# Aggregate Pattern of Time-dependent Adjustment Rules, II: Strategic Complementarity and Endogenous Nonsynchronization

by

Sau-Him Paul Lau *

Australian National University and Hong Kong Baptist University

April 2000

**Running Title:**
Strategic Complementarity and Endogenous Nonsynchronization

**Correspondence to:**
S.-H. P. Lau, Department of Economics, Hong Kong Baptist University, Kowloon Tong, Hong Kong.

Phone: (+852) 2339 5200
Fax: (+852) 2339 5580
E-mail: shlau@hkbu.edu.hk

* I am grateful to Steve Dowrick, Simon Grant, Stephen King, Jeff Kline, Jacek Krawczyk, Bill Neilson, Katya Sherstyuk, Hugh Sibly, Curtis Taylor, Chung Yi Tse, and anonymous referees for helpful comments and suggestions.

**Aggregate Pattern of Time-dependent Adjustment Rules, II: Strategic Complementarity and Endogenous Nonsynchronization**

**Abstract:**

This paper provides an explanation for an important institutional feature of staggered time-dependent adjustment rules assumed in a number of macroeconomic models (Fischer [19], Taylor [32]). It identifies strategic complementarity as the crucial factor leading to nonsynchronized decisions in a game-theoretic framework. The paper first shows that nonsynchronization is the equilibrium outcome in an infinite-horizon game in which strategic complementarity is present, whether the players choose predetermined or fixed actions. By pursuing Tirole's [33] interpretation of a nonsynchronized-move dynamic game as a series of games with 'symmetric Stackelberg leadership', it is further suggested that the relationship between strategic complementarity and the benefit to the Stackelberg *follower* provides the insight to the game-theoretic explanation of nonsynchronization. The results of this paper reveal a link between strategic complementarity and nonsynchronization - two important macroeconomic features. *Journal of Economic Literature* Classification Numbers: E30; L16

**Keywords**: interdependence; strategic complementarity; endogenous nonsynchronization

## 1. INTRODUCTION

An important institutional feature appearing in a number of macroeconomic models (Fischer [19], Taylor [32], Blanchard [6]) is that wage or price decisions are made at different times. With nonsynchronous decisions, fully anticipated monetary policies can affect the real economy even when expectations are rational. Moreover, fixed level of individual prices for a short duration may generate gradual adjustment of aggregate price level in response to nominal disturbances and therefore lead to persistent real effects. The seminal papers in this field emphasize the real and long-lasting effects of short-run disturbances under the hypothesis of rational expectations, and how institutional arrangements affecting the propagation mechanism of the economy. In these articles, the overlapping timing pattern is taken as an assumption, which is believed to be representative of certain stylized facts.

Given the crucial importance of nonsynchronous decisions, recent papers have tried to provide the microfoundations. Endogenous timing pattern is derived when each agent is allowed a choice of contract renewal dates. To highlight the difference, the outcome which results from an overlapping timing pattern is often contrasted with that of its polar opposite - complete synchronization - in which all agents make decisions at the same time. Fethke and Policano [17, 18] extend the model in Fischer [19] and Gray [22] and obtain the equilibrium pattern of wage negotiation. In their papers, staggered contracts allow the transmission of employment effects across sectors. On the other hand, Ball and Cecchetti [2] and Ball and Romer [4] examine price setting behavior in variants of the Blanchard-Kiyotaki model (Blanchard [6], Blanchard and Kiyotaki [8]). The former paper suggests that information extraction is the reason for nonsynchronization, while the latter emphasizes rapid adjustment to idiosyncratic shocks.

