Multi-Layer Transmission and Hybrid Relaying for Relay Channels with Multiple Out-of-Band Relays

Seok-Hwan Park, Osvaldo Simeone, Onur Sahin and Shlomo Shamai (Shitz)

Abstract

In this work, a relay channel is studied in which a source encoder communicates with a destination decoder through a number of out-of-band relays that are connected to the decoder through capacityconstrained digital backhaul links. This model is motivated by the uplink of cloud radio access networks. In this scenario, a novel transmission and relaying strategies are proposed in which multi-layer transmission is used, on the one hand, to adaptively leverage the different decoding capabilities of the relays and, on the other hand, to enable hybrid decode-and-forward (DF) and compress-and-forward (CF) relaying. The hybrid relaying strategy allows each relay to forward part of the decoded messages and a compressed version of the received signal to the decoder. The problem of optimizing the power allocation across the layers and the compression test channels is formulated. Albeit non-convex, the derived problem is found to belong to the class of so called complementary geometric programs (CGPs). Using this observation, an iterative algorithm based on the homotopy method is proposed that achieves a stationary point of the original problem by solving a sequence of geometric programming (GP), and thus convex, problems. Numerical results are provided that show the effectiveness of the proposed multi-layer hybrid scheme in achieving performance close to a theoretical (cutset) upper bound.

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Index Terms

Relay channel, multi-layer transmission, hybrid relaying, out-of-band relaying, cloud radio access networks.

I. INTRODUCTION

The multiple relay network, in which a source encoder wishes to communicate with a destination through a number of relays, as seen in Fig. 1, has been actively studied due to its wide range of applications. Most of the activity, starting from [1], focuses on Gaussian networks in which the first hop amounts to a Gaussian broadcast channel from source to relays and the second hop to a multiple access channel between relays and receivers. The literature on this subject is vast and includes the proposal of various transmission strategies, including *decode-and-forward* (DF) [1]-[3], *compress-and-forward* (CF) [1]-[9], *amplify-and-forward* (AF) [2][3][8] and hybrid AF-DF [2][8].

In this paper, we are concerned with a variation of the more classical multi-relay channel discussed above in which the relays are connected to the destination through digital backhaul links of finite-capacity. The motivation for this model comes from the application to so called cloud radio cellular networks, in which the base stations (BSs) act as relays connected to the central decoder via finite-capacity backhaul links [10][11]. This model was studied in [4]-[7][9][12] (see also review in [13]). References [4][6][7][9] focus on CF strategies, while [5] considers hybrid DF-CF strategies and [12] studies schemes based on *compute-and-forward*.

A. Contributions

In this paper, we propose a novel transmission and relaying strategy in which multi-layer transmission is used, on the one hand, in order to properly leverage the different decoding capabilities of the relays similar to [2], and, on the other hand, to enable hybrid DF and CF relaying. In the proposed hybrid relaying strategy, each relay forwards part of the decoded messages and a compressed version of the received signal. The multi-layer strategy is designed so as to facilitate decoding at the destination based on the information received from the relays. To this end, the proposed design is different from the classical broadcast coding approach of [14] in which each layer encodes an independent message. Instead, in the proposed scheme,

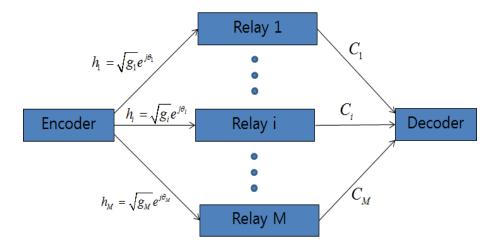


Figure 1. Illustration of the considered channel with multiple relays connected to the decoder via out-of-band digital backhaul links with given capacities.

each layer encodes an appropriately selected set of independent messages. It is emphasized that the hybrid DF-CF approach studied in [5] is based on single-layer transmission.

The problem of optimizing the power allocation across the layers and the compression test channels is formulated. Albeit non-convex, the derived problem is found to belong to the class of so called complementary geometric programs (CGPs) (see [15, Sec. 3.2] for more detail). Using this observation, an iterative algorithm based on the homotopy method is proposed that achieves a stationary point of the original problem by solving a sequence of geometric programming (GP) [16], and thus convex, problems. Numerical results are provided that show the effectiveness of the proposed multi-layer hybrid scheme in achieving performance close to a theoretical cutset upper bound [17, Theorem 1].

Notation: We use p(y|x) to denote conditional probability density function (pdf) of random variable X given Y. All logarithms are in base two unless specified. Given a sequence X_1, \ldots, X_m , we define a set $X_S = \{X_j | j \in S\}$ for a subset $S \subseteq \{1, \ldots, m\}$; we set X_{ϕ} as the empty set.

