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RESEARCH ARTICLE

# Design of a variable sampling interval exponentially weighted moving average median control chart in presence of measurement errors

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**Abstract**

In the literature, many control charts monitoring the median is designed under a perfect condition that there is no measurement error. This may make the practitioners confusing to apply these control charts because the measurement error is the true problem in practice. In this paper, we consider the effect of measurement error on the performance of the exponentially weighted moving average (EWMA) control chart combining with the variable sampling interval (VSI) strategy. A linear covariate error model is supposed to model the measurement error. The performance of the VSI EWMA median control chart is evaluated through the average time to signal. The numerical simulation shows that the measurement errors have a negative influence on the proposed chart.

**KEYWORDS**

control chart, EWMA, measurement errors, median, VSI

## 1 | INTRODUCTION

Statistical process control (SPC) is widely used in monitoring industrial manufacturing processes. Its goal is to improve product quality and to be sure that a process is in control. Among several techniques of SPC, a control chart is a useful tool to achieve this goal by helping manufacturers in detecting timely the out-of-control conditions. A control chart includes an upper control limit (UCL) and/or lower control limit (LCL) that allow defining whether or not a process is in control. The sample characteristics of interest are plotted on the control chart, and the number of samples plotted above the upper limit and/or under the lower limit is counted. Each type of control chart has its own rule to alarm a signal when the process is out of control.

In the literature, several control charts have been introduced, and the related references are abundant. One can name the Shewhart control chart as the first control chart designed by Dr. Shewhart.[1] This chart is very popular in SPC, and its wide application over a long period of time has motivated researchers to introduce many other control charts with higher performance in detecting process shifts. There are three main kind techniques in these new control charts, including new rules to alarm a signal such as the synthetic control chart (Hu et al.[2] and Tran et al.[3]) and the run rules control chart (Tran[4]); the adaptive strategies by changing parameters of the process such as the variable

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sampling interval (VSI) chart (Reynolds et al.[5]) and the variable sample size chart (Reynolds and Arnold[6]); and the advanced charts by considering the memories of the chart like the cumulative sum (CUSUM) control chart (Brook and Evans,[7] Wu et al.,[8] and Maravelakis[9]) and the exponentially weighted moving average (EWMA) control chart (Maravelakis et al.[10]). In addition, researchers recently suggest combining the adaptive strategies with the advanced control charts to provide new kinds of control charts like the VSI CUSUM control chart (Nguyen et al.[11]) and the VSI EWMA control chart (Nguyen et al.[12]). In general, these new hybrid control charts significantly improve the performance of the original charts.

The usual assumption used in many control charts is that there is no measurement error. However, this assumption seems to be not realistic in practice because the measurement error does exist for every measurement system no matter how good they are. Under the presence of measurement error, the performance of the control chart differs from the performance of the corresponding chart without considering the measurement error due to the change of the parameters of the model. This problem has been pointed out in various control charts by a lager number; see, for example, Nguyen et al.,[13-16] Linna et al.,[17] Costa and Castagliola,[18] Noorossana and Zerehsaz,[19] and Tran et al.[20,21]

Among several characteristics of interest from a process, the median is quite an important characteristic and the median (denoted by $\widetilde{X}$) chart is also a popular chart for monitoring the mean of a process. According to the discussion in Castagliola et al.,[22] the $\widetilde{X}$ chart is interested in monitoring a process running for a long time because it is robust against outliers or small deviations from normality. This property makes it more advantageous compared to the $\overline{X}$ chart in such a case. The study in Khoo[23] also points out that a sample median chart is suitable for detecting permanent shifts in the process. In the literature, the EWMA $\widetilde{X}$ chart is introduced by Castagliola.[24] This study is then extended in Tran et al.[25] by combining the EWMA chart with the VSI strategy. The Shewhart-$\widetilde{X}$ control chart is studied in Khoo.[23] The median control charts with estimated process parameters are also a major concern in several studies, such as Castagliola and Figueiredo[26] with the Shewhart-type chart, Castagliola et al.[22] with the EWMA-type chart, Saha et al.[27] with the run sum-type chart, and Khoo et al.[28] with VSI EWMA-type chart. Tran et al.[20] presents the performance of the Shewhart chart for a normal process under a linear covariate error model. The synthetic $\widetilde{X}$ control chart for monitoring the process mean with measurement errors is studied in Tran et al.[3]

The goal of this paper is to investigate the performance of the VSI EWMA-$\widetilde{X}$ control chart in the presence of measurement errors. As cited above, the VSI EWMA-$\widetilde{X}$ control chart has been designed in Tran et al.,[25] and it is shown that by combining with the VSI strategy, the VSI EWMA-$\widetilde{X}$ leads to impressive performance in detecting process mean shifts, and it improves significantly the performance of the EWMA-$\widetilde{X}$ control chart. However, in this study, the authors assumed that the characteristic of interest is measured exactly without error. To make the application of the VSI EWMA-$\widetilde{X}$ control chart more practical, it is necessary to consider the effect of measurement error on the chart's performance. Similar to previous studies, we apply the linear covariate error model introduced in Linna and Woodall[29] for the measurement error.

The rest of the paper is organized as follows. In Section 2, we provide briefly the linear covariate error model and the distribution of the sample median $\widetilde{X}$. Sections 3 and 4 present the implementation and the design of the VSI EWMA-$\widetilde{X}$ control chart. The effect of measurement error on the proposed control chart's performance is investigated in Section 5. An illustrative example based on the data form the production process of milk bottles is given in Section 6. Finally, Section 7 is for some concluding remarks.

## 2 | A BRIEF REVIEW OF THE LINEAR COVARIATE ERROR MODEL AND SAMPLE MEDIAN DISTRIBUTION

A brief review of the linear covariate error model (suggested in Linna and Woodall[29]) and the sample median distribution is presented in this section. To monitor the quality of characteristic $X$, suppose that a set of samples $\{X_{i,1}, X_{i,2}, \ldots, X_{i,n}\}$ of size $n$ is taken at the sampling period $i = 1, \ldots$. We assume that $X_{i,j}$ are independent and identically distributed from a normal distribution, $X_{i,j} \sim N\left(\mu_0 + \delta\sigma_0, \sigma_0^2\right)$, where $\delta$ represents the standardized shift size. The process is considered to be in control when $\delta = 0$; otherwise, it is is considered as out of control. The in-control $\mu_0$ and $\sigma_0$ are assumed to be known. Due to the measurement error problem, the *true* value $X_{i,j}$ is impossible to obtain. Instead, we can only obtain the *observed* value $\{X^*_{i,j,1}, X^*_{i,j,2}, \ldots, X^*_{i,j,m}\}$ where $X^*_{i,j,k}$ is the $k$th measurement of the item $j$ at the sampling time $i$ and $m \geq 1$ is the number of repeated measurements per item. As suggested in Linna and Woodall,[29] we have a linear covariate model as

$$X^*_{i,j,k} = A + BX_{i,j} + \varepsilon_{i,j,k},$$

where $A$ and $B$ are two constants estimated from phase I data and $\varepsilon_{i,j,k} \sim N(0, \sigma_M^2)$ is a normal random error, which is independent of $X_{i,j}$.

Let $\overline{X}^*_{i,j}$ denote mean of $m$ quantities $\{X^*_{i,j,1}, X^*_{i,j,2}, ..., X^*_{i,j,m}\}$, that is,

$$\begin{aligned} \overline{X}^*_{i,j} &= \frac{1}{m}\sum_{k=1}^{m} X^*_{i,j,k} \\ &= \frac{1}{m}\sum_{k=1}^{m} (A + BX_{i,j} + \varepsilon_{i,j,k}) \\ &= A + BX_{i,j} + \frac{1}{m}\sum_{k=1}^{m} \varepsilon_{i,j,k}. \end{aligned} \tag{1}$$

Then, $\overline{X}^*_{i,j}$ follows a normal distribution with mean

$$\mu^* = E(\overline{X}^*_{i,j}) = A + B(\mu_0 + \delta\sigma_0), \tag{2}$$

and standard variation

$$\sigma^* = \sigma(\overline{X}^*_{i,j}) = \sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}. \tag{3}$$

We now define the sample median of the mean values $\{\overline{X}^*_{i,1}, \overline{X}^*_{i,2}, ..., \overline{X}^*_{i,n}\}$ corresponding to subgroup $i = 1,2, ...,$ as

$$\widetilde{X}^*_i := \begin{cases} \overline{X}^*_{i,((n+1)/2)} & \text{if } n \text{ is odd,} \\ \dfrac{\overline{X}^*_{i,(n/2)} + \overline{X}^*_{i,(n/2+1)}}{2} & \text{if } n \text{ is even,} \end{cases} \tag{4}$$

where $\left\{\overline{X}^*_{i,(1)}, \overline{X}^*_{i,(2)}, ..., \overline{X}^*_{i,(n)}\right\}$ is the ordered sample of the mean values for each subgroup $i = 1,2,....$ According to the discussion in Tran et al.,[20] the sample size $n$ is usual to assume to be an odd value when working with the sample median. In the rest of the paper, we will consider the case where $n$ is odd. For this case, the cumulative distribution function (c.d.f.) $F_{\widetilde{X}^*}(x|n)$ of the sample median $\widetilde{X}^*_i$ is defined as (see Castagliola and Figueiredo[26])

$$\begin{aligned} F_{\widetilde{X}^*_i}(x|n) &= F_\beta\left(\Phi\left(\frac{x-\mu^*}{\sigma^*}\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) \\ &= F_\beta\left(\Phi\left(\frac{x - A - B(\mu_0 + \delta\sigma_0)}{\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}}\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right), \end{aligned} \tag{5}$$

where $\Phi(x)$ is the c.d.f. of the standard normal distribution and $F_\beta(x|a, b)$ is the c.d.f. of the beta distribution with parameters $(a, b) = \left(\frac{n+1}{2}, \frac{n+1}{2}\right)$.