A common characteristic of these papers is that the choices of individual agents, in terms of the contract renewal dates and the price (or wage) levels, will affect the payoffs of other agents in the economy. The presence of interdependence is a necessary condition for the pattern of price changes to be a relevant issue. In the absence of interdependence, an economic agent will not care about other agents' prices and the time of making such changes. Since the presence of interdependence among economic agents is common and is likely to be recognized by them, it is natural to investigate what will result when they take this factor into account and interact strategically. In a sense, this is what the above-mentioned microfoundation papers do. However, in these papers, strategic interaction among the agents only appears in the choice of contract renewal dates, but not in the choice of prices or wages, even though an agent's payoff depends on the choice variables of other agents. A distinctive feature of this paper and Lau [26] is that strategic interaction among the agents appears in both aspects. The basic idea is that the nature of the game may differ with respect to the timing pattern, as is found in the dynamic analysis of market structure by Maskin and Tirole [27, 28, 29]; see also the discussion in Blanchard and Fischer [7, p. 401].

Lau [26] studies two aspects of the labor market institutions (staggered versus synchronized wage setting, and coordinated versus noncooperative wage adjustment) in a model with strategic complementarity and negative externality. In particular, it is shown that wage setters prefer moving alternatingly when they interact noncooperatively. That paper mainly focuses on the relative ability of synchronization and nonsynchronization in overcoming macroeconomic externalities of wage adjustment, and does not consider whether and how the *Pareto superior* pattern will arise as the *equilibrium* outcome. This paper goes a step further by deriving formally that nonsynchronization is the equilibrium timing pattern in a dynamic model with strategic complementarity and positive externality, whether the players choose predetermined or fixed actions.[1] Strategic complementarity may arise from the production technology, the trading technology, or agents' demands in many macroeconomic models; see Cooper and John [12]. A question that arises naturally in the presence of strategic complementarity is whether the Pareto superior timing pattern of nonsynchronization (for a particular model) will arise as the equilibrium outcome, especially if the initial pattern is

[1]Following the terminology in macroeconomics (see Blanchard and Fischer [7, p. 389], Romer [31, p. 257]), price (or wage) adjustment in a multi-period contract is known as 'predetermined' if the price in each period of the contract may be different (as in Fischer [19]), but the adjustment is known as 'fixed' if the price is constant throughout the contract (as in Taylor [32]).

synchronization. The analysis in Cooper and John [12] suggests that when strategic complementarity is present, an inefficient equilibrium may perpetuate because no individual has an incentive to deviate from this equilibrium. The possibility of coordination failure may therefore prevent a Pareto superior outcome to be realized. The analysis of this paper suggests a simple way to solve the coordination problem in reaching the Pareto superior pattern of nonsynchronized moves, and as a result, nonsynchronization will emerge as the equilibrium outcome.

The intuition underlying the above result is that in a model with strategic complementarity and positive externality such as a differentiated-product price competition game, each player has an incentive to set a lower price to undercut his opponent when they move simultaneously, yet the effect of this action is offset by similar behavior of his opponent at the equilibrium. As a result, both players earn lower payoffs at the simultaneous-move equilibrium when compared to moving sequentially. With nonsynchronized moves, a player needs not worry about his action being undercut and can set a higher price. Moreover, such action induces his opponent to do so (because of strategic complementarity) in the future. As positive externality is present, both players benefit from the higher prices. The first part of this paper applies this idea and shows formally that nonsynchronization is the equilibrium outcome, whether predetermined or fixed prices are chosen. As there are some interesting differences between the two games with predetermined and fixed prices respectively, the precise implementation of the above idea in each game will be discussed in more details after the equilibrium is derived.

Generalizing from the analysis of the specific differentiated-product price competition model, this paper then aims to understand the factors leading to the equilibrium timing pattern under strategic and dynamic interaction. This question is motivated by the different endogenous timing patterns found in various games in Maskin and Tirole [27, 28, 29], De Fraja [14], Lau [26] and this paper. Based on the interpretation of a nonsynchronized-move dynamic game as a series of games with 'symmetric Stackelberg leadership' (Tirole [33, p. 343]), this paper extends the insight of the analysis of a Stackelberg game and suggests that the equilibrium timing pattern can be traced to two aspects of the players' payoff functions: the slope of the reaction function (i.e., strategic complementarity versus substitutability) and the sign of the externality effect. The importance of these two factors in determining different interesting phenomena has been first mentioned by Fudenberg and Tirole [20] and Bulow et al. [10]. The analysis of this paper enriches the applicability of these taxonomies to the determination of the aggregate pattern of time-dependent adjustment rules.

