Title

Opportunistic interference management: A new approach for multiantenna downlink cellular networks

Journal
Wireless Communications and Mobile Computing, 15(14)

ISSN
1530-8669

Authors
Kiskani, MK
Wang, Z
Sadjadpour, HR
et al.

Publication Date
2015-10-10

DOI
10.1002/wcm.2465

Peer reviewed

RESEARCH ARTICLE

Opportunistic Interference Management: A New Approach for Multi-Antenna Downlink Cellular Networks

Mohsen Karimzadeh Kiskani†, Zheng Wang†, Hamid R. Sadjadpour†, Jose A. Oviedo† and J. J. Garcia-Luna-Aceves ‡

ABSTRACT

A new approach for multi-antenna broadcast channels in cellular networks based on multiuser diversity concept is introduced. The technique called Opportunistic Interference Management (OIM), achieves Dirty Paper Coding (DPC) capacity asymptotically with minimum feedback required. When there are K antennas at the base station with M mobile users in the cell, the proposed technique only requires K integer numbers related to channel state information (CSI) between mobile users and base station. The encoding and decoding complexity of this scheme is the same as that of point-to-point communications which makes the implementation of this technique easy. An antenna selection scheme is proposed at the base station to reduce the minimum required mobile users significantly at the expense of reasonable increase in feedback. In order to guarantee fairness, a new algorithm is presented which incorporates OIM into existing GSM standard.

KEYWORDS

Interference Management, Diversity, Antenna Selection, Capacity, Interference

1. INTRODUCTION

Multiuser diversity scheme [7] is an alternative approach to more traditional techniques like time division multiple access (TDMA) to increase the capacity of wireless cellular networks. The main idea behind this approach is that the base station selects a mobile user that has the best channel condition by taking advantage of the time varying nature of fading channels, thus maximizing the signal-to-noise ratio (SNR). This technique has been recently studied for MIMO and downlink channels in [2] and [6]. Traditionally, fading and interference have been viewed as the two major impeding factors in increasing

M. K. Kiskani†, Z. Wang†, H. R. Sadjadpour†, J. A. Oviedo† and J.J. Garcia-Luna-Aceves‡, are with the Department of †Electrical and ‡Computer Engineering, University of California, Santa Cruz. Email: {mohsen, wzgold, hamid, xmando, jj}@soe.ucsc.edu

the capacity of wireless cellular networks. Opportunistic Interference Management (OIM) scheme is an approach that takes advantage of the fading in the channel to reduce the negative effects of interference.

Several schemes have been developed that achieve optimal dirty paper coding capacity by utilizing beamforming [10, 11]. Most recent studies [5, 9, 13, 19] have investigated the effect of partial finite-rate feedback on the capacity of MIMO broadcast channels in networks with limited number of users M.

We present the OIM technique for the downlink of wireless cellular networks in which d ($d \leq K$) independent data streams can be broadcasted to d ($d \ll M$) mobile stations with single antenna such that these data streams do not interfere with each other*. Furthermore, the mean value of d, i.e. $D = \mathbf{E}[d]$, can be any number up to the maximum value of K as long as M is large enough. Therefore, OIM is capable of achieving the maximum multiplexing gain as long as there is a minimum number of mobile stations in the network. The feedback requirement to transmit K independent data streams is proportional to K. The original multiuser diversity concept was based on searching for the best channel to communicate, while our approach shows that searching simultaneously for the best and worst channels can lead to significant capacity gains. This technique can asymptotically achieve the capacity of DPC when M is very large. OIM scheme does not require mobile stations to cooperate for synchronization during transmission. It achieves optimal K maximum multiplexing gain in the downlink of cellular systems as long as $K = \Theta(\log M)$. However, in most practical cellular networks, there may not be too many mobile users in a cell. Therefore, it is important to reduce the minimum required number of mobile users. This paper also introduces an antenna selection technique at the base station such that it reduces the minimum required number of mobile users significantly. This improvement is achieved at the expense of modest increase in the feedback requirement and additional computational complexity at the base station.

The remaining of this paper is organized as follows. Section 2 presents an overview of the related work. Section 3 introduces the OIM scheme and the model used in our analysis. Section 4 presents the theoretical analysis and corresponding numerical results. Section 5 focuses on the antenna selection scheme. Fairness issues and practical considerations are discussed in Section 6 and the paper is concluded in Section 7.

2. RELATED WORK

Knopp and Humblet [7] introduced the multiuser diversity approach that achieves the optimum capacity for the uplink of a wireless cellular network. It was shown that the channel with the highest SNR in the network can provide the maximum capacity by allocating all the power to this user instead of water-filling technique for power allocation. This result was extended in [15] for the downlink of cellular networks. Moreover, similar ideas was used in [17] to introduce the so-called "dumb antennas" concept by using opportunistic beamforming. The multiuser diversity concept was extended to mobile ad hoc networks (MANETs) in [4] which resulted in linear scaling of capacity for these networks. These approaches took advantage of multiuser diversity concept to address two major problems in wireless networks, namely, fading and interference.

*Note that d is a random variable.

DOI: 10.1002/wcm
Prepared using wcmauth.cls

Sharif and Hassibi introduced a technique [10,11] based on random beamforming concept to search for the best SINR in the network. Random beamforming technique uses isotropically random unitary matrices at the receiver side to create random beams. However, our technique utilizes deterministic vectors of identity matrix. In a sense, our algorithm is a special case of Hassibi's work when we no longer use random beams. However, this assumption allows us to derive new results. There are other differences between our approach and the design in [10, 11]. The feedback requirement in our scheme is proportional to K integers while this value is proportional to M complex numbers in [10, 11]. Moreover, our approach can be easily extended to distributed systems such as ad hoc networks [18] while random beamforming approaches cannot be extended to distributed systems.

DPC provides the maximum multiplexing and multiuser diversity gains which results in sum-rate capacity of $K \log \log M$. DPC requires full knowledge of CSI which makes the approach not practical. We propose a new scheduling approach which requires minimum feedback proportional to K and yet asymptotically retains the optimal multiplexing and multiuser diversity gains similar to DPC.

There are few papers in literature [3, 12, 13] with some similarities to our technique. In [3], a "1-bit" feedback was proposed instead of CSI information to the base station with the total feedback still proportional to M. The approach in [13] is asymptotically optimal and it also exhibits a good performance for practical network sizes. In this approach, it was shown that the total number of feedback can be bounded by using appropriate feedback to $K \log K$ bits. Further, OIM can be considered as a cognitive radio scheme that takes advantage of spatial orthogonalization. This viewpoint was derived independently in [12].

3. OPPORTUNISTIC INTERFERENCE MANAGEMENT

3.1. Network Model

This paper studies the optimal transmission in the downlink for the cellular networks when the base station (BS) sends independent data to different mobile stations (MS). When the BS has K antennas, it can at most transmit K data streams simultaneously. In this paper, we assume each MS has one antenna. The channel between the BS and MSs denoted as $\mathbf{H}$ is an $M \times K$ matrix with elements h_{ji}, where i $(1 \leq i \leq K)$ represents the index for BS antenna and j $(1 \leq j \leq M)$ represents the MS. The channel is a block fading model where the coefficients of the channel are constant during each coherence interval of T. The received signal in this model $\mathbf{Y}^{M\times 1}$ is given by

$$\mathbf{Y} = \mathbf{Hx} + \mathbf{n}, \quad (1)$$

where $\mathbf{x}$ is the transmit signal column vector of length K and $\mathbf{n}$ is the noise column vector of length M. The noise at the receiver of each antenna is i.i.d. with $\mathcal{CN}(0, \sigma_n^2)$ distribution. Table I summarizes the definitions of notations that are used in this paper.

3.2. The scheduling protocol

At the initial phase of communication, the BS antennas transmit K pilot signals while the MSs listen to these known signals. After this phase, MSs compute the SNR for each antenna. If certain conditions with respect to SNR are satisfied for a mobile node, that particular MS will be chosen by the BS. The MS satisfies the OIM condition when the SNR with respect to one transmit antenna is above a predetermined threshold SNR_{tr} and for the rest of $K-1$ antennas, it is below another predetermined threshold of INR_{tr}.