II. SYSTEM MODEL

We consider a relay channel in which a source encoder wishes to communicate with a destination decoder through a number M of relays as illustrated in Fig. 1. We denote the set of relays by $\mathcal{M} = \{1, \ldots, M\}$. The relays operate out of band in the sense that each *i*th relay is

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connected to the receiver via an orthogonal finite-capacity link of capacity C_i in bits per channel use (c.u.). The encoder transmits a signal X which is subject to power constraint $\mathbb{E}[|X|^2] \leq P$. Each relay *i* receives a signal Y_i which is given as

$$Y_i = h_i X + Z_i \tag{1}$$

with a complex channel coefficient $h_i = \sqrt{g_i} e^{j\theta_i}$ and independent additive white Gaussian noise (AWGN) $Z_i \sim C\mathcal{N}(0,1)$ for i = 1, ..., M. We assume that the channel coefficients $h_1, ..., h_M$ are constant over a transmission block and are perfectly known to all nodes. Without loss of generality, the channel powers $g_1, ..., g_M$ are assumed to be sorted such that

$$g_1 \leq \ldots \leq g_M. \tag{2}$$

III. MULTI-LAYER TRANSMISSION WITH HYBRID RELAYING

In this section, we propose a transmission strategy that is based on multi-layer transmission and hybrid relaying. Hybrid relaying is performed by having each relay forward part of the decoded messages, which amounts to partial decode-and-forward (DF), along with a compressed version of the received signal, thus adhering also to the compress-and-forward (CF) paradigm. The multi-layer strategy used at the source is designed so as to facilitate decoding at the destination based on the information received from the relays, as detailed below.

A. Multi-Layer Transmission

The amount of information decodable at the relays depends on the generally different fading powers g_1, \ldots, g_M . To leverage the different channel qualities, we enable flexible decoding at the relays by adopting a multi-layer transmission strategy at the encoder. This approach was also considered in [2] for the case of two relays that communicate to the decoder via multiple access Gaussian channels. We assume that the transmitter splits its message into M + 1independent submessages, say W_1, \ldots, W_{M+1} , with corresponding rates R_1, \ldots, R_{M+1} in bit/c.u., respectively. The idea is that message W_1 will be decoded by all relays, message W_2 only by relays $2, \ldots, M$, and so on. This way, relays with better channel conditions decode more information. Message W_{M+1} is instead decoded only at the destination.

To encode these messages, the encoded signal is given by

$$X = \sum_{k=1}^{M+1} \sqrt{P_k} X_k, \tag{3}$$

where the signals X_1, \ldots, X_{M+1} are independent and distributed as $\mathcal{CN}(0, 1)$, and the power coefficients P_1, \ldots, P_{M+1} are subject to the power constraint $\sum_{k=1}^{M+1} P_k \leq P$. The signal X_1 encodes message W_1 , signal X_2 encodes both message W_1 and W_2 , and so on, so that signal X_k encodes messages W_1, \ldots, W_k for $k = 1, \ldots, M$. Note that, unlike classical multi-layer transmission [14][18], here signal X_k does not only encode message W_k . The reason for this choice will be clarified below. Finally, signal X_{M+1} encodes message W_{M+1} .

Relay 1 decodes message W_1 from X_1 ; relay 2 first decodes message W_1 from X_1 and then message W_2 from X_2 using its knowledge of W_1 ; and so on, so that relay k decodes messages W_1, \ldots, W_k for $k = 1, \ldots, M$. From standard information-theoretic considerations, the following conditions are sufficient to guarantee that rates R_k are decodable by the relays [14]

$$R_k \le I\left(X_k; Y_k | X_1, \dots, X_{k-1}\right),\tag{4}$$

for k = 1, ..., M. This is because, by (3), condition (4) with k = 1, namely $R_1 \leq I(X_1; Y_1)$ ensures that not only relay 1 but all relays can decode message W_1 ; and, generalizing, the inequality (4) for a given k guarantees that not only relay k can decode message W_k after having decoded $W_1, ..., W_{k-1}$, but also all relays k + 1, ..., M can. The signal X_{M+1} , and thus message W_{M+1} is decoded by the destination only as it will be described in the next subsection.

B. Hybrid Relaying

As discussed, relay *i* decodes messages W_1, \ldots, W_i . Then, each *i*th relay transmits *partial* information about the decoded messages to the destination via the backhaul links. The rate at which this partial information is transmitted to the destination is selected so as to enable the latter to decode messages W_1, \ldots, W_M jointly based on all the signals received from the relays. This step will be detailed below. We denote as $C_i^{\text{DF}} \leq C_i$ the portion of the backhaul capacity devoted to the transmission of the messages decoded by relay *i*.