This paper is organized as follows. Section 2 introduces a simple dynamic model exhibiting strategic complementarity and positive externality, with each player choosing repeatedly both the commitment length and the level(s) of the strategic variable. In Section 3, the aggregate timing pattern is derived endogenously when the players choose predetermined actions. Section 4 analyzes the game with fixed actions. Section 5 then examines the relationship between strategic complementarity and the benefit to the follower in a Stackelberg game, and suggests that the relationship provides the insight for the explanation of nonsynchronization in repeated interaction. Section 6 provides discussion and extension. Section 7 concludes.

## 2. A SIMPLE DYNAMIC MODEL WITH STRATEGIC COMPLEMENTARITY AND POSITIVE EXTERNALITY

This section introduces an infinite-horizon game which models jointly the players' decisions of the strategic variables and the commitment length. As the title of this paper suggests, only time-dependent adjustment rules are considered in this paper.[2] However, the assumption that a player adjusts on a regular schedule of, say, every two periods (as in many papers with time-dependent adjustment) is relaxed in order to allow the aggregate timing pattern to be derived endogenously. Specifically, the action of any player is assumed to last for one or two periods.

---

[2]Most wage adjustment are characterized as time-dependent, and the majority of price adjustment may well be described as state-dependent. Blanchard and Fischer [7, p. 413] discuss these two major types of adjustment rules and suggest some possible factors leading to each type. Interestingly, Ball and Mankiw [3] develop a model including time-dependent and state-dependent pricing to provide an explanation for asymmetric adjustment of nominal prices. Note that the aggregate pattern of state-dependent adjustment rules is not analyzed in this paper since a different approach (as in Caplin and Leahy [11]) is required.

There are two identical players (A and B) in the game and they make a sequence of decisions. Each decision of a player consists of two dimensions: a commitment length of either one or two periods and the level(s) of the strategic variable. If the commitment length is two periods, the actions in the two periods are assumed to be predetermined in Section 3, but are assumed to be fixed in Section 4. The game begins (at period zero) with both players making their initial choices simultaneously, so as to maintain complete symmetry. The above formulation allows either synchronization or nonsynchronization to arise as possible eventual equilibrium outcome. Such a flexible specification (of allowing the two-dimensional choices to be made repeatedly) may appear, at first glance, to be quite intractable, but the recursive nature of the problem makes the solution possible.

When making a new decision, say at period $t$ $(t \geq 0)$, player i (i=A,B) maximizes his intertemporal payoff (which is the present discounted value of the stream of his current and future single-period payoffs):

$$\sum_{s=0}^{\infty} \beta^s U^i(x^i_{t+s}, x^j_{t+s}), \tag{2.1}$$

where $i,j = A, B$ $(i \neq j)$, $\beta \in (0,1)$ is the discount factor and $x^i_{t+s}$ is the choice of player i at period t+s. To simplify notation, the subscript referring to the time period may sometimes be omitted if it does not cause confusion. The strategic variable $x^i$ is assumed to lie in a compact action space $[\underline{x}, \overline{x}]$. The single-period payoff function $U^i(.,.)$ is twice continuously differentiable with $U^i_{ii} < 0$, where the subscripts on $U^i$ denote partial derivatives (e.g. $U^i_j \equiv \partial U^i / \partial x^j$, $U^i_{ij} \equiv \partial^2 U^i / \partial x^i \partial x^j$). As will be elaborated in Section 5, the equilibrium pattern of time-dependent adjustment rules depends crucially on the presence of positive versus negative externality, and strategic complementarity versus substitutability. The externality effect refers to the interaction of the players at the level of payoffs, and strategic complementarity or substitutability refers to their interaction at the level of strategies.[3] Many interesting macroeconomic models satisfy either (a) strategic complementarity and positive externality,[4] or (b) strategic complementarity and negative externality.[5]