## 3 | IMPLEMENTATION OF THE VSI EWMA-$\widetilde{X}$ CONTROL CHARTS WITH MEASUREMENT ERRORS

As in the EWMA-type chart, the observed characteristics of interest $\widetilde{X}^*_i$ are not monitored directly. Instead, we monitor its EWMA sequence

$$Z_i^* = (1-\lambda)Z_{i-1}^* + \lambda \widetilde{X}_i^*, i = 1,2,..., \tag{6}$$

where $\lambda$ is a smoothing parameter that can be assigned a value in [0,1] and the initial value $Z_0^* = \mu_0^* = A + B\mu_0$. Adjusting the value of $\lambda$ helps to change the memory of the EWMA charts to handle certain sizes of faults. The VSI EWMA-$\widetilde{X}$ control chart consists of the UCL, the LCL, the upper warning limit (UWL), and the lower warning limit (LWL), which are defined as follows.

$$LCL = A + B\mu_0 - K\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{7}$$

$$UCL = A + B\mu_0 + K\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{8}$$

$$LWL = A + B\mu_0 - W\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{9}$$

$$UWL = A + B\mu_0 + W\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m}\right)}, \tag{10}$$

where $K > 0$ is a control coefficient used to specify the width of the control limits and $W(0 < W < K)$ is an warning coefficient used to determine the width of the warning limits in the VSI-type chart. The closer to 0 is the warning limit coefficient $W$, the smaller is the warning region.

The proposed control chart is adaptive with respect to the time interval in which a sample is taken. The sampling interval varies depending on the information from the most recent sample collected. In particular, random samples are collected in intervals of variable length according to the function:

$$h(i) = \begin{cases} h_S \text{ if } Z_{i-1}^* \in R_S, \\ h_L \text{ if } Z_{i-1}^* \in R_L, \end{cases} \tag{11}$$

where $h(i)$ is the time practiced to take the $i$th sample and $h_S$ and $h_L$ represent the "short time" and "long time" interval, with the corresponding regions $R_S$ and $R_L$. Figure 1 shows a graphical view of the VSI EWMA-$\widetilde{X}$ chart. It is divided into three regions:

- The central region, that is, [$LWL$,$UWL$]. When the control statistic $Z_i^*$ falls within this region, the next sample should be taken after a "long time" interval $h_L$, because there is no evidence to say that the process is being shifted.

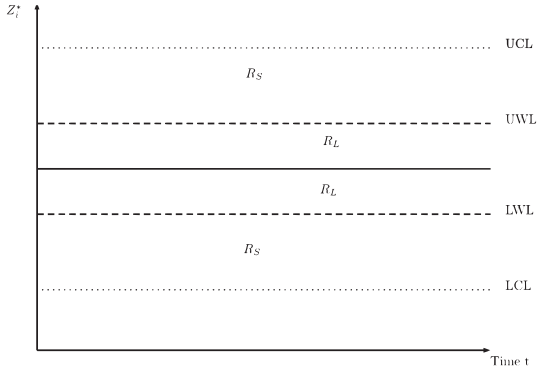



**FIGURE 1** Three regions of the variable sampling interval (VSI) exponentially weighted moving average (EWMA)-$\widetilde{X}$ control chart. LCL, lower control limit; LWL, lower warning limit; UCL, upper control limit; UWL, upper warning limit

- The warning region, that is, $[LCL,LWL) \cup (UWL,UCL]$. If the control statistic falls into this region, the next sample should be taken faster, after a "short time" $h_S$, because the process seems to be shifting into out-of-control conditions.
- Out-of-control regions, that is, $(-\infty,LCL) \cup (UCL, +\infty)$. A signal from the chart is given to alarm that the process is out of control.

## 4 | DESIGN OF OPTIMAL VSI EWMA-$\widetilde{X}$ CONTROL CHARTS WITH MEASUREMENT ERRORS

The performance of the VSI EWMA-$\widetilde{X}$ control chart is evaluated by the measure of average time to signal ($ATS$). It is the expected value of the time after the process starts until the first signal. When the process is in control, it is denoted by $ATS_0$. Conversely, when the process is out of control, it is denoted by $ATS_1$. For any VSI-type chart, the smaller the value of the $ATS_1$, the better the performance of the control chart. In this study, the $ATS$ is calculated by using the Markov chain method which is proposed by Brook and Evans[7] (see Appendix A1).

When samples are taken at fixed intervals of time (FSI), the $ATS$ is simply a multiple of the $ARL$. The sampling interval $h_F$ is fixed, and we have

$$ATS^{\mathrm{FSI}} = h_F \times ARL^{\mathrm{FSI}}. \tag{12}$$

Because of the variation of the interval sampling in the VSI-type chart, the $ATS$ is defined as

$$ATS^{\mathrm{VSI}} = E(h) \times ARL^{\mathrm{VSI}}, \tag{13}$$

where $E(h)$ is the expected sampling interval value.

The design of the proposed control chart is defined by finding related parameters that minimize the $ATS_1$. To make a fair comparison with the FSI charts, we suppose that the in-control values of both the average time to signal $ATS_0$ and the average sampling interval $E_0(h)$ are the same as $ARL_0$ and $h_F$. Without loss of generality, we assume that $h_F = 1$ time unit in the FSI-type chart. Moreover, according to the discussion in Nguyen et al.,[12] the short sampling interval $h_S$ and the warning coefficient $W$ are fixed. Thus, the design of the VSI EWMA-$\widetilde{X}$ control chart is now to find an optimal combination of parameters $\lambda^*$, $K^*$, and $h_L^*$ which minimize the out-of-control $ATS$ for predefined values of $ATS_0$, $W$, and $h_S$ and specific values of $n$, $\delta$, $m$,$B$, and $\eta$. In particular, the optimization problem consists in finding the optimal parameters $\lambda^*$, $K^*$, and $h_L^*$ such that

$$(\lambda^*,K^*,h_L^*) = \underset{(\lambda,K,h_L)}{\operatorname{argmin}}\, ATS(n, \lambda, K, W, h_L, h_S, \delta, B, m, \eta), \tag{14}$$

subject to the constraint

$$\begin{aligned} E_0(h) &= 1, \\ ATS(n,\lambda,K,W,h_L,h_S,B,m,\eta,\delta=0) &= ATS_0. \end{aligned} \tag{15}$$

## 5 | NUMERICAL RESULTS

In this section, we present the optimal parameters $\lambda^*$, $h_L^*$, and $K^*$ of the VSI EWMA-$\widetilde{X}$ control chart and the corresponding $ATS_1$ for some scenarios of other parameters. The following specific values are considered in our simulation:

- The shift size $\delta \in \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0\}$.

- The sample size $n \in \{3, 5, 15\}$.
- The warning limit coefficient $W \in \{0.2, 0.6\}$.
- The precision error ratio $\eta \in \{0, 0.1, 0.3\}$.
- The number of repeated measurements per item $m \in \{1, 5, 10\}$.
- The slope in the linear covariate model $B \in \{0.8, 1, 1.2\}$.

The optimal parameters ($\lambda^*$, $K^*$, and $h_L^*$) and the corresponding $ATS_1$ of the VSI EWMA-$\widetilde{X}$ control chart in the presence of the measurement error are given in Tables 1–3. Some conclusions can be drawn from these results as follows.

- Given the values of other parameters, the optimal values ($\lambda^*$, $K^*$, and $h_L^*$) depend on $\eta$. However, when $\delta \leq 0.2$, this dependence is insignificant as the values of $\lambda^*$, $K^*$, and $h_L^*$ are almost the same as $\delta \leq 0.2$. If $\delta > 0.2$, the values of ($\lambda^*$, $K^*$) decrease whereas the values of $h_L^*$ increase when $\eta$ increases. For example, with $n = 5$, $B = 1$, $m = 1$, $h_S = 0.1$, $W = 0.2$, and $\delta = 0.3$ in Table 2, we have ($\lambda^*$, $K^*$,) = (0.0837, 1.4212) and $h_L^* = 2.9729$ when $\eta = 0$, whereas ($\lambda^*$, $K^*$) = (0.0783, 1.4108) and $h_L^* = 2.9845$ when $\eta = 0.3$.

- In general, the precision error ratio $\eta$ has a negative effect on the chart's performance. The value of the $ATS_1$ decreases when $\eta$ increases. For example, given $n = 3$, $\eta = 0.28$, $m = 1$, $\delta = 0.1$, $W = 0.6$, and $h_S = 0.1$ in Table 1, we have $ATS_1 = 128.6$ when $\eta = 0.1$ and $ATS_1 = 135.5$ when $\eta = 0.3$.
- The choice of the short sampling interval $h_S$ effects the value of $ATS$ depends on $h_S$. In particular, the smaller value of $h_S$ leads to smaller values of $ATS_1$. For example, when $\delta = 0.2$, $n = 15$, $W = 0.6$, $\eta = 0.28$, $m = 1$, and $B = 1$ in Table 3, we have $ATS_1 = 16.4$ for $h_S = 0.1$ and $ATS_1 = 18.1$ for $h_S = 0.5$. However, when $\delta \geqslant 1.5$, the increase of $h_S$ leads to the decrease of the $ATS_1$.
- The influence of the warning coefficient $W$ on the performance of the proposed chart depends on the shift size $\delta$. In general, when $\delta \leqslant 0.3$, the smaller value of $W$ leads to the smaller value of the $ATS_1$, whereas the opposite is true when $\delta > 0.3$. For example, we have $ATS_1 = 103.3$ when $W = 0.2$ and $ATS_1 = 107.1$ when $W = 0.6$ given that $n = 5$, $B = 1$, $m = 1$, $h_S = 0.5$, $\eta = 0.28$, and $\delta = 0.1$ in Table 2.
- The value of the $ATS_1$ reduces as $B$ increases. For example, when $n = 15$, $\eta = 0.28$, $m = 1$, $\delta = 0.1$, $W = 0.6$, and $h_S = 0.5$, we have $ATS_1 = 54.3$ for $B = 0.8$ and $ATS_1 = 51.7$ for $B = 1.2$; see Table 3.
- When $m$ increases, the $ATS_1$ decreases significantly, especially when $\delta$ is small. For example, with the same value of $n = 3$, $W = 0.2$, $h_S = 0.1$, $B = 1$, $\eta = 0.28$, and $\delta = 0.1$, we have $ATS_1 = 130.6$ when $m = 1$ and $ATS_1 = 124.6$ when $m = 10$. That is to say, taking multiple measurements per item, in each sample, is an efficient strategy to compensate for the effect of measurement errors in the VSI EWMA-$\widetilde{X}$ control chart.