In the second phase of communication, the MSs that satisfy SNR criteria will notify the BS that they can receive packets during the rest of time period T. The channel access protocol for MS users to communicate with the BS or the case when two MSs both satisfy SNR condition for the same BS antenna will not be discussed in this paper. Note that by selecting appropriate threshold values for SNR_{tr} and INR_{tr} such that $\text{SNR}_{\text{tr}} \gg \text{INR}_{\text{tr}}$, then the BS can transmit multiple data streams from different antennas to different MSs. Each MS receives the intended packet with a strong signal while the rest of received packets are below INR_{tr} which can be treated as noise. The values of these thresholds depend on the application and can be achieved in the network if the total number of MS users M, is large enough.

There is a direct relationship between the average number of BS antennas with OIM condition, $D = \mathbf{E}(d)$, and the number of MSs, M. One advantage of OIM is the reduction in encoding and decoding complexity of this network. The OIM encoding and decoding reduces to point-to-point communications because of the decoupling of the channels. However, this advantage is at the expense of large number of MSs required to achieve this gain. The OIM system is shown in Figure 1. Without loss of generality, we have paired the i^{th} user with the antenna i at the BS. Solid (dotted) line in this figure represents strong (weak) channel. If there is no line between the BS and MS, the channel is considered a random parameter with some channel probability distribution function.

4. THEORETICAL ANALYSIS

We assume SNR_{ji} (INR_{ji}) denotes the signal-to-noise ratio (interference-to-noise ratio) between antenna j at BS to MS i. The OIM scheduling technique attempts to identify the eligible MS users d from M potential candidates that satisfy the following conditions.

$$\begin{aligned} \text{SNR}_{ii} &\geq \text{SNR}_{\text{tr}}, \quad 1 \leq i \leq d, \\ \text{INR}_{ji} &\leq \text{INR}_{\text{tr}}. \quad 1 \leq j \leq K, 1 \leq i \leq d, j \neq i \end{aligned} \tag{2}$$

The above condition in (2) states that any of the d MS users has a strong channel with an antenna of the BS and weak channels with the other $K-1$ antennas of BS as shown in Fig. 1. The BS will select those MS users with OIM condition based on the feedback it receives from MS users. The selection is based on the criterion to have the maximum number of active MS users in the second phase of communications. If two MS users have OIM condition for the same antenna at the BS, then one of them will be selected. If none of the MS users satisfy the conditions in (2), then the BS selects one of the MSs and only communicates with that MS which results in a multiplexing gain of one.

The downlink sum rate of this cellular network can be computed as

$$\begin{aligned} R_{\text{proposed}} &= \sum_{i=1}^{d} \log\left(1 + \text{SINR}_{ii}\right) \\ &= \sum_{i=1}^{d} \log\left(1 + \frac{\text{SNR}_{ii}}{\sum_{j=1, j\neq i}^{d-1} \text{INR}_{ji} + 1}\right) \\ &\geq d \log\left(1 + \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}} + 1}\right) \\ &= d \log(1 + \text{SINR}_{\text{tr}}), \end{aligned} \tag{3}$$

DOI: 10.1002/wcm
Prepared using wcmauth.cls

where SINR_{ii} and SINR_{tr} are defined as

$$\begin{aligned}\text{SINR}_{ii} &= \frac{\text{SNR}_{ii}}{\sum_{j=1,j\neq i}^{d-1}\text{INR}_{ji}+1}, \forall i=1,2,\cdots,d \\ \text{SINR}_{\text{tr}} &= \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}}+1},\end{aligned} \quad (4)$$

respectively.

We will first derive the multiplexing gain d. Next, we will develop the relationship between SINR_{tr} and M. Finally, the optimality of the approach will be proved.

The channel is considered to be Rayleigh fading but this approach can be implemented for any time-varying wireless channel.In i.i.d. Rayleigh fading channel, the probability distribution function (pdf) of SNR (or INR) [16] is given by

$$p(\mathrm{z}) = \frac{1}{\sigma}\exp\left(-\frac{z}{\sigma}\right), \quad z \geq 0 \quad (5)$$

where z is the SNR (or INR) value, $\mathbf{E_H}(z) = \sigma$, and $\mathbf{Var_H}(z) = \sigma^2$. Note that $\sqrt{\sigma/2}$ is the parameter for Rayleigh fading distribution which demonstrates the strength of the fading channel.

Another practical OIM condition can be described as:

$$\begin{aligned}\text{SNR}_{ii} &\geq \text{SNR}_{\text{tr}}, \quad 1 \leq i \leq d, \quad (6)\\ \gamma_{ji} = \frac{\text{INR}_{ji}}{\text{SNR}_{ii}} &\leq \gamma. \quad 1 \leq j \leq K, 1 \leq i \leq d, j \neq i\end{aligned}$$

Utilizing this condition, we can use the multi-user diversity in a more efficient way. The dependence between the random variables in γ_{ji} makes the analysis very complicated. Further, this model requires more feedback messages which results in higher computational complexity at the receiver to select the right MS users. Therefore, we will use the simpler model for analysis in this paper but we will compute $\text{Pr}(A)$ for this model in the appendix.

4.1. Exact Analysis

For any mobile station, let's define event A as the event that satisfies the conditions in (2) (or (6)). The channels between the BS antennas and the MS users are i.i.d. following an exponential distribution as in (5). Using the independence assumption between these channels, the probability of A can be derived as

$$\begin{aligned}\text{Pr}(A) &= \binom{K}{1}\int_{\text{SNR}_{\text{tr}}}^{\infty} p(z)dz \left(\int_0^{\text{INR}_{\text{tr}}} p(z)dz\right)^{K-1} \\ &= \binom{K}{1} e^{-\frac{\text{SNR}_{\text{tr}}}{\sigma}}\left(1-e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{K-1}. \end{aligned} \quad (7)$$

We intend to maximize this probability based on network parameters. Maximization of $\text{Pr}(A)$ is equivalent to minimization of M. Note that among all network parameters $K, \text{SNR}_{\text{tr}}, \text{INR}_{\text{tr}}$ and σ, the values of K and σ are related to the physical properties of the network and are not design parameters.

Let X be the random variable that defines the number of mobile stations satisfying (2). The probability of $X = x$ is computed as

$$\text{Pr}_A(X=x) = \binom{M}{x}(\text{Pr}(A))^x(1-\text{Pr}(A))^{M-x}. \quad (8)$$

Hence, with a probability of $(1-\text{Pr}(A))^M$, none of the mobile stations satisfy (2) which is equivalent of a multiplexing gain of one. Thus for our analysis, we assume that $x > 0$ and later on include the case of $x = 0$. We solve it by formulating it as a "bins and balls" problem. We have x balls that satisfy the OIM condition. The pdf of x is given in (8). Let's define event $\mathcal{B}$ as the event of associating y specific BS antennas (or bins) to $x > 0$ given MSs (or balls) which satisfy the OIM condition. This probability is denoted as $\text{Pr}_{\mathcal{B}}(d = y|X = x)$. This probability includes the cases that some of y antennas are associated to either

DOI: 10.1002/wcm

none or more than one of x MSs, i.e., some bins are empty and some bins have more than one ball in them. This conditional probability is given by

$$\Pr_{\mathcal{B}}(d = y|X = x) = \left(\frac{y}{K}\right)^{x}. \qquad y \leq K \tag{9}$$

Let's define another event $\mathcal{C}$ as the event of associating y specific BS antennas to x given MS users satisfying the OIM condition. Therefore, each BS antenna in this set is associated to at least one of the x mobile stations and denote it by $\Pr_{\mathcal{C}}(d = y|X = x)$. This probability is computed as

$$\Pr_{\mathcal{C}}(d = y|X = x) =$$

$$\begin{cases} \Pr_{\mathcal{B}}(d = 1|X = x), & \text{for } \ y = 1 \\ \Pr_{\mathcal{B}}(d = y|X = x) - \displaystyle\sum_{j=1}^{y-1} \binom{y}{j} \Pr_{\mathcal{C}}(d = j|X = x), & \\ \qquad \text{for } \ 1 < y \leq \min(x, K) & \\ 0. & \text{for } \ y > \min(x, K) \end{cases} \tag{10}$$

This iterative equation is initialized for $y = 1$ as $\Pr_{\mathcal{B}}(d = 1|X = x)$. Note $\Pr_{\mathcal{C}}(d = y|X = x)$ represents the probability of selection of a unique combination of y antennas. We define event $\mathcal{D}$ as the summation of all possible choices of $\Pr_{\mathcal{C}}(d = y|X = x)$. Event $\mathcal{D}$ is the event of associating y BS antennas chosen from all the BS antennas for the x MS users with OIM condition so that all the antennas in this set are associated to at least one of the x MS users. $\Pr_{\mathcal{D}}(d = y|X = x)$ is computed as

$$\Pr_{\mathcal{D}}(d = y|X = x) = \binom{K}{y} \Pr_{\mathcal{C}}(d = y|X = x). \tag{11}$$

The expected value of d is given in the following theorem and proved in the Appendix.