For the analysis in Sections 3 and 4, the single-period payoff function is assumed to be:

$$U^i(x_t^i, x_t^j) = x_t^i(h - x_t^i + g x_t^j), \tag{2.2}$$

where $g\,(0 < g < 1)$ and $h\,(h > 0)$ are constants. It is clear that $U_{ij}^i = g$ and $U_j^i = g x^i$. As a result, strategic complementarity and positive externality are present when $[\underline{x}, \overline{x}]$ is in the positive interval. The payoff function (2.2) may be interpreted as a player's profit in a differentiated-product price competition model.[6] When the demand of product i (i=A,B) is given by the bracketed term in the

---

[3]Positive (resp. negative) externality exists if an increase in a player's strategic variable leads to an increase (resp. decrease) in his opponent's total payoff, i.e., $U_j^i > 0$ (resp. $< 0$). Strategic complementarity (resp. substitutability) exists if an increase in a player's strategic variable leads to an increase (resp. decrease) in his opponent's marginal payoff, i.e., $U_{ij}^i > 0$ (resp. $< 0$); Eq. (5.2) makes clear that this condition is equivalent to a positive (resp. negative) dependence of a player's optimal strategy upon his opponent's.

[4]Strategic complementarity and positive externality are present in the trading externalities model of Diamond [15] and the production externalities model of Bryant [9]. In the Diamond [15] model, risk-neutral individuals face production decisions that arrive stochastically and have varying costs. To represent the advantage of specialized production and trade over self-sufficiency, it is assumed that individuals cannot consume their own products but trade their own output for that produced by others. Having made a decision to produce, agents then seek trading partners and consume the good so obtained. Utility depends negatively upon the cost of production and positively upon consumption. The payoff of a player is defined as the expected utility prior to the arrival of the production opportunity, and the strategic variable is the ex ante choice of a cutoff cost of production below which the individual will choose to produce. As more individuals produce, the expected payoff to an agent from producing will increase (i.e., positive externality) under the assumption that an increase in the number of potential trading partners makes trade easier. As more production opportunities are expected to be profitable, so an agent will increase his cutoff cost of production (i.e., strategic complementarity).

[5]Strategic complementarity and negative externality are present in the economic environment examined in Blanchard and Kiyotaki [8] and Lau [26]. In the household-firm version of the Blanchard and Kiyotaki [8] model, each of the monopolistically competitive firms set its price to maximize profit. A negative aggregate demand externality is present: a firm's profit will decrease with an increase in the prices of all other firms since the increase in prices will decrease the real money balances and thus aggregate demand. Also, strategic complementarity exists when the relative price effect dominates the aggregate demand effect; see Figure 3 and p. 659 of that paper.

[6]One may interpret that the two firms compete in prices by mailing price catalogs effective for one or two periods. However, it should be emphasized that this price setting game is used mainly to illustrate the two features of strategic complementarity and positive externality, and by no means implies that time-dependent price adjustment rules are necessarily more empirically relevant than state-dependent rules. The empirical evidence in Kashyap [23] suggests that either simple time-dependent rules or state-dependent rules capture some but not all of the essential aspects of catalog price setting. Alternatively, one may interpret the model as a real wage

right-hand side of (2.2) where $x^i$ represents price of good i, then with (normalized) zero production cost, the profit of firm i is given by (2.2). Parameter $h\,(h > 0)$ is the demand intercept, and parameter $g$ represents the degree of substitutability between the two goods (with the restriction $0 < g < 1$ capturing that they are imperfect substitutes). When price of product A increases, its demand is reduced but that for product B is increased, resulting in higher profit of firm B (i.e., positive externality). Moreover, an increase in the price of good A will raise the marginal profit of firm B and therefore, firm B's optimal price will increase (i.e., strategic complementarity).[7]