- The sample size $n$ has a strong influence on the chart performance: the larger the sample size $n$, the smaller the value of $ATS_1$. For example, in Tables 1 and 2, with $h_S = 0.1$, $m = 1$, $B = 1$, $W = 0.2$, $\delta = 0.1$, and $\eta = 0.1$, we have $ATS_1 = 124.8$ when $n = 3$ whereas $ATS_1 = 88.3$ when $n = 5$.

We also make a comparison between the performance of the VSI EWMA-$\widetilde{X}$ control chart in the presence of measurement errors with the performance of other control charts in the same conditions in the literature, involving the Shewhart-$\widetilde{X}$ control chart (Tran et al.[20]) and the synthetic $\widetilde{X}$ control chart (Tran et al.[3]). The compared results show that regardless of the presence of measurement errors, the VSI EWMA-$\widetilde{X}$ control chart leads to the best performance. For example, given the value of $n = 3$, $B = 1$,$m = 1$, $\eta = 0.1$, and $\delta = 0.1$, we have $ARL_1 = 335$ in the Shewhart-$\widetilde{X}$ control chart (tab. 1 in Tran et al.[20]), $ARL_1 = 319.8$ in the synthetic $\widetilde{X}$ control chart (tab. 3 in Tran et al.[3]) compared with $ATS_1 = 12.48$ (for the case $h_S = 0.1$, $W = 0.2$ in Table 1 in this study).

## 6 | ILLUSTRATIVE EXAMPLE

In this section, we illustrate the implementation of the VSI EWMA-$\widetilde{X}$ control chart under the presence of measurement errors. We consider a real context from the production process of milk bottles introduced in Castagliola et al.[22] This context is also widely applied in the literature; see, for example, Tran et al.[25] According to Castagliola et al.,[22] the weight $Y$ (in milliliters) of each bottle in the production process is the quality characteristic of interest. The weight $Y$ is supposed to follow a normal distribution with parameters $(\mu_0, \sigma_0^2)$. From phase I data, it is estimated that $\hat{\mu}_0 = 500.023$

**TABLE 1** Optimal couples $(\lambda^*, K^*)$ (first row of each block) and values of $(h_L^*, ATS)$ (second row of each block) of the variable sampling interval (VSI) exponentially weighted moving average (EWMA) control chart in the presence of measurement error for $n = 3$

| | $h_S = 0.1$ $m = 1$ $B = 1$ | | | | | | $h_S = 0.5$ $m = 1$ $B = 1$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ |
| 0.10 | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) |
| | (3.5157,123.9) | (3.5157,124.8) | (3.5157,131.6) | (1.4386,127.7) | (1.4386,128.6) | (1.4386,135.5) | (2.3976,133.8) | (2.3976,134.6) | (2.3976,141.4) | (1.2437,135.9) | (1.2437,136.8) | (1.2437,143.5) |
| 0.20 | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) |
| | (3.5157,38.5) | (3.5157,38.8) | (3.5157,41.6) | (1.4386,40.0) | (1.4386,40.4) | (1.4386,43.3) | (2.3976,46.4) | (2.3976,46.8) | (2.3976,49.8) | (1.2437,47.3) | (1.2437,47.7) | (1.2437,50.8) |
| 0.30 | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0511,1.6734) | (0.0500,1.6686) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) |
| | (3.5094,19.7) | (3.5094,19.9) | (3.5094,21.1) | (1.4642,19.4) | (1.4642,19.6) | (1.4642,20.9) | (2.3941,24.8) | (2.3950,24.9) | (2.3976,26.4) | (1.2579,24.6) | (1.2579,24.8) | (1.2579,26.3) |
| 0.50 | (0.1325,1.8589) | (0.1315,1.8577) | (0.1097,1.8273) | (0.1140,1.8339) | (0.1131,1.8325) | (0.1097,1.8273) | (0.1089,1.8259) | (0.1081,1.8247) | (0.1024,1.8151) | (0.1097,1.8273) | (0.1004,1.8116) | (0.0947,1.8009) |
| | (3.9019,9.6) | (3.9034,9.7) | (3.6220,9.9) | (1.5014,8.3) | (1.5020,8.3) | (1.5039,8.9) | (2.4575,11.8) | (2.4583,11.9) | (2.4646,12.6) | (1.2799,11.2) | (1.2655,11.3) | (1.2676,12.0) |
| 0.70 | (0.2142,1.9273) | (0.2127,1.9265) | (0.2018,1.9198) | (0.1952,1.9154) | (0.1938,1.9145) | (0.1828,1.9067) | (0.1742,1.9000) | (0.1731,1.8991) | (0.1643,1.8917) | (0.1690,1.8957) | (0.1678,1.8947) | (0.1587,1.8866) |
| | (3.8114,6.5) | (3.8126,6.5) | (3.8215,6.8) | (1.5037,4.8) | (1.5041,4.8) | (1.5071,5.1) | (2.5821,7.6) | (2.5828,7.7) | (2.5882,8.1) | (1.2841,6.7) | (1.2843,6.7) | (1.2860,7.1) |
| 1.00 | (0.3415,1.9769) | (0.3394,1.9764) | (0.3236,1.9721) | (0.3239,1.9721) | (0.3218,1.9715) | (0.3062,1.9668) | (0.2783,1.9572) | (0.2766,1.9566) | (0.2634,1.9514) | (0.2773,1.9569) | (0.2755,1.9562) | (0.2620,1.9508) |
| | (3.7442,4.8) | (3.7450,4.8) | (3.7509,4.9) | (1.5157,2.9) | (1.5159,2.9) | (1.5178,3.0) | (2.5395,5.0) | (2.5400,5.0) | (2.5439,5.2) | (1.2899,3.9) | (1.2901,4.0) | (1.2912,4.2) |
| 1.50 | (0.5498,2.0100) | (0.5498,2.0100) | (0.5287,2.0079) | (0.5218,2.0072) | (0.5190,2.0069) | (0.4977,2.0045) | (0.4745,2.0016) | (0.4711,2.0012) | (0.4465,1.9977) | (0.4681,2.0008) | (0.4650,2.0004) | (0.4416,1.9970) |
| | (3.6982,4.0) | (3.6982,4.0) | (3.7011,4.0) | (1.5017,1.9) | (1.5018,1.9) | (1.5028,1.9) | (2.5055,3.5) | (2.5059,3.5) | (2.5086,3.6) | (1.2801,2.3) | (1.2802,2.3) | (1.2810,2.4) |
| 2.00 | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.7137,2.0207) | (0.7100,2.0205) | (0.6818,2.0192) | (0.5498,2.0100) | (0.5498,2.0100) | (0.5498,2.0100) | (0.6890,2.0196) | (0.6849,2.0194) | (0.6536,2.0177) |
| | (3.5094,3.8) | (3.5094,3.8) | (3.5094,3.8) | (1.5318,1.6) | (1.5319,1.6) | (1.5325,1.7) | (2.4990,2.9) | (2.4990,2.9) | (2.4990,3.0) | (1.2957,1.7) | (1.2958,1.7) | (1.2962,1.8) |
| | $h_S = 0.1$ $\eta = 0.28$ $m = 1$ | | | | | | $h_S = 0.5$ $\eta = 0.28$ $m = 1$ | | | | | |
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ |
| 0.10 | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) |
| | (3.5157,134.3) | (3.5157,130.6) | (3.5157,128.6) | (1.4386,138.1) | (1.4386,134.5) | (1.4386,132.5) | (2.3976,144.0) | (2.3976,140.4) | (2.3976,138.4) | (1.2437,146.1) | (1.2437,142.6) | (1.2437,140.6) |
| 0.20 | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) |
| | (3.5157,42.7) | (3.5157,41.2) | (3.5157,40.4) | (1.4386,44.4) | (1.4386,42.8) | (1.4386,42.0) | (2.3976,51.0) | (2.3976,49.4) | (2.3976,48.5) | (1.2437,52.0) | (1.2437,50.3) | (1.2437,49.4) |
| 0.30 | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) |
| | (3.5094,21.5) | (3.5094,20.9) | (3.5094,20.5) | (1.4642,21.4) | (1.4642,20.7) | (1.4642,20.3) | (2.3976,27.0) | (2.3976,26.2) | (2.3976,25.7) | (1.2579,26.9) | (1.2579,26.1) | (1.2579,25.6) |
| 0.50 | (0.1097,1.8273) | (0.1253,1.8498) | (0.1274,1.8525) | (0.1097,1.8273) | (0.1097,1.8273) | (0.1097,1.8273) | (0.1002,1.8113) | (0.1032,1.8164) | (0.1048,1.8193) | (0.0926,1.7967) | (0.0955,1.8024) | (0.0972,1.8056) |
| | (3.6220,10.1) | (3.9137,10.1) | (3.9101,10.0) | (1.5039,9.1) | (1.5039,8.8) | (1.5039,8.6) | (2.4670,12.9) | (2.4637,12.5) | (2.4618,12.3) | (1.2684,12.3) | (1.2673,11.9) | (1.2667,11.7) |
| 0.70 | (0.1977,1.9171) | (0.2033,1.9207) | (0.2065,1.9227) | (0.1787,1.9036) | (0.1843,1.9078) | (0.1875,1.9101) | (0.1611,1.8888) | (0.1655,1.8927) | (0.1681,1.8949) | (0.1563,1.8844) | (0.1599,1.8877) | (0.1626,1.8901) |
| | (3.8251,6.9) | (3.8203,6.7) | (3.8176,6.6) | (1.5083,5.2) | (1.5067,5.1) | (1.5058,5.0) | (2.5904,8.2) | (2.5875,8.0) | (2.5859,7.9) | (1.2865,7.3) | (1.2858,7.1) | (1.2853,7.0) |
| 1.00 | (0.3177,1.9703) | (0.3258,1.9727) | (0.3304,1.9740) | (0.3003,1.9649) | (0.3083,1.9675) | (0.3129,1.9689) | (0.2585,1.9493) | (0.2652,1.9521) | (0.2690,1.9537) | (0.2490,1.9452) | (0.2639,1.9516) | (0.2678,1.9532) |
| | (3.7532,5.0) | (3.7500,4.9) | (3.7483,4.9) | (1.5186,3.1) | (1.5175,3.0) | (1.5170,3.0) | (2.5454,5.3) | (2.5433,5.2) | (2.5422,5.1) | (1.2735,4.3) | (1.2911,4.2) | (1.2907,4.1) |
| 1.50 | (0.5195,2.0070) | (0.5321,2.0083) | (0.5392,2.0090) | (0.4896,2.0036) | (0.5006,2.0049) | (0.5069,2.0056) | (0.4375,1.9963) | (0.4498,1.9982) | (0.4570,1.9993) | (0.4331,1.9956) | (0.4448,1.9975) | (0.4516,1.9985) |
| | (3.7025,4.1) | (3.7006,4.0) | (3.6996,4.0) | (1.5031,2.0) | (1.5026,1.9) | (1.5023,1.9) | (2.5097,3.6) | (2.5082,3.6) | (2.5074,3.6) | (1.2813,2.5) | (1.2809,2.4) | (1.2806,2.4) |
| 2.00 | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.5498,2.0100) | (0.6857,2.0194) | (0.6940,2.0198) | (0.5498,2.0100) | (0.5498,2.0100) | (0.5498,2.0100) | (0.6415,2.0170) | (0.6580,2.0179) | (0.6673,2.0184) |
| | (3.5094,3.9) | (3.5094,3.8) | (3.5094,3.8) | (1.5006,1.7) | (1.5324,1.7) | (1.5322,1.7) | (2.4990,3.0) | (2.4990,3.0) | (2.4990,3.0) | (1.2963,1.8) | (1.2961,1.8) | (1.2960,1.8) |

