0 - 1)$	$2G$

EGG, efficient Golay generator.

and G, the order of the Hadamard matrix used in the Walsh–Hadamard expansion. The required calculations to perform the same correlation using the EGPCC and a straightforward correlator are also shown. It is worth noticing that the differences between the operations required by each of the architectures increases with G. In the case of the proposed correlator, the products are constant with respect to G, because the architecture that implements the product $\boldsymbol{D'H}$ uses no multiplications.

Table II presents the amount of operations required for the simultaneous generation using the proposed architecture and that by García *et al*. Based on the table, it can be concluded that the proposed generator architecture requires fewer mathematical operations to generate a complete set of GPC sequences. It is noteworthy that even though the proposed generator has G more multiplexers than the other generator does, this increment is masked by a reduction of approximately $2G$ times fewer additions, products, and memory positions. Note that in this case, the products vary in terms of G, but the number of operations is still lower than those in [15].

5. CONCLUSIONS

This work presents optimized architectures for the simultaneous generation and correlation of a complete set of GPC sequences based on the particular mathematical properties of CSS and the previous contributions on that matter. The architectures proposed were compared with a

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straightforward correlator and simultaneous generators and correlators obtained by means of the replication of previously presented architectures, obtaining a notorious reduction in the number of required operations. Indeed, this is a major contribution to the practical application of GPC sequences in platforms of limited resources and/or in high demand processing applications.

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