For convenience in exposition, the above game will be referred to as the timing-and-price game, even though it is also applicable to other models exhibiting strategic complementarity and positive externality (such as the wage setting game mentioned in footnote 6). Moreover, the action space $[\underline{x}, \overline{x}]$ is taken to be $[x_N, x_C]$ where $0 < x_N < x_C$, $x_N$ is defined in (2.5) below, and $x_C = h/[2(1-g)]$, which is the 'cooperative' choice of each player if they jointly maximize their single-period payoffs. As symmetric single-period payoffs appear several times in this paper, it is useful to define:

$$U_{rr} = U^i(x_r, x_r) = hx_r - (1-g)x_r^2. \tag{2.3}$$

It is helpful to illustrate the fundamental strategic conflicts facing the two players in a static setting and to define some terms which will be useful later, before considering the outcome when the agents interact dynamically. First, the optimal static reaction function of player i (i=A,B) conditional on the choice $x^j$ of the other player, $R_i(x^j)$, is defined implicitly by:

$$U_i^i\left(R_i(x^j), x^j\right) = 0. \tag{2.4}$$

---

setting game. In each of the two sectors, the (real) wage is assumed to be chosen by the employees or the union representing them (in the spirit of the monopoly union model). The employer then chooses the employment level, taking the wage as given. In this case, the single-period payoff in (2.2) is the wage bill of a particular sector.

[7]To paraphrase Bulow et al. [10, p. 510], "if an oligopoly model assumes, say, linear demand and *differentiated-product price competition*, the real economic assumption may be that the products are *strategic complements*."

Moreover, each player's choice at the Nash equilibrium of this static non-cooperative game, $x^A = x^B \equiv x_N$, is given by:[8]

$$x_N = \frac{h}{2-g}, \tag{2.5}$$

and the corresponding single-period payoff is:

$$U_{NN} = U^i(x_N, x_N) = \frac{h^2}{(2-g)^2}. \tag{2.6}$$

## 3. EQUILIBRIUM TIMING PATTERN WITH PREDETERMINED PRICES

This section analyzes the timing-and-price game with predetermined prices. If a player commits for two periods, the prices for both periods may be different. To derive the equilibrium, it is helpful to perform a sequence of analysis such that the players assume that some conditions are (and will be) satisfied now (and in the future) for each problem. After the various simpler problems are analyzed, the validity of these conditions is verified along the equilibrium path. Specifically, Sub-section 3.1 obtains the players' equilibrium strategies and the corresponding payoffs if both players are and will be forever committing for two periods and moving alternatingly. Sub-section 3.2 derives a crucial condition such that no player will deviate from the current timing pattern of nonsynchronization (with both players making commitments for two periods) by committing to a particular action for one period when this condition holds. This condition is called the no-deviation-from-nonsynchronization (NDFN) condition. On the assumption that the NDFN condition holds, Sub-section 3.3 shows that there are two symmetric pure-strategy equilibria. Sub-section 3.4 derives a mixed-strategy equilibrium to solve the coordination problem in reaching nonsynchronization, and verifies that the NDFN condition is indeed satisfied, after the value of the game (to be defined precisely later) is determined along the time path corresponding to the mixed-strategy equilibrium.

---

[8]For the single-period payoff function (2.2), it is easy to show that $R_i(x^j) = (h+gx^j)/2$. It follows that: (a) the corresponding maximum single-period payoff of player i is given by $U^i\left(R_i(x^j), x^j\right) = (h+gx^j)^2/4$; and (b) solving the optimal static reaction functions of both players simultaneously gives (2.5).

Sub-section 3.5 briefly summarizes the equilibrium outcome of the game and discusses the intuition.

### *3.1. Equilibrium strategies and payoffs if the players move alternatingly forever*

In this sub-section, it is assumed that after a particular time period, all commitments made by both players last for two periods and that the players move alternatingly. Without loss of generality, it is also assumed that player A (resp. B) chooses his two prices (one for the current period and the other for the next) at every even (resp. odd) period. Therefore, both $x^A_{2k}$ and $x^A_{2k+1}$ are chosen at period $2k$, and both $x^B_{2k-1}$ and $x^B_{2k}$ are chosen at period $2k-1$.