(Continues)

**TABLE 1** (Continued)

| | $h_S = 0.1$ $\eta = 0.28$ $B = 1$ | | | | | | $h_S = 0.5$ $\eta = 0.28$ $B = 1$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ |
| 0.10 | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) |
| | (3.5157,130.6) | (3.5157,125.3) | (3.5157,124.6) | (1.4386,134.5) | (1.4386,129.1) | (1.4386,128.4) | (2.3976,140.4) | (2.3976,135.1) | (2.3976,134.5) | (1.2437,142.6) | (1.2437,137.3) | (1.2437,136.6) |
| 0.20 | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) | (0.0500,1.6686) |
| | (3.5157,41.2) | (3.5157,39.0) | (3.5157,38.8) | (1.4386,42.8) | (1.4386,40.6) | (1.4386,40.3) | (2.3976,49.4) | (2.3976,47.0) | (2.3976,46.7) | (1.2437,50.3) | (1.2437,47.9) | (1.2437,47.6) |
| 0.30 | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0500,1.6686) | (0.0509,1.6724) | (0.0511,1.6738) | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) |
| | (3.5094,20.9) | (3.5094,20.0) | (3.5094,19.9) | (1.4642,20.7) | (1.4642,19.7) | (1.4642,19.6) | (2.3976,26.2) | (2.3955,25.0) | (2.3948,24.9) | (1.2579,26.1) | (1.2579,24.9) | (1.2579,24.7) |
| 0.50 | (0.1253,1.8498) | (0.1310,1.8570) | (0.1317,1.8579) | (0.1097,1.8273) | (0.1126,1.8317) | (0.1133,1.8328) | (0.1032,1.8164) | (0.1077,1.8240) | (0.1083,1.8250) | (0.0955,1.8024) | (0.1000,1.8109) | (0.1006,1.8119) |
| | (3.9137,10.1) | (3.9043,9.7) | (3.9031,9.7) | (1.5039,8.8) | (1.5022,8.4) | (1.5018,8.3) | (2.4637,12.5) | (2.4588,12.0) | (2.4582,11.9) | (1.2673,11.9) | (1.2657,11.4) | (1.2655,11.3) |
| 0.70 | (0.2033,1.9207) | (0.2119,1.9260) | (0.2130,1.9266) | (0.1843,1.9078) | (0.1929,1.9139) | (0.1941,1.9147) | (0.1655,1.8927) | (0.1724,1.8985) | (0.1733,1.8993) | (0.1599,1.8877) | (0.1671,1.8941) | (0.1681,1.8949) |
| | (3.8203,6.7) | (3.8132,6.5) | (3.8123,6.5) | (1.5067,5.1) | (1.5043,4.9) | (1.5040,4.8) | (2.5875,8.0) | (2.5832,7.7) | (2.5827,7.7) | (1.2858,7.1) | (1.2844,6.8) | (1.2842,6.7) |
| 1.00 | (0.3258,1.9727) | (0.3382,1.9761) | (0.3399,1.9765) | (0.3083,1.9675) | (0.3206,1.9712) | (0.3223,1.9717) | (0.2652,1.9521) | (0.2756,1.9562) | (0.2769,1.9567) | (0.2639,1.9516) | (0.2745,1.9558) | (0.2759,1.9563) |
| | (3.7500,4.9) | (3.7454,4.8) | (3.7448,4.8) | (1.5175,3.0) | (1.5161,2.9) | (1.5159,2.9) | (2.5433,5.2) | (2.5403,5.0) | (2.5399,5.0) | (1.2911,4.2) | (1.2901,4.0) | (1.2900,4.0) |
| 1.50 | (0.5321,2.0083) | (0.5498,2.0100) | (0.5498,2.0100) | (0.5006,2.0049) | (0.5174,2.0068) | (0.5196,2.0070) | (0.4498,1.9982) | (0.4692,2.0010) | (0.4718,2.0013) | (0.4448,1.9975) | (0.4632,2.0001) | (0.4657,2.0005) |
| | (3.7006,4.0) | (3.6982,4.0) | (3.6982,4.0) | (1.5026,1.9) | (1.5019,1.9) | (1.5018,1.9) | (2.5082,3.6) | (2.5061,3.5) | (2.5058,3.5) | (1.2809,2.4) | (1.2803,2.3) | (1.2802,2.3) |
| 2.00 | (0.0514,1.6750) | (0.0514,1.6750) | (0.0514,1.6750) | (0.6857,2.0194) | (0.7079,2.0204) | (0.7108,2.0206) | (0.5498,2.0100) | (0.5498,2.0100) | (0.5498,2.0100) | (0.6580,2.0179) | (0.6826,2.0192) | (0.6858,2.0194) |
| | (3.5094,3.8) | (3.5094,3.8) | (3.5094,3.8) | (1.5324,1.7) | (1.5319,1.6) | (1.5319,1.6) | (2.4990,3.0) | (2.4990,2.9) | (2.4990,2.9) | (1.2961,1.8) | (1.2958,1.7) | (1.2958,1.7) |

**TABLE 2** Optimal couples $(\lambda^*, K^*)$ (first row of each block) and values of $(h_L^*, ATS)$ (second row of each block) of the variable sampling interval (VSI) exponentially weighted moving average (EWMA) control chart in the presence of measurement error for $n = 5$