Theorem 1

The average multiplexing gain is

$$\begin{aligned} D &= \mathbf{E}(d) & (12) \\ &= (1 - \Pr(A))^{M} + K\left(1 - \left(1 - \frac{\Pr(A)}{K}\right)^{M}\right). \end{aligned}$$

Theorem 2

Assuming that M is large enough $(M \gg 10)$, we have

$$M\Pr(A) \approx \sqrt{\frac{2K}{K-1}(D-1)}. \tag{13}$$

The proof for this theorem is provided in the Appendix. These theorems show that for a desired value of D and fixed K and M, we can compute $\Pr(A)$ and derive the design parameters SNR_{th} and INR_{th}.

4.2. Numerical Results

Our simulation results are compared with our analytical results in order to demonstrate the validity of our analysis. Fig. 2 shows that the simulation results are very close to the theoretical results. The simulation results are for different values of K when $\Pr(A)$ is fixed. Fig. 3 illustrates the relationship between M and D for $K = 3$ or 5, $\sigma = 10$ and $\text{INR}_{\text{th}} = 1$. The result implies that by increasing SINR_{tr}, the minimum number of MS users to achieve OIM condition increases exponentially. It has been shown that Shannon capacity achieving techniques such as Turbo code or LDPC can operate at very low SINR_{tr} which makes the OIM approach more practical. Further, the simulation results demonstrate that there is a tradeoff between the total number of the MS users M and the number of the nodes $K - D$ needed to communicate utilizing cooperation techniques such as distributed MIMO. For example for $K = 3$, the capacity of the network increases twofold with only 100 mobile stations in the network.

Wirel. Commun. Mob. Comput. 2010; **00**:1–17
DOI: 10.1002/wcm
Prepared using wcmauth.cls

4.3. Scaling Law Analysis

We now prove that the sum-rate of OIM scheme achieves asymptotically the optimum DPC capacity of $K \log\log M$. We can use equation (13) to minimize the required number of mobile users M in terms of $\Pr(A)$ when the average multiplexing gain is fixed to a constant value D. Minimizing M is equivalent of minimizing $(\Pr(A))^{-1}$ under the SINR_{tr} constraint.

$$\begin{aligned} &\text{minimize} && (\Pr(A))^{-1}, && (14)\\ &\text{subject to} && \text{SINR}_{\text{tr}} = \frac{\text{SNR}_{\text{tr}}}{(K-1)\text{INR}_{\text{tr}}+1}, && (15)\end{aligned}$$

This optimization problem can be rewritten as

$$\begin{aligned} &\min_{Eq.(15)} \left((\Pr(A))^{-1}\right) = \\ &\frac{1}{K}\min_{Eq.(15)}\left(\frac{e^{\frac{\text{SNR}_{\text{tr}}}{\sigma}}}{\left(1-e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{K-1}}\right) \overset{(a)}{=} \\ &\frac{1}{K}e^{\frac{\text{SINR}_{\text{tr}}}{\sigma}}\min_{\text{INR}_{\text{tr}}}\left(\frac{e^{(K-1)\frac{\text{SINR}_{\text{tr}}\text{INR}_{\text{tr}}}{\sigma}}}{\left(1-e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{K-1}}\right).\end{aligned}$$

Equality (a) is derived by replacing SNR_{tr} with INR_{tr} and SINR_{tr} using equation (15). In most practical applications, there is a required predetermined minimum value for SINR_{tr} and if this value is fixed, we can optimize the above equation based on INR_{tr}. The solution for INR^*_{tr} is

$$\text{INR}^*_{\text{tr}} = \sigma\log\left(1+\frac{1}{\text{SINR}_{\text{tr}}}\right). \qquad (16)$$

The maximum value of $\Pr^*(A)$ is found to be

$$\begin{aligned}\Pr^*(A) &= K\exp\left(-\frac{\text{SINR}_{\text{tr}}}{\sigma}\right)\\ &\times \frac{\text{SINR}_{\text{tr}}{}^{(K-1)\text{SINR}_{\text{tr}}}}{(1+\text{SINR}_{\text{tr}})^{(K-1)(1+\text{SINR}_{\text{tr}})}}. \qquad (17)\end{aligned}$$

Minimum value of M can be found by replacing $\Pr^*(A)$ in equation (13).

$$M^* \approx \frac{1}{\Pr^*(A)}\sqrt{\frac{2K}{K-1}(D-1)}, \qquad (18)$$

The asymptotic behavior of the network with respect to M is investigated. We also compute the maximum achievable capacity and scaling laws for this scheme. When M tends to infinity, $\Pr^*(A) \to 0$ and from equation (17), it implies that SINR_{tr} approaches infinity. Therefore,

$$\begin{aligned}&\Omega\left(\frac{M}{\sqrt{\frac{2K}{K-1}(D-1)}}\right) = \lim_{M\to\infty}\frac{1}{\Pr^*(A)} = \\ &\lim_{\text{SINR}_{\text{tr}}\to\infty}\frac{1}{K}(1+\text{SINR}_{\text{tr}})^{K-1}e^{\frac{\text{SINR}_{\text{tr}}}{\sigma}}\left(\frac{1+\text{SINR}_{\text{tr}}}{\text{SINR}_{\text{tr}}}\right)^{\text{SINR}_{\text{tr}}(K-1)}\\ &= \frac{1}{K}e^{K-1}\lim_{\text{SINR}_{\text{tr}}\to\infty}e^{\frac{\text{SINR}_{\text{tr}}}{\sigma}}(1+\text{SINR}_{\text{tr}})^{K-1}\\ &= O\left(\frac{1}{K}e^{K-1}e^{2\frac{\text{SINR}_{\text{tr}}}{\sigma}}\right).\end{aligned}$$

The SINR_{tr} asymptotic lower bound is given by

$$\begin{aligned}&\lim_{M\to\infty}\text{SINR}^{\max}_{\text{tr}}\\ &= \Omega\left(\frac{\sigma}{2}\log\left(e^{-(K-1)}\sqrt{\frac{K(K-1)}{2(D-1)}}M\right)\right)\\ &= \Omega(\log M). \qquad (19)\end{aligned}$$

Thus, $\text{SINR}^{\max}_{\text{tr}}$ scales at least with $\Omega(\log M)$. Let's assume $\text{SINR}_{\text{tr}} = \Theta\left(\frac{\sigma}{4}\log M\right) = \Theta\left(\frac{\sigma}{2}\log M^{1/2}\right)$ and by considering above equations, we arrive at $M^{1/2} = O\left(e^{K-1}\sqrt{\frac{2(D-1)}{K(K-1)}}\right) = O\left(e^{2K}\right)$. This result implies that $K = \Omega(\log M)$ is achievable. Thus the scaling laws of OIM scheme is

$$R_{\text{proposed}} = \Omega(K\log\log M). \qquad (20)$$

We will demonstrate by simulation that when SINR_{tr} increases proportional to $\Theta(\log M)$, maximum multiplexing gain of K is achievable provided that M grows to infinity. We define SINR_{tr} as

$$\text{SINR}_{\text{tr}} = \frac{\sigma}{c_0} \log\left(\left(\frac{1}{e}\right)^{K-1} M\right), \quad (21)$$

where c_0 is a constant value. In practical cellular systems, it is possible that the minimum number of mobile users may not be available in a cell. Note that it is easy to show that for any value of K, M and σ, the designer can select the appropriate value for SINR_{tr} such that the maximum multiplexing gain is achieved at the expense of reduced rate for each individual mobile user, i.e., $D = K$.