Beside the rate allocated to the transmission of (part of) the decoded messages, relay *i* utilizes the residual backhaul link to send a compressed version \hat{Y}_i of the received signal Y_i . The compression strategy at relay *i* is characterized by the test channel $p(\hat{y}_i|y_i)$ according to conventional rate-distortion theory arguments (see, e.g., [19]). Moreover, since the received signals at different relays are correlated with each other, it is beneficial to adopt a distributed source coding strategy. Here, similar to [7][9][20], we use successive decoding via Wyner-Ziv

compression with a given order $\hat{Y}_{\pi(1)} \to \ldots \to \hat{Y}_{\pi(M)}$, where $\pi(i)$ is a given permutation of the relays' indices \mathcal{M} . Thus, the decoder can successfully retrieve the descriptions $\hat{Y}_1, \ldots, \hat{Y}_M$ if the conditions [21]

$$I\left(Y_{\pi(i)}; \hat{Y}_{\pi(i)} | \hat{Y}_{\{\pi(1),\dots,\pi(i-1)\}}\right) \le C_{\pi(i)}^{\rm CF}$$
(5)

are satisfied for all i = 1, ..., M, where we defined $C_i^{\text{CF}} \leq C_i$ as the capacity allocated by relay i to communicate the compressed received signal \hat{Y}_i to the decoder. It is recalled that (5) is the rate needed to compress $Y_{\pi(i)}$ as $\hat{Y}_{\pi(i)}$ given that the destination has side information given by the previously decompressed signals $\hat{Y}_{\pi(1)}, \ldots, \hat{Y}_{\pi(i-1)}$.

Without claim of optimality, we assume Gaussian test channel $p(\hat{y}_i|y_i)$, so that the compressed signal \hat{Y}_i can be expressed as

$$\hat{Y}_i = Y_i + Q_i,\tag{6}$$

where the compression noise $Q_i \sim C\mathcal{N}(0, \sigma_i^2)$ is independent of the received signal Y_i to be compressed. We observe that assumption of the Gaussian test channels (6) does not involve any loss of optimality if the relays are allowed to perform only the CF strategy [6][22][23]. We remark that the compression strategy (6) at relay *i* is characterized by a single parameter σ_i^2 .

C. Decoding

The destination decoder is assumed to first recover the descriptions $\hat{Y}_1, \ldots, \hat{Y}_M$ from the signals received by the relays. This step is successful as long as conditions (5) are satisfied. Having obtained $\hat{Y}_{\mathcal{M}} = {\hat{Y}_1, \ldots, \hat{Y}_M}$, the destination decodes jointly the messages W_1, \ldots, W_M based on the partial information about these messages received from the relays and on the compressed received signals $\hat{Y}_{\mathcal{M}}$. Finally, message W_{M+1} is decoded. The following lemma describes the set of tuples (R_1, \ldots, R_{M+1}) that is achievable via this strategy.

Lemma 1. A rate tuple (R_1, \ldots, R_{M+1}) is achievable by the proposed multi-layer strategy with hybrid relaying if the following conditions are satisfied for some values of $C_i^{\text{DF}} \in [0, C_i]$,

i = 1, ..., M:

$$R_i \le I(X_i; Y_i | X_1, \dots, X_{i-1}), \ i = 1, \dots, M,$$
(7)

$$C_{\pi(i)}^{\rm DF} + I\left(Y_{\pi(i)}; \hat{Y}_{\pi(i)} | \hat{Y}_{\{\pi(1),\dots,\pi(i-1)\}}\right) \le C_{\pi(i)}, \ i = 1,\dots,M,$$
(8)

$$\sum_{j=k}^{M} R_j \le \sum_{j=k}^{M} C_j^{\text{DF}} + I\left(X_{\{k,\dots,M\}}; \hat{Y}_{\mathcal{M}} | X_{\{1,\dots,k-1\}}\right), \ k = 1,\dots,M,$$
(9)

and
$$R_{M+1} \leq I\left(X_{M+1}; \hat{Y}_{\mathcal{M}} | X_{\mathcal{M}}\right).$$
 (10)

Proof: The constraint (7) corresponds to (4) and guarantees correct decoding at the relays. Constraint (8) follows from (5) and the backhaul constraint. The inequalities in (9) ensure that the messages W_1, \ldots, W_M are correctly decoded by the destination based on the partial information received from the relays and the compressed signals \hat{Y}_M . This is a consequence of well-known results on the capacity of multiple access channels with transmitters encoding given subsets of messages [24] (see also [25]), as recalled in Appendix A. We observe here that the sufficiency of (9) for correct decoding hinges on the fact that signal X_k encodes messages W_1, \ldots, W_k for $k = 1, \ldots, M$, and not merely W_k as in the more conventional multi-layer approach [18][14]. Finally, constraint (10) ensures the correct decoding of message W_{M+1} based on all the decoded signals X_M and the compressed received signals \hat{Y}_M .