| | $h_S = 0.1$ $m = 1$ $B = 1$ | | | | | | $h_S = 0.5$ $m = 1$ $B = 1$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ |
| 0.10 | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) |
| | (2.8875,87.6) | (2.8875,88.3) | (2.8875,94.0) | (1.2549,94.3) | (1.2549,95.0) | (1.2549,100.8) | (2.0486,97.7) | (2.0486,98.5) | (2.0486,104.1) | (1.1416,101.4) | (1.1416,102.2) | (1.1416,107.9) |
| 0.20 | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0516,1.3400) | (0.0516,1.3400) | (0.0516,1.3400) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) |
| | (2.8875,25.8) | (2.8875,26.0) | (2.8875,27.7) | (1.2692,28.2) | (1.2692,28.4) | (1.2692,30.4) | (2.0486,32.3) | (2.0486,32.5) | (2.0486,34.6) | (1.1416,33.7) | (1.1416,33.9) | (1.1416,36.1) |
| 0.30 | (0.0837,1.4212) | (0.0831,1.4200) | (0.0783,1.4108) | (0.0644,1.3788) | (0.0639,1.3774) | (0.0613,1.3705) | (0.0710,1.3951) | (0.0705,1.3939) | (0.0666,1.3845) | (0.0619,1.3719) | (0.0614,1.3705) | (0.0613,1.3705) |
| | (2.9729,13.5) | (2.9742,13.6) | (2.9845,14.5) | (1.2750,14.2) | (1.2754,14.3) | (1.2774,15.2) | (2.1122,17.6) | (2.1130,17.7) | (2.1187,18.8) | (1.1539,17.9) | (1.1541,18.0) | (1.1541,19.1) |
| 0.50 | (0.1799,1.5225) | (0.1786,1.5218) | (0.1692,1.5157) | (0.1576,1.5075) | (0.1576,1.5075) | (0.1576,1.5075) | (0.1483,1.5002) | (0.1473,1.4994) | (0.1397,1.4929) | (0.1388,1.4921) | (0.1378,1.4911) | (0.1299,1.4836) |
| | (3.0466,6.6) | (3.0475,6.6) | (3.0553,6.9) | (1.3074,6.0) | (1.3074,6.0) | (1.3074,6.4) | (2.1528,8.4) | (2.1534,8.5) | (2.1580,9.0) | (1.1630,8.1) | (1.1632,8.1) | (1.1645,8.6) |
| 0.70 | (0.2264,1.5462) | (0.2264,1.5462) | (0.2264,1.5462) | (0.2629,1.5598) | (0.2610,1.5592) | (0.2466,1.5541) | (0.2264,1.5462) | (0.2264,1.5462) | (0.2204,1.5436) | (0.2304,1.5479) | (0.2288,1.5472) | (0.2264,1.5462) |
| | (3.0162,4.6) | (3.0162,4.6) | (3.0162,4.7) | (1.3092,3.5) | (1.3094,3.5) | (1.3112,3.7) | (2.1201,5.4) | (2.1201,5.4) | (2.1219,5.7) | (1.1741,4.8) | (1.1742,4.8) | (1.1744,5.1) |
| 1.00 | (0.4404,1.5960) | (0.4379,1.5957) | (0.4186,1.5932) | (0.4240,1.5939) | (0.4215,1.5936) | (0.4024,1.5909) | (0.3684,1.5854) | (0.3660,1.5850) | (0.3485,1.5817) | (0.3721,1.5860) | (0.3697,1.5856) | (0.3520,1.5823) |
| | (3.1654,3.7) | (3.1658,3.7) | (3.1694,3.8) | (1.3177,2.1) | (1.3178,2.1) | (1.3188,2.2) | (2.2114,3.7) | (2.2118,3.7) | (2.2144,3.9) | (1.1667,2.8) | (1.1668,2.9) | (1.1674,3.0) |
| 1.50 | (0.7056,1.6146) | (0.7018,1.6145) | (0.6728,1.6134) | (0.6621,1.6129) | (0.6587,1.6127) | (0.6332,1.6115) | (0.6450,1.6121) | (0.6408,1.6119) | (0.6093,1.6103) | (0.6369,1.6117) | (0.6329,1.6115) | (0.6024,1.6099) |
| | (3.1387,3.3) | (3.1389,3.3) | (3.1405,3.3) | (1.3112,1.5) | (1.3112,1.5) | (1.3116,1.5) | (2.1902,2.7) | (2.1903,2.7) | (2.1916,2.8) | (1.1731,1.7) | (1.1731,1.7) | (1.1734,1.8) |
| 2.00 | (0.0516,1.3400) | (0.0516,1.3400) | (0.0516,1.3400) | (0.8649,1.6184) | (0.8616,1.6183) | (0.8356,1.6179) | (0.8634,1.6184) | (0.8602,1.6183) | (0.8349,1.6179) | (0.8540,1.6182) | (0.8508,1.6182) | (0.8253,1.6178) |
| | (2.8817,3.1) | (2.8817,3.1) | (2.8817,3.1) | (1.3094,1.3) | (1.3094,1.3) | (1.3095,1.4) | (2.1853,2.3) | (2.1853,2.3) | (2.1856,2.4) | (1.1719,1.3) | (1.1719,1.3) | (1.1720,1.4) |
| | $h_S = 0.1$ $\eta = 0.28$ $m = 1$ | | | | | | $h_S = 0.5$ $\eta = 0.28$ $m = 1$ | | | | | |
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ |
| 0.10 | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) |
| | (2.8875,96.2) | (2.8875,93.2) | (2.8875,91.5) | (1.2549,103.1) | (1.2549,100.0) | (1.2549,98.3) | (2.0486,106.4) | (2.0486,103.3) | (2.0486,101.6) | (1.1416,110.2) | (1.1416,107.1) | (1.1416,105.4) |
| 0.20 | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0516,1.3400) | (0.0516,1.3400) | (0.0516,1.3400) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) |
| | (2.8875,28.4) | (2.8875,27.5) | (2.8875,27.0) | (1.2692,31.3) | (1.2692,30.2) | (1.2692,29.6) | (2.0486,35.4) | (2.0486,34.3) | (2.0486,33.7) | (1.1416,37.0) | (1.1416,35.8) | (1.1416,35.1) |
| 0.30 | (0.0766,1.4072) | (0.0790,1.4121) | (0.0804,1.4149) | (0.0613,1.3705) | (0.0613,1.3705) | (0.0617,1.3714) | (0.0652,1.3808) | (0.0672,1.3858) | (0.0683,1.3886) | (0.0613,1.3705) | (0.0613,1.3705) | (0.0613,1.3705) |
| | (2.9885,14.8) | (2.9831,14.3) | (2.9800,14.1) | (1.2774,15.6) | (1.2774,15.1) | (1.2771,14.8) | (2.1209,19.2) | (2.1179,18.6) | (2.1162,18.3) | (1.1541,19.5) | (1.1541,18.9) | (1.1541,18.6) |
| 0.50 | (0.1657,1.5133) | (0.1705,1.5165) | (0.1732,1.5184) | (0.1576,1.5075) | (0.1576,1.5075) | (0.1576,1.5075) | (0.1369,1.4903) | (0.1408,1.4938) | (0.1430,1.4957) | (0.1270,1.4807) | (0.1310,1.4847) | (0.1333,1.4869) |
| | (3.0584,7.1) | (3.0542,6.9) | (3.0519,6.8) | (1.3074,6.6) | (1.3074,6.4) | (1.3074,6.2) | (2.1598,9.1) | (2.1573,8.9) | (2.1559,8.7) | (1.1651,8.8) | (1.1643,8.6) | (1.1639,8.4) |
| 0.70 | (0.2264,1.5462) | (0.2264,1.5462) | (0.2264,1.5462) | (0.2413,1.5521) | (0.2486,1.5549) | (0.2528,1.5564) | (0.2162,1.5417) | (0.2220,1.5443) | (0.2252,1.5457) | (0.2117,1.5396) | (0.2264,1.5462) | (0.2264,1.5462) |
| | (3.0162,4.8) | (3.0162,4.7) | (3.0162,4.7) | (1.3119,3.8) | (1.3109,3.7) | (1.3104,3.6) | (2.1233,5.8) | (2.1214,5.6) | (2.1204,5.6) | (1.1647,5.2) | (1.1744,5.1) | (1.1744,5.0) |
| 1.00 | (0.4114,1.5922) | (0.4213,1.5936) | (0.4269,1.5943) | (0.3952,1.5898) | (0.4051,1.5913) | (0.4107,1.5921) | (0.3420,1.5804) | (0.3509,1.5821) | (0.3560,1.5831) | (0.3453,1.5810) | (0.3544,1.5828) | (0.3596,1.5838) |
| | (3.1709,3.8) | (3.1689,3.8) | (3.1678,3.8) | (1.3192,2.3) | (1.3187,2.2) | (1.3184,2.2) | (2.2154,3.9) | (2.2140,3.9) | (2.2132,3.8) | (1.1677,3.1) | (1.1673,3.0) | (1.1672,3.0) |
| 1.50 | (0.6618,1.6129) | (0.6769,1.6135) | (0.6854,1.6139) | (0.6235,1.6111) | (0.6367,1.6117) | (0.6442,1.6121) | (0.5972,1.6096) | (0.6137,1.6105) | (0.6230,1.6110) | (0.5908,1.6092) | (0.6067,1.6101) | (0.6156,1.6106) |
| | (3.1412,3.3) | (3.1403,3.3) | (3.1398,3.3) | (1.3118,1.5) | (1.3116,1.5) | (1.3114,1.5) | (2.1922,2.8) | (2.1914,2.8) | (2.1910,2.8) | (1.1736,1.8) | (1.1734,1.8) | (1.1733,1.8) |
| 2.00 | (0.0516,1.3400) | (0.0516,1.3400) | (0.0516,1.3400) | (0.8253,1.6178) | (0.8393,1.6180) | (0.8470,1.6181) | (0.8247,1.6177) | (0.8385,1.6180) | (0.8461,1.6181) | (0.8151,1.6176) | (0.8290,1.6178) | (0.8366,1.6180) |
| | (2.8817,3.1) | (2.8817,3.1) | (2.8817,3.1) | (1.3096,1.4) | (1.3095,1.4) | (1.3095,1.4) | (2.1858,2.4) | (2.1856,2.4) | (2.1855,2.4) | (1.1720,1.4) | (1.1720,1.4) | (1.1720,1.3) |

(Continues)

**TABLE 2** (Continued)

| | $h_S = 0.1$ $\eta = 0.28$ $B = 1$ | | | | | | $h_S = 0.5$ $\eta = 0.28$ $B = 1$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ |
| 0.10 | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) |
| | (2.8875,93.2) | (2.8875,88.7) | (2.8875,88.2) | (1.2549,100.0) | (1.2549,95.4) | (1.2549,94.9) | (2.0486,103.3) | (2.0486,98.9) | (2.0486,98.3) | (1.1416,107.1) | (1.1416,102.6) | (1.1416,102.0) |
| 0.20 | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0516,1.3400) | (0.0516,1.3400) | (0.0516,1.3400) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) | (0.0500,1.3341) |
| | (2.8875,27.5) | (2.8875,26.1) | (2.8875,25.9) | (1.2692,30.2) | (1.2692,28.6) | (1.2692,28.4) | (2.0486,34.3) | (2.0486,32.7) | (2.0486,32.5) | (1.1416,35.8) | (1.1416,34.1) | (1.1416,33.9) |
| 0.30 | (0.0790,1.4121) | (0.0827,1.4194) | (0.0832,1.4203) | (0.0613,1.3705) | (0.0636,1.3766) | (0.0640,1.3777) | (0.0672,1.3858) | (0.0702,1.3932) | (0.0706,1.3941) | (0.0613,1.3705) | (0.0613,1.3705) | (0.0615,1.3708) |
| | (2.9831,14.3) | (2.9750,13.7) | (2.9739,13.6) | (1.2774,15.1) | (1.2756,14.3) | (1.2753,14.3) | (2.1179,18.6) | (2.1134,17.8) | (2.1128,17.7) | (1.1541,18.9) | (1.1541,18.1) | (1.1540,18.0) |
| 0.50 | (0.1705,1.5165) | (0.1779,1.5213) | (0.1789,1.5219) | (0.1576,1.5075) | (0.1576,1.5075) | (0.1576,1.5075) | (0.1408,1.4938) | (0.1468,1.4989) | (0.1475,1.4996) | (0.1310,1.4847) | (0.1372,1.4906) | (0.1380,1.4913) |
| | (3.0542,6.9) | (3.0481,6.6) | (3.0473,6.6) | (1.3074,6.4) | (1.3074,6.0) | (1.3074,6.0) | (2.1573,8.9) | (2.1537,8.5) | (2.1532,8.5) | (1.1643,8.6) | (1.1633,8.2) | (1.1631,8.1) |
| 0.70 | (0.2264,1.5462) | (0.2264,1.5462) | (0.2264,1.5462) | (0.2486,1.5549) | (0.2599,1.5588) | (0.2614,1.5593) | (0.2220,1.5443) | (0.2264,1.5462) | (0.2264,1.5462) | (0.2264,1.5462) | (0.2279,1.5468) | (0.2292,1.5473) |
| | (3.0162,4.7) | (3.0162,4.6) | (3.0162,4.6) | (1.3109,3.7) | (1.3096,3.5) | (1.3094,3.5) | (2.1214,5.6) | (2.1201,5.4) | (2.1201,5.4) | (1.1744,5.1) | (1.1743,4.9) | (1.1742,4.8) |
| 1.00 | (0.4213,1.5936) | (0.4364,1.5955) | (0.4384,1.5958) | (0.4051,1.5913) | (0.4201,1.5934) | (0.4220,1.5937) | (0.3509,1.5821) | (0.3647,1.5847) | (0.3665,1.5851) | (0.3544,1.5828) | (0.3684,1.5854) | (0.3702,1.5857) |
| | (3.1689,3.8) | (3.1661,3.7) | (3.1657,3.7) | (1.3187,2.2) | (1.3179,2.1) | (1.3178,2.1) | (2.2140,3.9) | (2.2120,3.8) | (2.2117,3.7) | (1.1673,3.0) | (1.1669,2.9) | (1.1668,2.9) |
| 1.50 | (0.6769,1.6135) | (0.6996,1.6144) | (0.7026,1.6145) | (0.6367,1.6117) | (0.6568,1.6127) | (0.6594,1.6128) | (0.6137,1.6105) | (0.6385,1.6118) | (0.6417,1.6120) | (0.6067,1.6101) | (0.6307,1.6114) | (0.6338,1.6116) |
| | (3.1403,3.3) | (3.1390,3.3) | (3.1389,3.3) | (1.3116,1.5) | (1.3113,1.5) | (1.3112,1.5) | (2.1914,2.8) | (2.1904,2.7) | (2.1903,2.7) | (1.1734,1.8) | (1.1731,1.7) | (1.1731,1.7) |
| 2.00 | (0.0516,1.3400) | (0.0516,1.3400) | (0.0516,1.3400) | (0.8393,1.6180) | (0.8597,1.6183) | (0.8623,1.6183) | (0.8385,1.6180) | (0.8584,1.6183) | (0.8609,1.6183) | (0.8290,1.6178) | (0.8490,1.6182) | (0.8515,1.6182) |
| | (2.8817,3.1) | (2.8817,3.1) | (2.8817,3.1) | (1.3095,1.4) | (1.3094,1.3) | (1.3094,1.3) | (2.1856,2.4) | (2.1853,2.3) | (2.1853,2.3) | (1.1720,1.4) | (1.1719,1.3) | (1.1719,1.3) |