When SINR_{tr} increases logarithmically with M, OIM can achieve the maximum multiplexing gain for different values of c_o based on (21) (see Fig. 4).

It is noteworthy to point out that for a constant value of M, when the fading coefficient σ increases, OIM provides higher multiplexing gain up to a point where by increasing σ, the probability of having strong channels decreases such that OIM does not perform well any more. Figures 5 and 6 illustrate this fact.

When $K = 1$, then our approach is similar to that of [7]. Moreover if $M \to \infty$ and $D = K$, then our scheme has the same asymptotic scaling laws capacity result as that of [10]. OIM requires a large number of mobile stations in order to perform well. However, we will introduce some techniques later to reduce the minimum required number of mobile stations for OIM.

4.4. Feedback requirements

A natural question regarding OIM scheme is the number of MS users that will send feedback to the base station. Clearly, this number is a random variable, which we denote by X. We demonstrate that the probability that X MS users satisfy the OIM criteria can be arbitrarily close to 1 if we select proper SINR_{tr} based on network parameters such as M and the fading parameter σ.

For any mobile station, the probability that it satisfies the OIM condition is $\Pr(A)$, i.e., the mobile station has a very strong channel with a single base station antenna and very weak channels with all other base station antennas. The number of the mobile stations satisfying the interference management criteria is a random variable X satisfying binomial distribution whose probability density function (pdf) is given by (8). Therefore, the cumulative distribution function can be expressed as

$$\Pr(X \leq K) = \sum_{i=0}^{K} \binom{M}{i} (P(A))^i (1 - P(A))^{M-i}. \quad (22)$$

To prove that this value can be very close to one, note that for large values of M, if we want to have a predetermined multiplexing gain of D with a fixed number of antennas in the base station, theorem 2 states that the value of $M\Pr(A)$ remains constant. This result implies that the random variable X can be approximated by a Poisson distribution with parameter

$$\lambda = M\Pr(A) = \sqrt{\frac{2K}{K-1}(D-1)}. \quad (23)$$

Therefore,

$$\Pr(X \leq K) \approx \sum_{i=0}^{K} e^{-\lambda} \frac{\lambda^k}{k!}. \quad (24)$$

Since λ is always less than or equal to $\sqrt{2K}$, it is also smaller than K (for example for $K = 12$, $\lambda < 5$). It is not hard to verify that the sum of the first K terms of the probability distribution for a Poisson random variable with parameter $\lambda \leq \sqrt{2K}$ is very close to one. In other words,

Wirel. Commun. Mob. Comput. 2010; **00**:1–17 © 2010 John Wiley & Sons, Ltd.
DOI: 10.1002/wcm
Prepared using wcmauth.cls

$\Pr(X \leq K) \geq \eta$ where η is very close to one. Table (II) shows the values of η for different values of K and $\lambda \leq \sqrt{2K}$. As suggested by this table, the probability that the number of MS users satisfying OIM condition being less than K is lower bounded by η which is very close to one.

For any given values of K, M and σ, the appropriate value for SINR$_{tr}$ can be selected with probability close to 1 such that the value of random variable X is less than K (see numerical results in Fig. 7). Since the number of MS users in a cell is known to the BS, the BS can adjust the value of SINR$_{tr}$ such that there are enough MS users in the network that qualify the OIM constraints. OIM approach is an improvement compared to the dirty paper coding or techniques introduced in [10, 11], which require $K \times M$ and M CSI feedback information respectively. In these approaches, by increasing M, the required feedback information also increases. However, OIM requires $\Theta(K)$ CSI feedback independent of the number of MS users with probability arbitrarily close to 1 as long as the SINR$_{tr}$ is selected appropriately.

5. ANTENNA SELECTION ALGORITHM

Antenna selection [8] is a powerful technique to capture many advantages of the MIMO systems. In this approach, the selection of the antenna is based on computing the SNR for all possible channels and choosing the highest SNR among the BS antennas. In OIM, we assume that the objective is to achieve a multiplexing gain of D while there are K antennas at the base station with $K \gg D$.

In our proposed antenna selection technique, a parameter $L \leq K-1$ is defined as the minimum number of channels between a BS antenna and a MS such that their associated INR is less than or equal to INR$_{tr}$. In other words, any MS that satisfies the following conditions sends a feedback to the BS.

$$\begin{aligned} \mathrm{SNR}_{ii} &\geq \mathrm{SNR}_{\mathrm{tr}}, \quad \text{for some } i,\ 1 \leq i \leq K \\ \mathrm{INR}_{ji} &\leq \mathrm{INR}_{\mathrm{tr}}, \quad \text{for at least L MS users} \end{aligned} \tag{25}$$

Since there are a large number of antennas at the BS, there will be an increase in the number of pilot signals transmitted from the BS. If A' represents the event that a specific MS statisfies the new conditions, then we have

$$\Pr(A') \geq \binom{K}{L}(K-L)e^{-\frac{\mathrm{SNR}_{\mathrm{tr}}}{\sigma}}\left(1-e^{-\frac{\mathrm{INR}_{\mathrm{tr}}}{\sigma}}\right)^{L}. \tag{26}$$

The right hand side in equation (26) corresponds to the case when a specific MS has exactly L weak channels and one strong channel with BS antennas.

Under the new assumption, each MS with at least L weak and one strong channels will send its information to the BS. Each MS should notify the strong and weak channels to the BS. Therefore, the new approach requires more feedback exchange between MS users and the BS. Notice that it is possible for a mobile station to have more than one strong channel and it is the task of the base station to choose one strong channel based on the total information it receives.

It is reasonable to assume that the BS has a large number of antennas. The challenge is to select a subset of these MS users such that they have OIM condition when operated simultaneously. The objective is to search amongst these MS users that send their information to the BS and select the set of mobile users with maximum multiplexing gain. In this set, every mobile user should have weak channels with all of the other antennas that have strong channels associated to other mobile stations.

There are mainly two ways to carry out this search. The optimum solution is the exhaustive search among all possible combinations of MS users and finding the highest multiplexing gain. The optimum (exhaustive) search is carried by utilizing backtracking algorithm [1]. Backtracking is an algorithm that finds some (or all) solutions to a computational problem, that incrementally builds candidates to the solution, and drops any partial candidate as soon as it determines that it cannot lead to the best and valid solution. Backtracking algorithm searches among all different combinations of antennas and selects the ones that combined result provides the maximum parallel transmissions. This approach requires significant computational complexity and long delay which is not practical for implementation in the base stations.

We propose a sub-optimal approach based on antenna selection techniques. In this approach, after the mobile stations report their OIM to the BS, we use a greedy search algorithm to select the mobile stations. First, we choose the one with the maximum number of weak channels with BS antennas. Then we create a table with the first row having the information related to the number of antennas with weak channels and one strong channel†. Then based on this set, we choose the next mobile user that covers the largest subset of this set provided that its strong channel location is different from the first MS user. We continue this algorithm until there is no MS that can be added to this table. The set of MS users in this table can receive parallel data flows with OIM condition. The OIM approach requires less than K feedback with high probability. The new antenna selection approach requires more feedback information from the MS users but results in smaller number of MS users for the same multiplexing gain as compared to OIM approach. This technique provides a trade-off between the multiplexing gain and feedback information.

Algorithm 1 Optimal search algorithm

procedure OIMReport
  for $i = 1 \to M$ **do**
    if (25) and (25) hold **then** MS i reports to the BS.
      return
ReportedSet ← Set of all reported MS to the BS
$d_{\max} \leftarrow 1$
SelectedSet ← Pick a random MS from ReportedSet
procedure OptimalSearch
  for $\forall A \subseteq$ ReportedSet **do**
    $d(A) \leftarrow$ multiplexing gain using the subset A
    if $d(A) \geq d_{\max}$ **then**
      $d_{\max} \leftarrow d(A)$
      SelectedSet ← A

Algorithm 2 Sub-Optimal search algorithm

procedure SubOIMReport
  for $i = 1 \to M$ **do**
    if (25) and (25) hold **then** MS i reports to the BS.
      return
ReportedSet ← Set of all reported MS to the BS
SelectedMSSet ← An element of ReportedSet with the largest number of week channels with BS antennas.
$d_{\max} \leftarrow$ Multiplexing gain of SelectedMSSet.
$S \leftarrow$ Set of BS antennas that have weak channels with all members of SelectedMSSet.
procedure SubOptimalSearch
  while $S \neq \emptyset$ **do**
    SelectedMS ← An element of ReportedSet that has at least one strong channel with one of the elements of S and also has the maximum number of weak channels with other elements of S.
    Append SelectedMS to the SelectedMSSet.
    $S \leftarrow$ Set of BS antennas that have weak channels with all members of SelectedMSSet.