IV. OPTIMIZATION

In this section, we are interested in optimizing the power allocation P_1, \ldots, P_{M+1} , the compression test channels characterized by the compression noise variances $\sigma_1^2, \ldots, \sigma_M^2$ and the backhaul capacity allocation between DF and CF relaying, with the aim of maximizing the sum-rate $R_{\text{sum}} = \sum_{k=1}^{M+1} R_k$. Based on Lemma 1, this problem is formulated as

$$\begin{array}{l}
\text{maximize} & \sum_{k=1}^{M+1} R_k \\
\pi, \{P_k, R_k \ge 0\}_{k=1}^{M+1}, & \sum_{k=1}^{M+1} R_k \\
\{\sigma_i^2, C_i^{\text{DF}} \ge 0\}_{i=1}^M \\
\text{s.t.} & (7) - (10), \\
& \sum_{k=1}^{M+1} P_k \le P.
\end{array}$$
(11)

In (11), the optimization space includes the ordering π used for decompression at the decoder, along with the mentioned power and backhaul allocations and the compression noises. Due to the inclusion of the ordering π , the problem is combinatorial. Therefore, in this section, we focus on the optimization of the other variables for fixed ordering π . Optimization of π will then have to be generally performed using an exhaustive search procedure or using a suitable heuristic method.

Under the assumption of the multi-layer transmission (3), the Gaussian test channels (6) and given ordering π , the problem (11) can be written as

$$\begin{array}{l} \underset{\{R_{i}, C_{i}^{\text{DF}} \geq 0, \ \beta_{i} \in [0, 1]\}_{i=1}^{M}, \\ \{P_{i} \geq 0\}_{i=1}^{M+1} \\ \text{s.t.} \quad R_{i} \leq \log\left(\frac{1+g_{i}\bar{P}_{i}}{1+g_{i}\bar{P}_{i+1}}\right), \ i = 1, \dots, M, \end{array}$$
(12a) (12a)

$$C_{i}^{\rm DF} + \log\left(\frac{1 + \bar{P}_{1}\bar{\beta}_{\pi^{-1}(i)}}{1 + \bar{P}_{1}\bar{\beta}_{\pi^{-1}(i)-1}}\right) - \log\left(1 - \beta_{i}\right) \le C_{i}, \ i = 1, \dots, M$$
(12c)

$$\sum_{j=k}^{M} R_{j} \leq \sum_{j=k}^{M} C_{j}^{\text{DF}} + \log\left(\frac{1+\bar{P}_{k}\bar{\beta}_{M}}{1+P_{M+1}\bar{\beta}_{M}}\right), \ k = 1, \dots, M,$$
(12d)

$$\bar{P}_1 \le P,$$
 (12e)

where we have defined variables $\beta_i = 1/(1 + \sigma_i^2) \in [0, 1]$ for i = 1, ..., M, the cumulative powers $\overline{P}_k = \sum_{j=k}^{M+1} P_j$ for k = 1, ..., M + 1, the cumulative variables $\overline{\beta}_i = \sum_{j=1}^i g_{\pi(j)} \beta_{\pi(j)}$ for i = 1, ..., M and the function $\pi^{-1}(j)$ returns the position of the index $j \in \{1, ..., M\}$ in the ordering π . The problem (12) is not easy to solve due to the non-convexity of the constraints (12b)-(12d). In Sec. IV-A, we propose an iterative algorithm to find a stationary point of the problem (12).

A. Proposed Algorithm

Here we propose an iterative algorithm for finding a stationary point of problem (12). We first simplify the problem by proving the following lemma.

Lemma 2. Imposing equalities on the constraints (12b) and (12c) induces no loss of optimality.

Proof: Suppose that the constraints (12b) or (12c) are not satisfied with equality. Then, we can decrease the transmission powers P_1, \ldots, P_{M+1} or increase the backhaul usage until the constraints are tight without decreasing the achievable rate.