**TABLE 3** Optimal couples $(\lambda^*, K^*)$ (first row of each block) and values of $(h_L^*, ATS)$ (second row of each block) of the variable sampling interval (VSI) exponentially weighted moving average (EWMA) control chart in the presence of measurement error for $n = 15$

| | $h_S = 0.1$ $m = 1$ $B = 1$ | | | | | | $h_S = 0.5$ $m = 1$ $B = 1$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ | $\eta = 0$ | $\eta = 0.1$ | $\eta = 0.3$ |
| 0.10 | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) |
| | (1.8808,35.0) | (1.8808,35.3) | (1.8808,37.9) | (1.0318,47.4) | (1.0318,47.8) | (1.0318,50.8) | (1.4893,42.7) | (1.4893,43.1) | (1.4893,45.9) | (1.0176,49.6) | (1.0176,50.0) | (1.0176,53.1) |
| 0.20 | (0.0931,0.8556) | (0.0924,0.8549) | (0.0869,0.8495) | (0.0867,0.8493) | (0.0867,0.8493) | (0.0764,0.8375) | (0.0812,0.8432) | (0.0805,0.8425) | (0.0757,0.8366) | (0.0764,0.8375) | (0.0764,0.8375) | (0.0764,0.8375) |
| | (1.9431,10.8) | (1.9439,10.9) | (1.9501,11.6) | (1.0413,15.4) | (1.0413,15.5) | (1.0394,16.5) | (1.5318,14.5) | (1.5322,14.7) | (1.4993,15.5) | (1.0219,17.1) | (1.0219,17.2) | (1.0219,18.3) |
| 0.30 | (0.1748,0.9041) | (0.1736,0.9036) | (0.1640,0.8998) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1483,0.8928) | (0.1473,0.8923) | (0.1394,0.8883) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1190,0.8762) |
| | (1.9554,5.7) | (1.9560,5.7) | (1.9606,6.1) | (1.0469,7.7) | (1.0469,7.8) | (1.0469,8.3) | (1.5384,7.9) | (1.5388,7.9) | (1.5415,8.4) | (1.0261,9.0) | (1.0261,9.0) | (1.0249,9.6) |
| 0.50 | (0.3485,0.9403) | (0.3464,0.9401) | (0.3307,0.9382) | (0.2990,0.9338) | (0.2965,0.9335) | (0.2785,0.9306) | (0.2956,0.9333) | (0.2938,0.9331) | (0.2799,0.9308) | (0.2832,0.9313) | (0.2811,0.9310) | (0.2652,0.9282) |
| | (1.9838,3.0) | (1.9841,3.0) | (1.9865,3.1) | (1.0515,3.3) | (1.0515,3.3) | (1.0519,3.5) | (1.5515,3.8) | (1.5517,3.9) | (1.5533,4.1) | (1.0288,4.0) | (1.0288,4.0) | (1.0291,4.3) |
| 0.70 | (0.5118,0.9524) | (0.5089,0.9523) | (0.4879,0.9512) | (0.4982,0.9518) | (0.4949,0.9516) | (0.4700,0.9502) | (0.4573,0.9495) | (0.4541,0.9493) | (0.4310,0.9477) | (0.4668,0.9501) | (0.4635,0.9499) | (0.4388,0.9483) |
| | (1.9688,2.3) | (1.9689,2.3) | (1.9702,2.4) | (1.0529,1.9) | (1.0529,2.0) | (1.0531,2.1) | (1.5402,2.6) | (1.5403,2.6) | (1.5414,2.7) | (1.0295,2.4) | (1.0295,2.4) | (1.0296,2.5) |
| 1.00 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.7396,0.9588) | (0.7363,0.9587) | (0.7109,0.9583) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.7325,0.9587) | (0.7287,0.9586) | (0.6999,0.9581) |
| | (1.9633,2.0) | (1.9633,2.0) | (1.9633,2.1) | (1.0562,1.2) | (1.0562,1.3) | (1.0562,1.3) | (1.5352,1.9) | (1.5352,1.9) | (1.5352,1.9) | (1.0312,1.4) | (1.0312,1.4) | (1.0312,1.5) |
| 1.50 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.9638,0.9604) | (0.9621,0.9604) | (0.9476,0.9604) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.9649,0.9604) | (0.9633,0.9604) | (0.9500,0.9604) |
| | (1.9633,2.0) | (1.9633,2.0) | (1.9633,2.0) | (1.0560,1.1) | (1.0560,1.1) | (1.0560,1.1) | (1.5352,1.6) | (1.5352,1.6) | (1.5352,1.6) | (1.0311,1.1) | (1.0311,1.1) | (1.0311,1.1) |
| 2.00 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) |
| | (1.9633,2.0) | (1.9633,2.0) | (1.9633,2.0) | (1.0523,1.1) | (1.0523,1.1) | (1.0523,1.1) | (1.5352,1.5) | (1.5352,1.5) | (1.5352,1.5) | (1.0290,1.0) | (1.0290,1.0) | (1.0290,1.0) |
| | $h_S = 0.1$ $\eta = 0.28$ $m = 1$ | | | | | | $h_S = 0.5$ $\eta = 0.28$ $m = 1$ | | | | | |
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ | $B = 0.8$ | $B = 1$ | $B = 1.2$ |
| 0.10 | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) |
| | (1.8808,38.9) | (1.8808,37.5) | (1.8808,36.7) | (1.0318,52.1) | (1.0318,50.4) | (1.0318,49.5) | (1.4893,47.0) | (1.4893,45.5) | (1.4893,44.6) | (1.0176,54.3) | (1.0176,52.6) | (1.0176,51.7) |
| 0.20 | (0.0849,0.8474) | (0.0877,0.8503) | (0.0893,0.8519) | (0.0764,0.8375) | (0.0867,0.8493) | (0.0867,0.8493) | (0.0740,0.8345) | (0.0763,0.8374) | (0.0764,0.8375) | (0.0678,0.8260) | (0.0764,0.8375) | (0.0764,0.8375) |
| | (1.9525,11.9) | (1.9492,11.5) | (1.9474,11.3) | (1.0394,16.9) | (1.0413,16.4) | (1.0413,16.1) | (1.5006,15.9) | (1.4989,15.4) | (1.4988,15.1) | (1.0208,18.7) | (1.0219,18.1) | (1.0219,17.8) |
| 0.30 | (0.1605,0.8983) | (0.1653,0.9004) | (0.1681,0.9015) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1365,0.8867) | (0.1405,0.8889) | (0.1428,0.8901) | (0.1163,0.8743) | (0.1365,0.8868) | (0.1365,0.8868) |
| | (1.9625,6.2) | (1.9600,6.0) | (1.9586,5.9) | (1.0469,8.5) | (1.0469,8.2) | (1.0469,8.1) | (1.5425,8.6) | (1.5411,8.3) | (1.5403,8.2) | (1.0250,9.9) | (1.0261,9.5) | (1.0261,9.4) |
| 0.50 | (0.3248,0.9375) | (0.3329,0.9385) | (0.3375,0.9390) | (0.2719,0.9294) | (0.2810,0.9310) | (0.2862,0.9318) | (0.2747,0.9299) | (0.2818,0.9311) | (0.2859,0.9318) | (0.2627,0.9277) | (0.2674,0.9286) | (0.2720,0.9294) |
| | (1.9875,3.2) | (1.9862,3.1) | (1.9855,3.1) | (1.0521,3.6) | (1.0519,3.5) | (1.0517,3.5) | (1.5540,4.1) | (1.5531,4.0) | (1.5526,4.0) | (1.0291,4.4) | (1.0290,4.2) | (1.0289,4.2) |
| 0.70 | (0.4800,0.9508) | (0.4908,0.9514) | (0.4969,0.9517) | (0.4605,0.9497) | (0.4735,0.9504) | (0.4808,0.9509) | (0.4225,0.9471) | (0.4341,0.9480) | (0.4408,0.9484) | (0.4295,0.9476) | (0.4422,0.9485) | (0.4494,0.9490) |
| | (1.9707,2.4) | (1.9700,2.4) | (1.9696,2.3) | (1.0532,2.1) | (1.0531,2.1) | (1.0530,2.0) | (1.5418,2.7) | (1.5412,2.7) | (1.5409,2.7) | (1.0297,2.6) | (1.0296,2.5) | (1.0296,2.5) |
| 1.00 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.7010,0.9582) | (0.7145,0.9584) | (0.7220,0.9585) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6886,0.9579) | (0.7039,0.9582) | (0.7125,0.9584) |
| | (1.9633,2.1) | (1.9633,2.1) | (1.9633,2.1) | (1.0562,1.3) | (1.0562,1.3) | (1.0562,1.3) | (1.5352,2.0) | (1.5352,1.9) | (1.5352,1.9) | (1.0312,1.5) | (1.0312,1.5) | (1.0312,1.5) |
| 1.50 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.9414,0.9604) | (0.9498,0.9604) | (0.9542,0.9604) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.9443,0.9604) | (0.9520,0.9604) | (0.9561,0.9604) |
| | (1.9633,2.0) | (1.9633,2.0) | (1.9633,2.0) | (1.0560,1.1) | (1.0560,1.1) | (1.0560,1.1) | (1.5352,1.6) | (1.5352,1.6) | (1.5352,1.6) | (1.0311,1.1) | (1.0311,1.1) | (1.0311,1.1) |
| 2.00 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) |
| | (1.9633,2.0) | (1.9633,2.0) | (1.9633,2.0) | (1.0523,1.1) | (1.0523,1.1) | (1.0523,1.1) | (1.5352,1.5) | (1.5352,1.5) | (1.5352,1.5) | (1.0290,1.0) | (1.0290,1.0) | (1.0290,1.0) |