5.1. Feedback Analysis

Using equations (7) and (26) in case of antenna selection, we have

$$\frac{\Pr(A')}{\Pr(A)} \geq \binom{K}{L} \frac{K-L}{K} \left(1 - e^{-\frac{\mathrm{INR_{tr}}}{\sigma}}\right)^{L-k+1}. \quad (27)$$

Since the right hand side of (27) is a tight lower bound for large values of M, the value of $\lambda = M\Pr(A)$ remains constant and similarly, the value of $\lambda' = M\Pr(A')$ also remains constant. Therefore, in case of antenna selection

†If there are more than one mobile user with the largest number of antennas satisfying the OIM constraint, then we choose all of them and do the parallel search for all of them to find the best solution.

Wirel. Commun. Mob. Comput. 2010; **00**:1–17 © 2010 John Wiley & Sons, Ltd.
DOI: 10.1002/wcm
Prepared using wcmauth.cls

with optimal search also the number of MSs satisfying the OIM conditions follows a Possion distribution with the lower bound of the parameter defined as

$$\lambda' = M\text{Pr}(A') \approx \lambda \binom{K}{L} \frac{K-L}{K} \left(1 - e^{-\frac{\text{INR}_{\text{tr}}}{\sigma}}\right)^{L-k+1}. \tag{28}$$

From (28), if L is decreased, λ' will increase and therefore for any arbitrary positive integer m, the following probability is decreased.

$$\text{Pr}(X \leq m) \approx \sum_{i=0}^{m} e^{-\lambda'} \frac{\lambda'^i}{i!} \tag{29}$$

This result implies that the number of feedbacks to the BS will increase. Notice that in case of optimal or sub-optimal search, the amount of feedback stays the same.

5.2. Numerical Results

Fig. 8 compares the performance of our sub-optimal search to that of the optimal search for different values of L and when $\text{SNR}_{tr} = 40$, $K = 10$, $\text{INR}_{tr} = 2$. If L is more than 5, the difference between the optimal and sub-optimal techniques is so small that can not be distinguished from the figure. Therefore, when L is relatively large compared to K, the sub-optimal search has a performance close to optimal search with a much lower computational complexity. However, when L is reduced, the difference becomes noticeable. For example for $L = 4$, the difference is approximately a multiplexing gain of 0.5 for $M = 850$. Thus, the use of sup-optimal search for small values of L will not lead to a result close to the optimal search.

The simulation results demonstrate that antenna selection technique reduces the required number of MS users significantly. For example, for $D = 2$ and $K = 10$ when $\text{SNR}_{tr} = 40$, $\text{INR}_{tr} = 2$ and $\sigma = 10$, the optimal search only requires 540 mobile users and the sub-optimal search requires 1290 mobile users when $L = 4$. However, with the same set of parameters in the original OIM (when $L = K - 1 = 9$) we need at least 53 million mobile users to be able to get a multiplexing gain of two.

Table (III) demonstrates the simulation results for the antenna selection technique when $K = 10$. The number in the second and third columns represent the number of mobile users required to achieve multiplexing gain of 2 for the optimal and sub-optimal search, respectively. From this table, it can be concluded that in general, few nodes usually send their feedback information which makes this technique practical. Further, as K increases the minimum required number of mobile users to achieve certain multiplexing gain decreases significantly compared to the original OIM scheme. These small numbers of mobile users are quite practical in wireless cellular networks. For example, we can achieve multiplexing gain of 2 with only 15 (20) mobile users using optimal (sub-optimal) search. These results are obtained based on the network parameters of $\text{SNR}_{tr} = 40, \text{INR}_{tr} = 2,$ and $\sigma = 10$.

6. PRACTICAL RELATED ISSUES

The selection of MS users in OIM is based on their channel condition. However, current cellular systems assign channels to users based on schemes such as time-division. Further, if a MS is very close to the BS with line of sight, this MS will not be selected by OIM approach. Therefore, there is the fairness issue that we should resolve. We will address these two problems in this section for TDMA systems. The approach can be also extended to other schemes like CDMA.

6.1. Fairness under TDMA Scheme

The issue of fairness and Quality of Service (QoS) is addressed here for TDMA users while allowing other users to use OIM scheme without interrupting the primary users. In TDMA approach, the signal vector is given by

$$\mathbf{R}^{\mathrm{T}}_{\mathrm{TDMA}} = \mathbf{S}^{\mathrm{T}}_{\mathrm{TDMA}} h_{\mathrm{TDMA}} + \sum_{i=1}^{d} S_i h_i \mathbf{V}^{\mathrm{T}} + \mathbf{n}^{\mathrm{T}}, \qquad (30)$$

where $\mathbf{R}_{\mathrm{TDMA}}$ is the received signal vector by MS and $\mathbf{S}_{\mathrm{TDMA}}$ is the transmitted signal vector by the BS antenna that is not participating in OIM scheme. The superscript T represents transpose of a vector, S_i and V^T are the signal transmitted by the OIM scheme and a vector with unit weight. $\mathbf{n}$ is the additive Gaussian noise vector with zero mean i.i.d. elements and variance of σ_n. h_{TDMA} and h_i are the channel between base station and mobile users that are participating in TDMA and OIM scheme respectively.

The received vector is multiplied by another vector $\mathbf{U}$ that is orthonormal to $\mathbf{V}$, i.e., $\mathbf{U}\mathbf{V}^T = 0$. Hence, the received signal is equal to

$$\begin{aligned} \mathbf{U}\mathbf{R}^{\mathrm{T}}_{\mathrm{TDMA}} &= \mathbf{U}\mathbf{S}^{\mathrm{T}}_{\mathrm{TDMA}} h_{\mathrm{TDMA}} + \sum_{i=1}^{d} S_i h_i \mathbf{U}\mathbf{V}^T + \mathbf{U}\mathbf{n}^{\mathrm{T}} \\ &= \mathbf{U}\mathbf{S}^{\mathrm{T}}_{\mathrm{TDMA}} h_{\mathrm{TDMA}} + \mathbf{n}'. \qquad (31) \end{aligned}$$

After we multiply the received vector by $\mathbf{U}$, the OIM signals will no longer be in the new value and there is no interference from OIM signals to the TDMA signals. Note that there is no relationship between the vector $\mathbf{V}$ and the channel. We will later describe the criterion for selecting this vector. For block fading channel, this vector only requires to be of length 2. Notice that by using the new approach, the actual rate of OIM signals is reduced by a factor proportional to the length of the vector $\mathbf{V}$. However, the rate of TDMA signal is still one symbol per channel use.

If the wireless channel is block fading, then $\mathbf{U} = [u_1, u_2]$ and $\mathbf{V} = [v_1, v_2]$ are enough for implementation. For the remaining of the paper, QPSK signals are assumed for transmission. Since our operation of multiplying by the vector $\mathbf{U}$ results in combining multiple QPSK signals, it is desirable that we have the maximum separation among the values of the new signal in the two dimensional constellation. This criterion improves the decoding of the received TDMA symbols.

One appropriate choice for the combination of two QPSK signals is a 16-QAM symbol. We can achieve this objective by utilizing the results from [14]. For a 16-QAM symbol, we have

$$\text{16-QAM} = \sum_{i=0}^{1} 2^i \left(\frac{\sqrt{2}}{2}\right) (j^{x_i}) \exp\left(\frac{\pi j}{4}\right), \qquad (32)$$

where $x_i \in Z_4 = \{0, 1, 2, 3\}$. The QPSK constellation is realized as QPSK $= j^{x_i}$. Thus by using shift and rotation operations, we can create M-QAM constellations from QPSK symbols. It is easy from Eq. (32) to see that the normalized values of vectors $\mathbf{U}$ and $\mathbf{V}$ are

$$\mathbf{U} = \sqrt{\frac{2}{5}} \exp\left(\frac{\pi j}{4}\right) \left[\frac{\sqrt{2}}{2}, \sqrt{2}\right] \qquad (33)$$

and

$$\mathbf{V} = \sqrt{\frac{2}{5}} \left[\sqrt{2}, -\frac{\sqrt{2}}{2}\right], \qquad (34)$$

respectively. Since the vector $\mathbf{U}$ is normalized, then the variance of Gaussian noise remains the same.