With Lemma 2 and some algebraic manipulations, the problem (12) can be written as

$$\underset{\{\bar{P}_{i}\geq 0\}_{i=1}^{M+1},\{\bar{\beta}_{i},\gamma_{i}\geq 0\}_{i=1}^{M}}{\text{minimize}} \frac{1}{1+P_{M+1}\bar{\beta}_{M}} \prod_{i=1}^{M} \frac{1+g_{i}\bar{P}_{i+1}}{1+g_{i}\bar{P}_{i}}$$
(13a)

s.t.
$$\frac{1 + \bar{P}_{M+1}\bar{\beta}_M}{2\sum_{i=k}^M C_i \left(1 + \bar{P}_k\bar{\beta}_M\right)} \prod_{i=k}^M \left\{ \frac{\left(1 + g_i\bar{P}_i\right) \left(1 + \bar{P}_1\bar{\beta}_{\pi^{-1}(i)}\right)}{\gamma_i \left(1 + g_i\bar{P}_{i+1}\right) \left(1 + \bar{P}_1\bar{\beta}_{\pi^{-1}(i)-1}\right)} \right\} \le 1, \ k = 1, \dots, M$$
(13b)

$$\frac{1+\bar{P}_1\bar{\beta}_{\pi^{-1}(i)}}{2^{C_i}\left(1+\bar{P}_1\bar{\beta}_{\pi^{-1}(i)-1}\right)\gamma_i} \le 1, \ i=1,\dots,M,$$
(13c)

$$\frac{\bar{P}_1}{P} \le 1,\tag{13d}$$

$$\frac{\bar{P}_{i+1}}{\bar{P}_i} \le 1, \ \frac{\bar{\beta}_{i-1}}{\bar{\beta}_i} \le 1, \ i = 1, \dots, M,$$
(13e)

$$\frac{\bar{\beta}_i}{g_{\pi(i)} + \bar{\beta}_{i-1}} \le 1, \ \frac{g_i \gamma_i + \beta_{\pi^{-1}(i)}}{g_i \gamma_i + \bar{\beta}_{\pi^{-1}(i)-1}} \le 1, \ i = 1, \dots, M,$$
(13f)

where we characterized the problem over the cumulative variables $\{\bar{P}_i\}_{i=1}^{M+1}$ and $\{\bar{\beta}_i\}_{i=1}^M$, and introduced auxiliary variables $\gamma_i = 1 - (\bar{\beta}_{\pi^{-1}(i)} - \bar{\beta}_{\pi^{-1}(i)-1})/g_i$ for $i = 1, \ldots, M$.

Problem (13) is not a standard GP [16] since the denominators in the left-hand side of (13b), (13c) and (13f) are not monomials. However, the problem is a class of CGP problems [15, Sec. 3.2], and thus a stationary point of (13) can be found by applying the homotopy method [15, Sec. 3.2], which solves a sequence of GPs¹ obtained by locally approximating the posynomial denominators as monomial expressions (see, e.g., [15, Lemma 3.1]). The resulting algorithm is summarized in Table Algorithm 1.

V. SPECIAL CASES

Here we discuss some relevant special cases of the proposed scheme.

¹A GP can be converted into an equivalent convex problem (see [16, Sec. 4.5.3] for more detail).

Algorithm 1 Homotopy method for problem (13)

1. Initialize the variables $\{\bar{P}_i^{(1)} \ge 0\}_{i=1}^{M+1}, \{\bar{\beta}_i^{(1)} \ge 0\}_{i=1}^M$ to an arbitrary feasible point and set n = 1.

2. Update the variables $\{\bar{P}_i^{(n+1)} \ge 0\}_{i=1}^{M+1}, \{\bar{\beta}_i^{(n+1)} \ge 0\}_{i=1}^M$ as a solution of the following GP problem:

$$\begin{split} \underset{\{\bar{P}_{i}^{(n+1)} \ge 0\}_{i=1}^{M+1}, \{\bar{\beta}_{i}^{(n+1)}, \gamma_{i} \ge 0\}_{i=1}^{M}}{\text{minimize}} & \frac{1}{f\left(P_{M+1}^{(n+1)}\bar{\beta}_{M}^{(n+1)}, P_{M+1}^{(n)}\bar{\beta}_{M}^{(n)}\right)} \prod_{i=1}^{M} \frac{1 + g_{i}\bar{P}_{i+1}^{(n+1)}}{f\left(g_{i}\bar{P}_{i}^{(n+1)}, g_{i}\bar{P}_{i}^{(n)}\right)} & (14) \\ \text{s.t.} & \prod_{i=k}^{M} \left\{ \frac{\left(1 + g_{i}\bar{P}_{i+1}^{(n+1)}, g_{i}\bar{P}_{i+1}^{(n)}\right) \left(1 + \bar{P}_{1}^{(n+1)}\bar{\beta}_{\pi^{-1}(i)}^{(n+1)}\right)}{\gamma_{i}f\left(g_{i}\bar{P}_{i+1}^{(n+1)}, g_{i}\bar{P}_{i+1}^{(n)}\right) f\left(\bar{P}_{1}^{(n+1)}\bar{\beta}_{\pi^{-1}(i)-1}^{(n+1)}, \bar{P}_{1}^{(n)}\bar{\beta}_{\pi^{-1}(i)-1}^{(n)}\right)} \right\} \\ & \times \frac{1 + \bar{P}_{M+1}^{(n+1)}\bar{\beta}_{M}^{(n+1)}, g_{i}\bar{p}_{M}^{(n)}}{2\sum_{i=k}^{i}C_{i}f\left(\bar{P}_{k}^{(n+1)}\bar{\beta}_{M}^{(n+1)}, \bar{P}_{k}^{(n)}\bar{\beta}_{M}^{(n)}\right)} \le 1, \ k = 1, \dots, M, \\ \frac{1 + \bar{P}_{1}^{(n+1)}\bar{\beta}_{\pi^{-1}(i)}^{(n+1)}}{P} \le 1, \ \frac{\bar{P}_{1}^{(n+1)}}{P} \le 1, \ i = 1, \dots, M, \\ \frac{\bar{P}_{1}^{(n+1)}}{\bar{P}_{i}^{(n+1)}} \le 1, \ \frac{\bar{\beta}_{i-1}^{(n+1)}}{\bar{\beta}_{i}^{(n+1)}} \le 1, \ i = 1, \dots, M, \\ \frac{\bar{\beta}_{i}^{(n+1)}}{g_{\pi(i)}f\left(\bar{\beta}_{i-1}^{(n+1)}/g_{\pi(i)}, \bar{\beta}_{i-1}^{(n)}/g_{\pi(i)}\right)} \le 1, \ i = 1, \dots, M, \\ \frac{g_{i}\gamma_{i} + \bar{\beta}_{\pi^{-1}(i)}^{(n+1)}}{g_{i}(j\left(\bar{\beta}_{i-1}^{(n+1)}/g_{\pi(i)}, \bar{\beta}_{\pi^{-1}(i)-1}^{(n)}/g_{i}\right)} \le 1, \ i = 1, \dots, M, \end{split}$$

where the function $f(s, \hat{s})$ is a monomial function of s defined as [15, Lemma 3.1]

$$f(s, \hat{s}) = c(\hat{s})s^{a(\hat{s})}$$
 (15)

with $a(\hat{s}) = \hat{s}(1+\hat{s})^{-1}$ and $c(\hat{s}) = \hat{s}^{-a}(1+\hat{s})$. 3. Stop if some convergence criterion is satisfied. Otherwise, set $n \leftarrow n+1$ and go to Step 2.

A. Compress-and-Forward

If we impose that the encoder uses only the highest layer X_{M+1} , i.e., $X = \sqrt{P}X_{M+1}$ in lieu of the more general (3), the proposed hybrid scheme reduces to a pure CF scheme with successive decoding as studied in [7][9]. Optimization of the test channels β_1, \ldots, β_M under this assumption and given ordering π can be simplified to

$$\underset{\beta_{1},\dots,\beta_{M}\geq 0}{\operatorname{maximize}} \log\left(1+P\sum_{j=1}^{M}g_{j}\beta_{j}\right)$$
s.t.
$$\log\left(\frac{1+P\bar{\beta}_{i}}{1+P\bar{\beta}_{i-1}}\right) - \log\left(1-\beta_{\pi(i)}\right) \leq C_{\pi(i)}, \ i=1,\dots,M,$$

$$(16)$$

whose solutions $\beta_1^{\text{opt}}, \ldots, \beta_M^{\text{opt}}$ are directly given, using Lemma 2, as

$$\beta_{\pi(i)}^{\text{opt}} = \frac{\left(2^{C_{\pi(i)}} - 1\right)\left(1 + P\bar{\beta}_{i-1}\right)}{2^{C_i}\left(1 + P\bar{\beta}_{i-1}\right) + Pg_{\pi(i)}}, \ i = 1, \dots, M.$$
(17)

B. Decode-and-Forward

The DF strategy is a special case of the proposed hybrid relaying scheme obtained by fixing $\beta_1 = \ldots = \beta_M = 0$ and $P_{M+1} = 0$. A similar approach was studied in [2, Sec. V-B] for M = 2 assuming Gaussian channels for relay-to-destination links. A stationary point of the problem can be obtained by adopting the homotopy method in Algorithm 1 with minor modifications. As an interesting special case, we consider DF with single-layer transmission in which multi-layer transmission is not leveraged.