(Continues)

**TABLE 3** (Continued)

| | $h_S = 0.1$ $\eta = 0.28$ $B = 1$ | | | | | | $h_S = 0.5$ $\eta = 0.28$ $B = 1$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W = 0.2$ | | | $W = 0.6$ | | | $W = 0.2$ | | | $W = 0.6$ | | |
| $\delta$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ | $m = 1$ | $m = 5$ | $m = 10$ |
| 0.10 | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) | (0.0500,0.7943) |
| | (1.8808,37.5) | (1.8808,35.5) | (1.8808,35.2) | (1.0318,50.4) | (1.0318,48.0) | (1.0318,47.7) | (1.4893,45.5) | (1.4893,43.3) | (1.4893,43.0) | (1.0176,52.6) | (1.0176,50.2) | (1.0176,49.9) |
| 0.20 | (0.0877,0.8503) | (0.0920,0.8545) | (0.0925,0.8551) | (0.0867,0.8493) | (0.0867,0.8493) | (0.0867,0.8493) | (0.0763,0.8374) | (0.0802,0.8421) | (0.0807,0.8426) | (0.0764,0.8375) | (0.0764,0.8375) | (0.0764,0.8375) |
| | (1.9492,11.5) | (1.9444,10.9) | (1.9438,10.9) | (1.0413,16.4) | (1.0413,15.6) | (1.0413,15.5) | (1.4989,15.4) | (1.5324,14.7) | (1.5321,14.6) | (1.0219,18.1) | (1.0219,17.3) | (1.0219,17.2) |
| 0.30 | (0.1653,0.9004) | (0.1728,0.9033) | (0.1738,0.9037) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1405,0.8889) | (0.1467,0.8920) | (0.1475,0.8924) | (0.1365,0.8868) | (0.1365,0.8868) | (0.1365,0.8868) |
| | (1.9600,6.0) | (1.9564,5.8) | (1.9559,5.7) | (1.0469,8.2) | (1.0469,7.8) | (1.0469,7.8) | (1.5411,8.3) | (1.5390,8.0) | (1.5387,7.9) | (1.0261,9.5) | (1.0261,9.1) | (1.0261,9.0) |
| 0.50 | (0.3329,0.9385) | (0.3453,0.9399) | (0.3469,0.9401) | (0.2810,0.9310) | (0.2952,0.9333) | (0.2971,0.9336) | (0.2818,0.9311) | (0.2927,0.9329) | (0.2942,0.9331) | (0.2674,0.9286) | (0.2799,0.9308) | (0.2815,0.9311) |
| | (1.9862,3.1) | (1.9843,3.0) | (1.9841,3.0) | (1.0519,3.5) | (1.0515,3.4) | (1.0515,3.3) | (1.5531,4.0) | (1.5518,3.9) | (1.5517,3.9) | (1.0290,4.2) | (1.0288,4.0) | (1.0288,4.0) |
| 0.70 | (0.4908,0.9514) | (0.5074,0.9522) | (0.5096,0.9523) | (0.4735,0.9504) | (0.4930,0.9515) | (0.4956,0.9516) | (0.4341,0.9480) | (0.4524,0.9492) | (0.4548,0.9493) | (0.4422,0.9485) | (0.4617,0.9498) | (0.4642,0.9499) |
| | (1.9700,2.4) | (1.9690,2.3) | (1.9689,2.3) | (1.0531,2.1) | (1.0529,2.0) | (1.0529,2.0) | (1.5412,2.7) | (1.5404,2.6) | (1.5403,2.6) | (1.0296,2.5) | (1.0295,2.4) | (1.0295,2.4) |
| 1.00 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.7145,0.9584) | (0.7345,0.9587) | (0.7370,0.9587) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.7039,0.9582) | (0.7266,0.9586) | (0.7295,0.9586) |
| | (1.9633,2.1) | (1.9633,2.0) | (1.9633,2.0) | (1.0562,1.3) | (1.0562,1.3) | (1.0562,1.3) | (1.5352,1.9) | (1.5352,1.9) | (1.5352,1.9) | (1.0312,1.5) | (1.0312,1.4) | (1.0312,1.4) |
| 1.50 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.9498,0.9604) | (0.9611,0.9604) | (0.9624,0.9604) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.9520,0.9604) | (0.9624,0.9604) | (0.9636,0.9604) |
| | (1.9633,2.0) | (1.9633,2.0) | (1.9633,2.0) | (1.0560,1.1) | (1.0560,1.1) | (1.0560,1.1) | (1.5352,1.6) | (1.5352,1.6) | (1.5352,1.6) | (1.0311,1.1) | (1.0311,1.1) | (1.0311,1.1) |
| 2.00 | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) | (0.6511,0.9571) |
| | (1.9633,2.0) | (1.9633,2.0) | (1.9633,2.0) | (1.0523,1.1) | (1.0523,1.1) | (1.0523,1.1) | (1.5352,1.5) | (1.5352,1.5) | (1.5352,1.5) | (1.0290,1.0) | (1.0290,1.0) | (1.0290,1.0) |

and $\hat{\sigma}_0 = 0.9616$. In the implementation of the proposed chart, we suppose a sample of the size $n = 5$ is taken for each subgroup, and we choose the value of $ATS_0$ equal to 370.4. Similar to Tran et al.,[20] we assume $\eta = 0.28$, $A = 0$, $B = 1$, and $m = 1$ for the parameters of the linear covariate error model. We also consider a shift of $0.5\sigma_0$ in the mean as an alarm that the process is out of control. Then, with the sampling interval $h_S = 0.5$ and $W = 0.2$, the optimal parameters $(\lambda^*, K^*)$ are $\lambda^* = 0.1467$ and $K^* = 1.4989$. From the formulas (7)–(10), we obtain

$$\begin{aligned}
LCL &= 500.0230 - 1.4989\sqrt{\frac{0.1467}{2-0.1467}\left(1+0.28^2\right)} \times 0.9616 = 499.6019,\\
UCL &= 500.0230 + 1.4989\sqrt{\frac{0.1467}{2-0.1467}\left(1+0.28^2\right)} \times 0.9616 = 500.4441,\\
LWL &= 500.023 - 0.3\sqrt{\frac{0.1467}{2-0.1467}\left(1+0.28^2\right)} \times 0.9616 = 499.9387,\\
UWL &= 500.023 + 0.3\sqrt{\frac{0.1467}{2-0.1467}\left(1+0.28^2\right)} \times 0.9616 = 500.1073.
\end{aligned}$$

We simulate phase II data from a normal distribution. In particular, the first 10 subgroups are simulated from $N(\hat{\mu}_0, \hat{\sigma}_0^2)$, and the last 10 subgroups are simulated from $N(\hat{\mu}_0 + 0.5 * \hat{\sigma}_0, \hat{\sigma}_0^2)$, that is, with a shift of the size $\delta = 0.5$, leading to higher milk weight. The simulated phase II data are presented in Table 4. The rightmost two columns of the Table 4 show the corresponding sample median values $\widetilde{T}_i^*$ and the EWMA sequence $Z_i^*$, whereas the second and the third columns present the corresponding sampling interval and the total time. Figure 2 plot the VSI EWMA-$\widetilde{X}$ control chart in the presence of measurement errors corresponding to the phase II data set in Table 4. As can be seen from this figure, from sample 13, the process is detected to be out of control (corresponding to the bold values in Table 4).

**TABLE 4** Phase II dataset (SI: sampling interval)

| Sample | SI | Total time | Phase II ($X_{ij}$) | | | | | $\widetilde{X}_i^*$ | $Z_i^*$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.63 | 1.63 | 501.4833 | 500.1706 | 499.9020 | 498.8775 | 499.6307 | 499.9020 | 500.0052 |
| 2 | 1.63 | 3.26 | 500.1050 | 499.9799 | 498.6100 | 501.0395 | 498.8064 | 499.9799 | 500.0015 |
| 3 | 1.63 | 4.89 | 499.5145 | 501.4646 | 501.1188 | 501.3637 | 499.7332 | 501.1188 | 500.1654 |
| 4 | 0.5 | 5.39 | 499.5620 | 499.8151 | 500.2803 | 500.7398 | 497.1229 | 499.8151 | 500.1140 |
| 5 | 0.5 | 5.89 | 499.7502 | 499.8326 | 499.0275 | 499.8758 | 499.8762 | 499.8326 | 500.0727 |
| 6 | 1.63 | 7.52 | 498.5258 | 499.6318 | 500.6597 | 500.4488 | 499.8711 | 499.8711 | 500.0432 |
| 7 | 1.63 | 9.15 | 499.8433 | 500.9569 | 500.0851 | 499.6945 | 498.8933 | 499.8433 | 500.0138 |
| 8 | 1.63 | 10.78 | 499.4935 | 499.7990 | 501.0783 | 500.8580 | 500.4979 | 500.4979 | 500.0849 |
| 9 | 1.63 | 12.41 | 500.2730 | 500.9816 | 499.2352 | 500.5186 | 498.6644 | 500.2730 | 500.1125 |
| 10 | 0.5 | 12.91 | 500.8658 | 502.7562 | 499.4933 | 501.3502 | 499.5705 | 500.8658 | 500.2230 |
| 11 | 0.5 | 13.41 | 501.1827 | 500.6837 | 501.3678 | 500.0807 | 502.1853 | 501.1827 | 500.1931 |
| 12 | 0.5 | 13.91 | 502.1790 | 500.6295 | 500.1285 | 499.6191 | 502.1439 | 500.6295 | 500.2571 |
| 13 | 0.5 | 14.41 | 501.3198 | 503.0769 | 502.9001 | 501.9779 | 498.7582 | 501.9779 | **500.5096** |
| 14 | 0.5 | 14.91 | 501.7116 | 499.0937 | 501.5189 | 501.9622 | 501.3381 | 501.5189 | **500.6577** |
| 15 | 0.5 | 15.41 | 501.6141 | 500.1646 | 502.7406 | 501.7982 | 501.1249 | 501.6141 | **500.7980** |
| 16 | 0.5 | 15.91 | 500.9030 | 499.9101 | 498.3796 | 501.3495 | 501.8195 | 500.9030 | **500.8134** |
| 17 | 0.5 | 16.41 | 497.5061 | 500.1663 | 501.0336 | 501.0758 | 497.7357 | 500.1663 | **500.7184** |
| 18 | 0.5 | 16.91 | 499.8274 | 499.9869 | 500.5583 | 500.3271 | 500.5652 | 500.3271 | **500.6610** |
| 19 | 0.5 | 17.41 | 499.5205 | 499.9218 | 500.1015 | 502.3268 | 501.0534 | 500.1015 | **500.5790** |
| 20 | 0.5 | 17.91 | 501.1493 | 502.6016 | 498.8458 | 500.9987 | 499.9475 | 500.9987 | **500.6405** |