Note that with this signaling at the base station, the fairness and Quality of Service (QoS) for all the users are guaranteed in a time-division approach while other users

Wirel. Commun. Mob. Comput. 2010; **00**:1–17 © 2010 John Wiley & Sons, Ltd.
DOI: 10.1002/wcm
Prepared using wcmauth.cls

can utilize the channel by taking advantage of the OIM scheme.

6.2. Signaling requirement

The OIM approach will simplify the problem of distributed MIMO system into multiple single-input single-output (SISO) systems. Therefore, the encoding and decoding computational complexity of this approach is much simpler than the MIMO system. This is another additional advantage of using OIM in the cellular systems.

7. CONCLUSION

In this paper, we have proposed Opportunistic Interference Management (OIM) technique that has similar capacity as DPC asymptotically. This approach uses multiuser diversity in wireless networks by taking advantage of significant variations of wireless channels to minimize the negative effects of interference. Further, OIM simplifies the encoding and decoding of distributed MIMO systems into multiple parallel SISO systems. We have also shown how to use antenna selection concept to reduce the minimum required number of MS users in the network to achieve high multiplexing gains. It has been shown through simulation that with as few as 15 mobile users one can achieve some multiplexing gain in the downlink of wireless cellular systems.

Our future work will concentrate on improving the minimum required mobile users for higher multiplexing gains than what we have derived with antenna selection technique. Extension of this technique to ad hoc networks will be also investigated in the future.

REFERENCES

1. Gilles Brassard and Paul Bratley. *Fundamentals of Algorithmics*. Prentice-Hall, 1995.
2. Jinho Choi. Multiuser diversity beamforming for downlink channels using modulation division multiplexing. *Wireless Communications and Mobile Computing*, 12(17):1528 – 1537, December 2012.
3. J. Diaz, O. Simeone, and Y. Bar-Ness. Asymptotic analysis of reduced-feedback strategies for MIMO Gaussian broadcast channels. *IEEE Transactions on Information Theory*, 54(3):1308 – 1316, March 2008.
4. M. Grossglauser and D. Tse. Mobility increases the capacity of ad hoc wireless networks. *IEEE/ACM Transactions on Networking*, 10(4):477–486, August 2002.
5. K. Huang, R. W. Heath, and J. G. Andrews. Space division multiple access with a sum feedback rate constraint. *IEEE Transactions on Signal Processing*, 55(7):38793891, July 2007.
6. C. Hung, C. Chiang, N. Yen, and S. Lin. Performance of multiuser diversity in mimo systems on arbitrary nakagami-m fading channels. *Wireless Communications and Mobile Computing*, 12(8):729 – 738, June 2012.
7. R. Knopp and P. Humblet. Information capacity and power control in single-cell multiuser communications. In *Proc. of IEEE ICC 1995*, Seattle, Washington, USA, June 18-22 1995.
8. S. Sanayei and A. Nosratinia. Antenna selection in MIMO systems. *IEEE Commmunications Magazine*, 42(10):68–73, October 2004.
9. S. Sanayei and A. Nosratinia. Opportunistic beamforming with limited feedback. *IEEE Transactions on Wireless Commununications*, 6(8):2765 2771,

August 2007.

10. M. Sharif and B. Hassibi. On the capacity of MIMO broadcast channels with partial side information. *IEEE Transactions on Information Theory*, 51(2):506–522, February 2005.
11. M. Sharif and B. Hassibi. A comparison of time-sharing, DPC, and beamforming for MIMO broadcast channels with many users. *IEEE Transactions on Communications*, 55(1):11–15, January 2007.
12. C. Shen, T. Liu, and M. P. Fitz. On the average rate performance of hybrid-arq in quasi-static fading channels. *IEEE Transactions on Communications*, 57(11):3339–3352, 2009.
13. A. Tajer and X. Wang. Opportunistic multi-antenna downlink transmission with finite-rate feedback. In *Proc. of Allerton 2008*, Monticello, Illinois, USA, September 23-26 2008.
14. B. Tarokh and H. R. Sadjadpour. Construction of OFDM M-QAM sequences with low peak-to-average power ratio. *IEEE Transactions on Communications*, 51(1):25– 28, January 2003.
15. D. Tse. Optimal power allocation over parallel Gaussian broadcast channels. In *Proc. of IEEE ISIT 1997*, Ulm, Germany, June 29 - July 4 1997.
16. D. Tse and P. Viswanath. *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.
17. P. Viswanath, D. Tse, and R. Laroia. Opportunistic beamforming using dumb antennas. *IEEE Transactions on Information Theory*, 48(6):1277–1294, June 2002.
18. Z. Wang, M. Ji, H. Sadjadpour, and J. J. Garcia-Luna-Aceves. Opportunistic interference management increases the capacity of ad hoc networks. *IEEE SECON 2010 conference*, 2010.
19. T. Yoo, N. Jindal, and A. Goldsmith. Multi-antenna downlink channels with limited feedback and user selection. *IEEE Journal on Selected Areas in Communications*, 27(7):14781491, September 2007.

.1. A more practical OIM model

In order to compute $\text{Pr}(A)$ using the model in (6), we need to find the probability that for any i, $1 \leq i \leq K$, the criterion in (6) holds. For a specific i, let $y_j = \text{INR}_{ji}$ for $j = 1, ..., k$, $j \neq i$ and $u = \text{SNR}_{ii}$. We can use the Jacobian transformation to prove that the joint probability distribution $f_{u,\gamma_{1i},...,\gamma_{Ki}}(u, \gamma_{1i}, ..., \gamma_{Ki})$ is equal to $u^K f_{u,y_1,...,y_K}(u, u\gamma_{1i}, ..., u\gamma_{Ki})$. Now we use the independence between y_j's and u to write this joint probability distribution as

$$f(u, \gamma_{1i}, ..., \gamma_{Ki}) = \frac{u^K}{\sigma^{K+1}} e^{-\frac{u}{\sigma}(1+\sum_{j=1}^{K} \gamma_{ji})}. \tag{35}$$

Therefore, we have

$$\begin{aligned} &\text{Pr}(A) = \\ &K \int_{\text{SNR}_{th}}^{\infty} \int_0^{\gamma} ... \int_0^{\gamma} f(u, \gamma_{1i}, ..., \gamma_{Ki}) \mathrm{d}\gamma_{Ki} ... \mathrm{d}\gamma_{1i} \mathrm{d}u \\ &= \frac{K}{\sigma} \int_{\text{SNR}_{th}}^{\infty} \left(1 - e^{-\frac{u}{\sigma}\gamma}\right)^K \mathrm{d}u. \end{aligned} \tag{36}$$

Notice that the only difference between the models in (6) and in (2) is in $\text{Pr}(A)$. All other equations of (8), (9) (10) and (11) remain unchanged and therefore theorems 1 and 2 will be valid.