Using single-layer transmission, the following rate is achievable by optimizing the selection of the transmitted layer:

$$\max_{i \in \mathcal{M}} \min\left\{ \log\left(1 + g_i P\right), \sum_{j=i}^{M} C_j \right\}.$$
(18)

We remark that in (18) we have used the fact, as in the more general result of Lemma 1, that all relays i, \ldots, M are able to decode message W_i and thus the message can be distributed across the backhaul links in order to be delivered to the destination.

VI. NUMERICAL RESULTS

In this section, we present numerical results to investigate the advantage of the proposed multi-layer transmission scheme with hybrid relaying studied in Sec. III-IV as compared to the

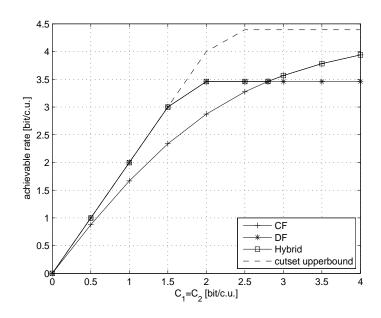


Figure 2. Achievable rates versus the backhaul capacity $C_1 = C_2$ in a symmetric network with M = 2, $P = 0 \,dB$ and $g_1 = g_2 = 10 \,dB$.

more conventional schemes reviewed in Sec. V. For reference, we also compare the achievable rates with the cutset upper bound [17, Theorem 1]

$$R_{\text{cutset}} = \min_{\mathcal{S} \subseteq \{1, \dots, M\}} \left\{ \sum_{j \in \mathcal{S}} C_j + \log \left(1 + P \sum_{j \in \mathcal{S}^c} g_j \right) \right\}.$$
 (19)

For ease of interpretation, we focus on the case with two relays, i.e., M = 2. We mark singlelayer schemes with the label 'SL' and multi-layer schemes with 'ML'. For CF related schemes, the optimal ordering π^{opt} in problem (11) was found via exhaustive search and was observed to be $\pi = (1, 2)$ for all the simulated cases.

In Fig. 2, we examine the performance in a symmetric setting by plotting the rate versus the backhaul capacities $C_1 = C_2$ when $P = 0 \,\mathrm{dB}$ and $g_1 = g_2 = 10 \,\mathrm{dB}$. It is seen that in this symmetric set-up, the optimized hybrid scheme ends up reducing to either the DF or the CF strategy at small and large backhaul capacity, respectively. Note that we have not distinguished between the single-layer and multi-layer strategies in the figure since they showed the same performance when the relays experience the same fading power, i.e., $g_1 = g_2$. This is expected since multi-layer strategies are relevant only when the two relays have different decoding capabilities.

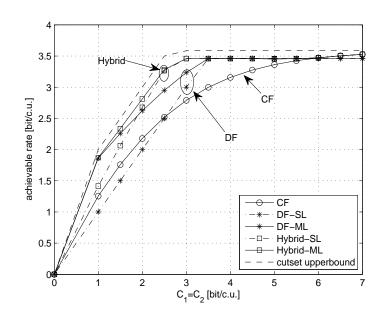


Figure 3. Achievable rates versus the backhaul capacity $C_1 = C_2$ per relay with M = 2, P = 0 dB and $[g_1, g_2] = [0, 10] dB$.

In Fig. 3, we observe the performance versus the backhaul capacity $C_1 = C_2$ with P = 0 dBand asymmetric channel powers $[g_1, g_2] = [0, 10] \text{ dB}$. Unlike the symmetric setting in Fig. 2, the multi-layer strategy is beneficial compared to the single-layer (SL) transmission for both DF and Hybrid schemes². Moreover, unlike the setting of Fig. 2, the hybrid relaying strategy shows a performance advantage with respect to all other schemes. This is specifically the case for intermediate values of the backhaul capacities $C_1 = C_2$. It should also be mentioned that, as $C_1 = C_2$ increases, the performance of DF schemes is limited by the capacity of the better decoder, namely $\log_2(1 + 10) = 3.46$ bit/c.u., while CF, and thus also the hybrid strategy, are able, for $C_1 = C_2$ large enough, to achieve the cutset bound.

Finally, in Fig. 4, we plot the achievable rates versus the channel power g_2 of the better relay when P = 0 dB, $g_1 = 0 \text{ dB}$ and $C_1 = C_2 = 2$ bit/c.u.. As expected, the performance gain of multi-layer transmission over the single-layer schemes is more pronounced as g_2 increases, since a better channel to relay 2 allows to support larger rates for both rates of both DF layers. In fact, single-layer transmission uses only the DF layer decoded exclusively by relay 2 according to (18). For the same reason, the rate of single-layer DF is limited by the backhaul capacity

²Not being based on relay decoding, CF operates only with one layer.