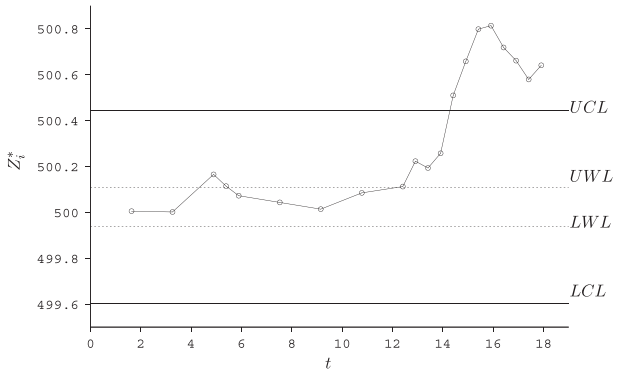



**FIGURE 2** The variable sampling interval (VSI) exponentially weighted moving average (EWMA)-$\widetilde{X}$ control chart in the presence of measurement errors corresponding to the Phase II data set in Table 4. LCL, lower control limit; LWL, lower warning limit; UCL, upper control limit; UWL, upper warning limit

## 7 | CONCLUDING REMARKS

In this paper, we have investigated the effect of measurement error on the VSI EWMA-$\widetilde{X}$ control chart. The obtained results show that the measurement error has a negative influence on the proposed chart performance. These negative impacts can be reduced by increasing either the sample size $n$ or the number of multiple measurements per item. We have also compared the performance of the VSI EWMA-$\widetilde{X}$ control chart with other median charts in the literature in the same condition of the presence of measurement errors. It is shown that our proposed control chart in this study leads to better performance compared with the others.

## ORCID

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## APPENDIX A

In order to calculate the *ATS* of the VSI EWMA-$\widetilde{X}$ control chart in the presence of measurement errors, we use a Markov chain method proposed in Brook and Evans.[7] By this method, the control limit [*LCL*, *UCL*] is divided into $2p+1$ equal-length subintervals $(H_j-\Delta, H_j+\Delta]$, $j\in\{-p,\dots,0,\dots,+p\}$, centered at

$$H_j=\frac{UCL+LCL}{2}+2j\Delta \text{ where } \Delta=\frac{UCL-LCL}{2(2p+1)}.$$

Using formulas (7) and (8) for *LCL* and *UCL*, we obtain

$$H_j=A+B\mu_0+2j\frac{K\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2+\frac{\sigma_M^2}{m}\right)}}{(2p+1)}, \tag{A1}$$

$$\Delta=\frac{K\sqrt{\frac{\lambda}{2-\lambda}\left(B^2\sigma_0^2+\frac{\sigma_M^2}{m}\right)}}{(2p+1)}. \tag{A2}$$

Each subinterval $(H_j-\Delta, H_j+\Delta]$, $j\in\{-p,\dots,0,\dots,+p\}$, represents a transient state of a Markov chain. For each $j\in\{-p,\dots,0,\dots,+p\}$, if $Z_i^*\in(H_j-\Delta,H_j+\Delta]$, then the Markov chain is in the transient state $j$ for sample $i$. If $Z_i^*\notin(H_j-\Delta,H_j+\Delta]$, then the Markov chain reached the absorbing state $(-\infty, LCL]\cup[UCL,+\infty)$. We assume that $H_j$ is the representative value of state $j$. Let $\mathbf{Q}$ be the $(2p+1, 2p+1)$ submatrix of probabilities $Q_{j,k}$ corresponding to the $2p+1$ transient states defined above, that is,

$$\mathbf{Q}=\begin{pmatrix} Q_{-p,-p} & \cdots & Q_{-p,-1} & Q_{-p,0} & Q_{-p,+1} & \cdots & Q_{-p,+p}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ Q_{-1,-p} & \cdots & Q_{-1,-1} & Q_{-1,0} & Q_{-1,+1} & \cdots & Q_{-1,+p}\\ Q_{0,-p} & \cdots & Q_{0,-1} & Q_{0,0} & Q_{0,+1} & \cdots & Q_{0,+p}\\ Q_{+1,-p} & \cdots & Q_{+1,-1} & Q_{+1,0} & Q_{+1,+1} & \cdots & Q_{+1,+p}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ Q_{+p,-p} & \cdots & Q_{+p,-1} & Q_{+p,0} & Q_{+p,+1} & \cdots & Q_{+p,+p} \end{pmatrix}. \tag{A3}$$

By definition, we have $Q_{j,k}=P(Z_i^*\in(H_k-\Delta,H_k+\Delta]|Z_{i-1}^*=H_j)$ or, equivalently, $Q_{j,k}=P(Z_i^*\le H_k+\Delta|Z_{i-1}^*=H_j)-P(Z_i^*\le H_k-\Delta|Z_{i-1}^*=H_j)$. Replacing $Z_i^*=(1-\lambda)Z_{i-1}^*+\lambda\widetilde{X}_i^*$, $Z_{i-1}^*=H_j$ and isolating $\widetilde{X}_i^*$ gives

$$\begin{aligned} Q_{j,k} &= P\left(\widetilde{X}_i^*\le\frac{H_k+\Delta-(1-\lambda)H_j}{\lambda}\right)-P\left(\widetilde{X}_i^*\le\frac{H_k-\Delta-(1-\lambda)H_j}{\lambda}\right)\\ &= F_{\widetilde{X}_i^*}\left(\frac{H_k+\Delta-(1-\lambda)H_j}{\lambda}\middle| n\right)-F_{\widetilde{X}_i^*}\left(\frac{H_k-\Delta-(1-\lambda)H_j}{\lambda}\middle| n\right). \end{aligned}$$

From (5), (A1), and (A2), it can easily be proven that

$$Q_{j,k}=F_\beta\left(\Phi\left(\frac{(2k+1-2j(1-\lambda))K\sqrt{\frac{\lambda}{2-\lambda}}}{\lambda(2p+1)}-\frac{B\delta\sigma_0}{\sqrt{B^2\sigma_0^2+\frac{\sigma_M^2}{m}}}\right)\middle|\frac{n+1}{2},\frac{n+1}{2}\right)$$

$$-F_\beta\left(\Phi\left(\frac{(2k-1-2j(1-\lambda))K\sqrt{\frac{\lambda}{2-\lambda}}}{\lambda(2p+1)}-\frac{B\delta\sigma_0}{\sqrt{B^2\sigma_0^2+\frac{\sigma_M^2}{m}}}\right)\middle|\frac{n+1}{2},\frac{n+1}{2}\right).$$

Let us denote $\eta=\frac{\sigma_M}{\sigma_0}$ the precision error ratio. The probability $Q_{j,\,k}$ can be rewritten in the form

$$\begin{aligned}Q_{j,k} &= F_\beta\left(\Phi\left(\frac{(2k+1-2j(1-\lambda))K\sqrt{\frac{\lambda}{2-\lambda}}}{\lambda(2p+1)}-\frac{B\delta}{\sqrt{B^2+\frac{\eta^2}{m}}}\right)\middle|\frac{n+1}{2},\frac{n+1}{2}\right)\\ &-F_\beta\left(\Phi\left(\frac{(2k-1-2j(1-\lambda))K\sqrt{\frac{\lambda}{2-\lambda}}}{\lambda(2p+1)}-\frac{B\delta}{\sqrt{B^2+\frac{\eta^2}{m}}}\right)\middle|\frac{n+1}{2},\frac{n+1}{2}\right).\end{aligned}\tag{A4}$$

Let $\mathbf{q}=(q_{-p},\ldots,q_0,\ldots,q_p)^T$ be the $(2p+1)$ vector of initial probabilities associated with the $2p+1$ transient states, where

$$q_j=\begin{cases}0 & \text{if } Z_0^*\notin(H_j-\Delta,H_j+\Delta],\\ 1 & \text{if } Z_0^*\in(H_j-\Delta,H_j+\Delta].\end{cases}\tag{A5}$$

Then, similar to Tran et al.,[25] the $ATS_1$ can be calculated by the following expression:

$$ATS_1=\mathbf{q}^T(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{g},\tag{A6}$$

where $\mathbf{g}$ is the vector of sampling intervals corresponding to the discretized states of the Markov chain and the $j$th element $g_j$ of the vector $\mathbf{g}$ is the sampling interval when the control statistic is in state $j$ (represented by $H_j$), that is,

$$g_j=\begin{cases}h_L & \text{if } LWL<H_j<UWL,\\ h_S & \text{otherwise.}\end{cases}\tag{A7}$$

Also, the average sampling interval of the VSI EWMA-$\widetilde{X}$ chart is given as

$$E(h)=\frac{\mathbf{q}^T(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{g}}{\mathbf{q}^T(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{1}}.\tag{A8}$$