.2. Proof of theorem 1

Lemma 1

Suppose a_n is a recursive sequence with $a_1 = 1$ and

$$a_n = n^x - \sum_{j=1}^{n-1} \binom{n}{j} a_j, \quad for \quad n > 1 \tag{37}$$

where x is an arbitrary real number. Then

$$a_n = \sum_{j=1}^{n} \binom{n}{j} (-1)^{n-j} j^x. \tag{38}$$

Proof

We use induction to prove the lemma. For $n = 1$, the correctness of the lemma can be easily verified. Suppose for $i = 1, 2, ..., n-1$ the lemma is correct. Our goal is to show that it is also true for $i = n$. If we plug the corresponding values of a_i from (38) for $i = 1, 2, ..., n-1$ and use (37), we have

$$\begin{aligned} a_n &= n^x - \sum_{i=1}^{n-1} \binom{n}{i} a_i \\ &= n^x - \sum_{i=1}^{n-1} \sum_{j=1}^{i} \binom{n}{i} \binom{i}{j} (-1)^{i-j} j^x \\ &= n^x - \sum_{j=1}^{n-1} \sum_{i=j}^{n-1} \binom{n}{i} \binom{i}{j} (-1)^{i-j} j^x \\ &= n^x - \sum_{j=1}^{n-1} \sum_{i=j}^{n-1} \binom{n}{j} \binom{n-j}{i-j} (-1)^{i-j} j^x \\ &= n^x - \sum_{j=1}^{n-1} \binom{n}{j} j^x \sum_{i=j}^{n-1} \binom{n-j}{i-j} (-1)^{i-j} \\ &= n^x - \sum_{j=1}^{n-1} \binom{n}{j} j^x \sum_{m=0}^{n-j-1} \binom{n-j}{m} (-1)^{m} \\ &= n^x + \sum_{j=1}^{n-1} \binom{n}{j} (-1)^{n-j} j^x \\ &= \sum_{j=1}^{n} \binom{n}{j} (-1)^{n-j} j^x. \end{aligned}$$

□

We will use this lemma to prove theorem 1. Let $a_y = K^x \Pr_C(d = y|X = x)$ and use (10) to arrive at

$$\begin{aligned} &a_1 = 1, \\ &a_y = y^x - \sum_{j=1}^{y-1} \binom{y}{j} a_j, \quad \text{for} \quad y > 1. \end{aligned} \tag{39}$$

Using lemma 1, the recursive sequence a_y can be rewritten as

$$a_y = \sum_{j=1}^{y} \binom{y}{j} (-1)^{y-j} j^x, \quad \text{for} \qquad 1 \le y \le \min(x, K). \tag{40}$$

Therefore, for $1 \le y \le \min(x, K)$ we have

$$\Pr_{\mathcal{C}}(d = y|X = x) = \sum_{j=1}^{y} \binom{y}{j} (-1)^{y-j} \left(\frac{j}{K}\right)^x, \tag{41}$$

(42)

and

$$\Pr_{\mathcal{D}}(d = y|X = x) = \sum_{j=1}^{y} \binom{K}{y} \binom{y}{j} (-1)^{y-j} \left(\frac{j}{K}\right)^x. \tag{43}$$

Equation (10) implies that $\Pr_{\mathcal{C}}(d = y|X = x)$ and $\Pr_{\mathcal{D}}(d = y|X = x)$ are zero for $y > \min(x, K)$. Now that we have closed form formulas for these probabilities, we can compute the expected value of d.

$$\begin{aligned}
\mathbf{E}(d|X=x) &= \sum_{y=1}^{K} y\mathrm{Pr}_{\mathcal{D}}(d=y|X=x)\\
&= \sum_{y=1}^{\min(x,K)} \sum_{j=1}^{y} y\binom{K}{y}\binom{y}{j}(-1)^{y-j}\left(\frac{j}{K}\right)^x\\
&= \sum_{j=1}^{\min(x,K)} \sum_{y=j}^{\min(x,K)} y\binom{K}{y}\binom{y}{j}(-1)^{y-j}\left(\frac{j}{K}\right)^x\\
&= \sum_{j=1}^{\min(x,K)} \sum_{y=j}^{\min(x,K)} y\binom{K}{j}\binom{K-j}{y-j}(-1)^{y-j}\left(\frac{j}{K}\right)^x\\
&= \sum_{j=1}^{\min(x,K)} \binom{K}{j}\left(\frac{j}{K}\right)^x \sum_{y=j}^{\min(x,K)} y\binom{K-j}{y-j}(-1)^{y-j}\\
&= \sum_{j=1}^{\min(x,K)} \binom{K}{j}\left(\frac{j}{K}\right)^x \sum_{m=0}^{\min(x,K)-j} (m+j)\\
&\times \binom{K-j}{m}(-1)^m \qquad (44)
\end{aligned}$$

If $x \geq K$, then $\min(x,K)=K$ and thus in the above derivation, we can use the following equation

$$\begin{aligned}
&\textstyle\sum_{m=0}^{K-j}(m+j)\binom{K-j}{m}(-1)^m\\
&= j\delta[K-j]-\delta[K-j-1], \qquad (45)
\end{aligned}$$

to obtain

$$\begin{aligned}
&\mathbf{E}(d|X=x) =\\
&\sum_{j=1}^{K}\binom{K}{j}\left(\frac{j}{K}\right)^x(j\delta[K-j]-\delta[K-j-1])\\
&= K-K\left(\frac{K-1}{K}\right)^x. \qquad (46)
\end{aligned}$$

However if $x < K$, then $\min(x,K)=x$ and we can use the equation

$$\begin{aligned}
&\sum_{m=0}^{x-j}(m+j)\binom{K-j}{m}(-1)^m\\
&= \frac{(-1)^{x-j}(xK-xj-j)}{(K-j-1)}\binom{K-j-1}{x-j}, \qquad (47)
\end{aligned}$$

to arrive at

$$\begin{aligned}
&\mathbf{E}(d|X=x) =\\
&\sum_{j=1}^{x}\binom{K}{j}\left(\frac{j}{K}\right)^x\left(\frac{(-1)^{x-j}(xK-xj-j)}{(K-j-1)}\right)\\
&\times\binom{K-j-1}{x-j} = K-K\left(\frac{K-1}{K}\right)^x. \qquad (48)
\end{aligned}$$

So far, we have proved that for every $x>0$, then

$$\mathbf{E}(d|X=x) = K-K\left(\frac{K-1}{K}\right)^x. \qquad (49)$$

Note that when $x=0$, the base station can select any one of the mobile stations and starts communicating with it with, thus $\mathbf{E}(d|X=0)=1$. Now we can proceed to compute D.

$$\begin{aligned}
&D = \mathbf{E}(d) =\\
&\sum_{x=0}^{M}\sum_{y=1}^{K} y\mathrm{Pr}_{\mathcal{D}}(d=y|X=x)\mathrm{Pr}_A(X=x)\\
&= \mathrm{Pr}_A(X=0)+\\
&\quad \sum_{x=1}^{M}\sum_{y=1}^{K} y\mathrm{Pr}_{\mathcal{D}}(d=y|X=x)\mathrm{Pr}_A(X=x)\\
&= (1-\mathrm{Pr}(A))^M + \sum_{x=1}^{M}\mathbf{E}(d|X=x)\mathrm{Pr}_A(X=x)\\
&= (1-\mathrm{Pr}(A))^M + \sum_{x=1}^{M}\left(K-K\left(\frac{K-1}{K}\right)^x\right)\times\\
&\qquad\qquad \binom{M}{x}(\mathrm{Pr}(A))^x(1-\mathrm{Pr}(A))^{M-x}\\
&= (1-\mathrm{Pr}(A))^M + K\left(1-\left(1-\frac{\mathrm{Pr}(A)}{K}\right)^M\right) \qquad (50)
\end{aligned}$$

.3. Proof of theorem 2

Proof

Using Taylor series representation for power series and

Wirel. Commun. Mob. Comput. 2010; **00**:1–17 © 2010 John Wiley & Sons, Ltd.
DOI: 10.1002/wcm
Prepared using wcmauth.cls

also using theorem 1, we arrive at

$$
\begin{aligned}
D &= 1 + \frac{M(M-1)}{2}(1 - \frac{1}{K})(\Pr(A))^2 \\
&+ O((\Pr(A))^3). \qquad (51)
\end{aligned}
$$

Since $\Pr(A)$ is small, the last term can be neglected compared with the second term and if M is large enough $(M >> 10)$, then $M(M-1) \approx M^2$ and the theorem follows immediately. $\square$

Notation	Definition
A	The event that a specific MS satisfies OIM conditions (2)
A'	The event that a specific MS satisfies OIM conditions (25) and (25).
$\mathcal{B}$	The event of associating y BS antennas to x given mobile stations.
$\mathcal{C}$	The event of associating y BS antennas to x given mobile stations without overlap.
$\mathcal{D}$	The event of associating y BS antennas to x given MSs without overlap such that each antenna is associated to at least one MS.
M	Number of mobile stations
K	Number of BS antennas
L	Minimum required number of weak antennas in antenna selection algorithm
N_{th}	Threshold number of MSs to report to BS in antenna selection algorithm
SNR_{ji}	signal-to-noise ratio for the channel between BS antenna j and MS i.
SNR_{th}	Threshold value of SNR for strong channels.
INR_{th}	Threshold value of SNR for weak channels.
d	Multiplexing gain
D	Average multiplexing gain
γ_{ji}	Ratio of INR_{ji} divided by SNR_{ii}.
γ	Threshold value for weak channels in (6)
σ	Rayleigh fading channel parameter
X	Number of MS reporting feedback to the BS
λ	Poisson parameter for random variable X

Table I. Definitions and notations used in this paper.