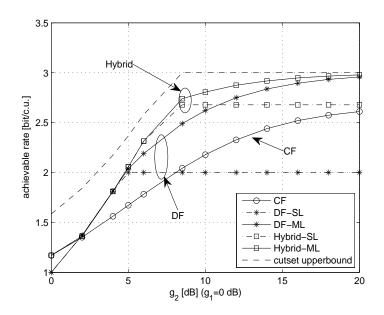


Figure 4. Achievable rates versus the channel power g_2 with M = 2, P = 0 dB, $g_1 = 0 dB$ and $C_1 = C_2 = 2$ bit/c.u.

 C_2 of relay 2. Moreover, hybrid relaying is advantageous over all conventional schemes for intermediate values of g_2 .

VII. CONCLUSIONS

We have studied transmission and relaying techniques for the relay channels with multiple out-of-band relays, which are connected to the destination via orthogonal finite-capacity backhaul links. We proposed a novel transmission and relaying strategies whereby multi-layer transmission is used at the encoder and hybrid DF-CF relaying is adopted at the relays. The multi-layer transmission is designed so as to adaptively leverage the different decoding capabilities of the relays and to enable the hybrid relaying strategy. As a result, the proposed multi-layer strategy is different from the classical broadcast coding approach of [14], which aims at coping with uncertain fading conditions at the transmitter (see also [8] for an application to a multi-relay setting).

We aimed at maximizing the achievable rate, which is formulated as a non-convex problem. However, based on the observation that the problem falls in the class of so called Complementary Geometric Programs (CGPs), we have proposed an iterative algorithm based on the homotopy method which attains a stationary point of the problem. From numerical results, it was shown that the proposed multi-layer transmission with the hybrid relaying strategy outperforms more conventional decode-and-forward, compress-and-forward and single-layer strategies, especially in the regime of moderate backhaul capacities and asymmetric channel gains from the source to the relays.

APPENDIX A

PROOF OF LEMMA 1

Here, we show that conditions (9) are sufficient for correct decoding of messages W_1, \ldots, W_M at the decoder. To see this, we observe that the destination, when decoding messages W_1, \ldots, W_M , can be regarded as the decoder of a multiple access channel with M sources. Specifically, source khas messages W_1, \ldots, W_k for $k = 1, \ldots, M$ and has two inputs to the channel to the destination, namely the signal X_k and the information sent at rate C_k^{DF} on the noiseless backhaul link. We denote the latter as T_k , where $T_k \in \{1, \ldots, 2^{C_k^{\text{DF}}}\}$ so that the overall channel input of the source k is given by $\tilde{X}_k = (X_k, T_k)$. The destination observes \hat{Y}_M and T_1, \ldots, T_M . We emphasize that both X_k and T_k in \tilde{X}_k depend on all messages W_1, \ldots, W_k .

As a result, we have an equivalent multiple access channel in which each source has a specific subset of all the messages and a hierarchy exists among the sources so that source k has all the messages also available to sources $1, \ldots, k-1$. Therefore, using the results in [24][25], the following conditions guarantee correct decoding of messages W_1, \ldots, W_M

$$\sum_{j=k}^{M} R_{j} \leq I\left(\tilde{X}_{\{k,\dots,M\}}; \hat{Y}_{\mathcal{M}}, T_{\{1,\dots,M\}} | \tilde{X}_{\{1,\dots,k-1\}}\right),$$
(20)

for k = 1, ..., M. The achievability of rates (20) is ensured for any joint distribution of the inputs $\{\tilde{X}_k\}_{k=1}^M$ [24][25]. To proceed, we take \tilde{X}_k to be independent according to the discussion around (3), and also take X_k to be independent of T_k for all k = 1, ..., M. It is not hard to see that this choice maximizes the mutual informations in (20). Under these assumptions, we can write the right-hand side of (20) as

$$I\left(X_{\{k,\dots,M\}}, T_{\{k,\dots,M\}}; \hat{Y}_{\mathcal{M}}, T_{\{1,\dots,M\}} | X_{\{1,\dots,k-1\}}, T_{\{1,\dots,k-1\}}\right)$$

$$=I\left(X_{\{k,\dots,M\}}; \hat{Y}_{\mathcal{M}} | X_{\{1,\dots,k-1\}}\right) + H\left(T_{\{k,\dots,M\}}\right)$$

$$=I\left(X_{\{k,\dots,M\}}; \hat{Y}_{\mathcal{M}} | X_{\{1,\dots,k-1\}}\right) + \sum_{j=k}^{M} C_{j}^{\mathrm{DF}},$$

$$(21)$$

by the chain rule for mutual informations [17, Theorem 2.5.2]. This proves that inequalities (20) reduce to (9) with the given choices.

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