$\lambda = M\Pr(A)$	K	η
2	6	0.996
2	8	0.999
2	10	1
3	8	0.997
3	10	0.9997
4	10	0.997
4	12	0.999

Table II. The probability that the number of mobile stations satisfying OIM conditions be less than K is very close to one.

L	(Optimal)	(Sub-Optimal)
$L=1$	15	20
$L=2$	23	29
$L=3$	48	61
$L=4$	151	181
$L=5$	651	780
$L=6$	4400	4700

Table III. Minimum required number of mobile stations in order to have $(D=2)$ when $(K=10)$

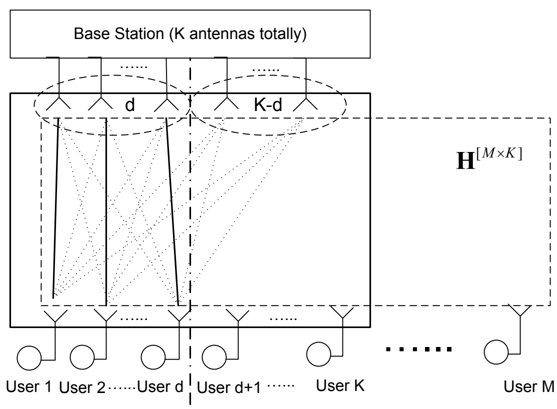

Figure 1. Wireless cellular network model

DOI: 10.1002/wcm
Prepared using wcmauth.cls

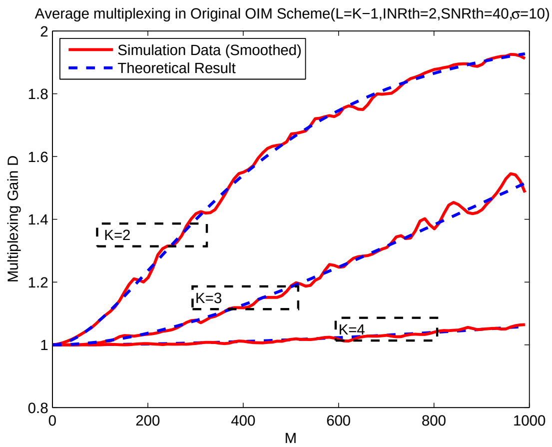

Figure 2. OIM simulations are consistent with analytical results.

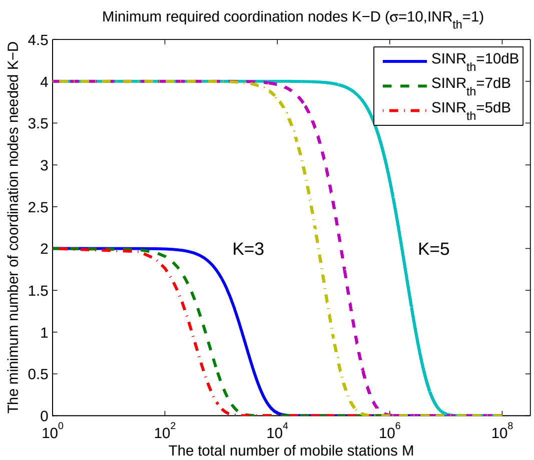

Figure 3. Number of cooperation nodes needed for different values of SINR

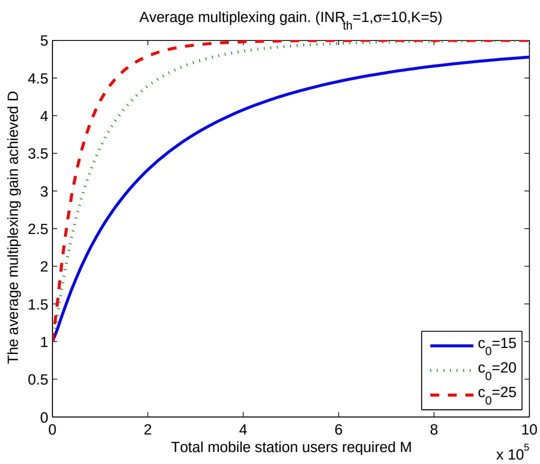

Figure 4. DPC capacity and maximum multiplexing gain are achieved simultaneously.

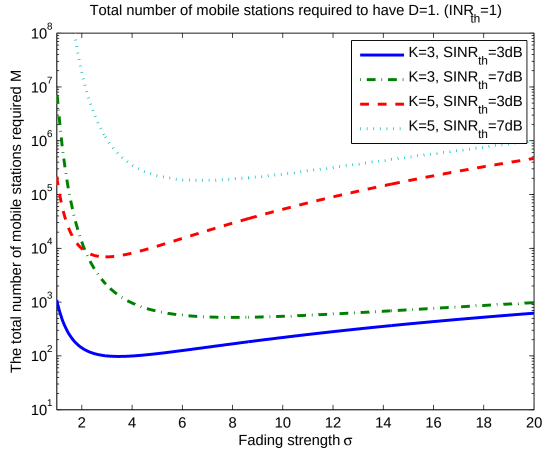

Figure 5. Different fading channel environments and total number of mobile stations M required

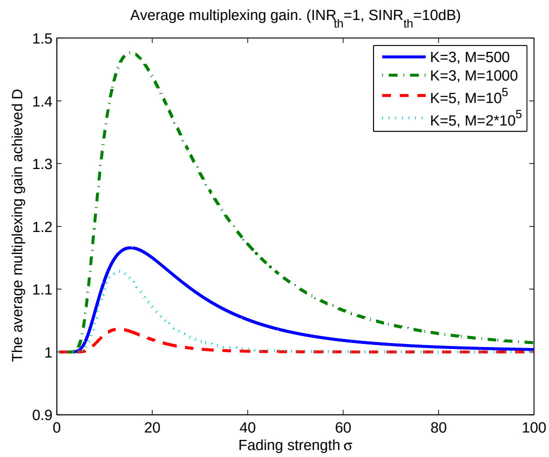

Figure 6. Relationship between fading strength and multiplexing gain.

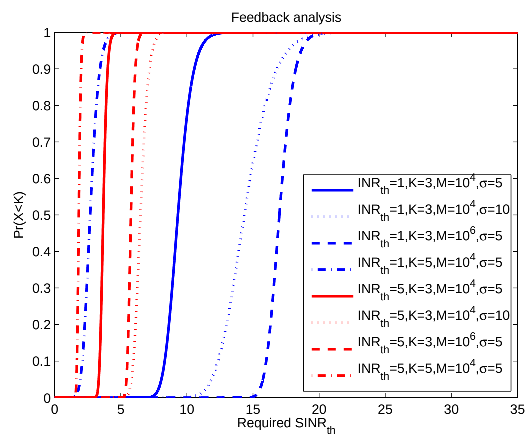

Figure 7. The feedback is at most K with almost sure.

DOI: 10.1002/wcm

Prepared using wcmauth.cls

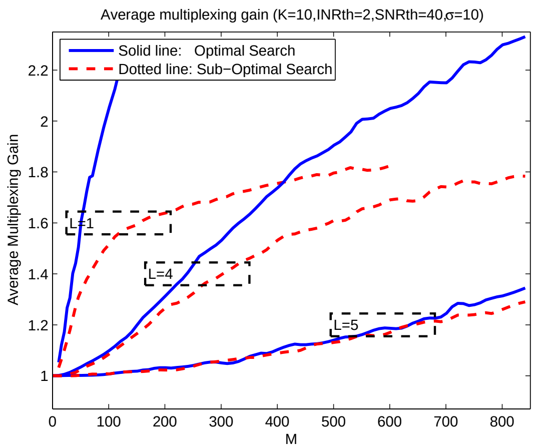

Figure 8. Trade off between complexity and minimum number of users required for K=10 ($N_{th} = M/10$).

DOI: 10.1002/wcm