It can easily be seen that at, say, an even period when player A moves, the action of player B for the current period has been chosen (in the previous period), and this action of player B affects player A's current-period payoff. Therefore, player A acts as a Stackelberg follower (temporarily) in an even period, with his opponent as the Stackelberg leader. Besides setting a price for the current period, player A also chooses another price for the next period, knowing that his opponent's choice will be made in the next period. The roles of the two players are just reversed in an odd period.

The sequential nature of the players' moves together with the assumption of predetermined (but not necessarily constant) prices means that the equilibrium of this game can be solved period by period. The subgame perfect equilibrium at a particular period with player j (j=B when t=2k, or j=A when t=2k+1) as the leader and player i (i=A when t=2k, or i=B when t=2k+1) as the follower, is derived as follows. Taking the action of leader j as given, follower i chooses $x^i$ to maximize (2.2). The optimal reaction function of follower i, $R_i(x^j)$, is defined in (2.4). Anticipating the future action of follower i, leader j chooses $x^j$ to maximize:

$$U^j_{RF}(x^j) \equiv U^j\left(x^j,\ R_i(x^j)\right), \tag{3.1}$$

where $U^j_{RF}(x^j)$ may be interpreted as the 'reduced-form' single-period payoff function of leader j.

With the single-period payoff function (2.2), it can be shown that the optimal choice of leader j is given by $x^j = x_L$ where

$$x_L = \frac{(2+g)h}{2(2-g^2)}. \tag{3.2}$$

Combining (2.2), (2.4) and (3.2), the optimal choice of follower i is given by $x^i = x_F$ where

$$x_F = R_i(x_L) = \frac{h+gx_L}{2} = \frac{(4+2g-g^2)h}{4(2-g^2)}. \tag{3.3}$$

As a result, the single-period payoff of leader j and that of follower i are given by:

$$U^j\big(x_L, R_i(x_L)\big) = U^j(x_L, x_F) = \frac{(2+g)^2h^2}{8(2-g^2)} \equiv U_L, \tag{3.4}$$

$$U^i\big(R_i(x_L), x_L\big) = U^i(x_F, x_L) = \frac{(4+2g-g^2)^2h^2}{16(2-g^2)^2} \equiv U_F. \tag{3.5}$$

Combining (2.6), (3.4) and (3.5), it can be shown that

$$U_{NN} < U_L < U_F. \tag{3.6}$$

(All detailed derivations are contained in an Appendix available from the author upon request.)

For subsequent analysis, it is helpful to define two value functions for each player, with one function at the period when he moves and the other when his opponent moves. The value function $V^i(x^j)$ (resp. $W^i(x^i)$), $i,j=A,B$ and $i \neq j$, is defined as player i's intertemporal payoff at the beginning of a period in which he (resp. his opponent) moves, with the expectation that whenever each player moves in the future, the player will commit for two periods and choose $R_i(x^j)$ and $x_L$ in the first and second periods respectively. Note that the variable $x^j$ in $V^i(x^j)$ or the variable $x^i$ in $W^i(x^i)$ has already been set in the previous period. According to the results of dynamic programming, the optimal choices and the value functions are mutually consistent in the following manner:

$$V^i(x^j) = \max_{x^i, x^i_{+1}}\left[U^i(x^i, x^j) + \beta W^i(x^i_{+1})\right] = U^i\big(R_i(x^j), x^j\big) + \beta W^i(x_L), \tag{3.7}$$

$$W^i(x^i) = U^i\big(x^i, R_j(x^i)\big) + \beta V^i(x_L). \tag{3.8}$$

*3.2. The no-deviation-from-nonsynchronization condition*

This sub-section considers the most important step in solving the timing-and-price game. It aims to obtain the condition such that if the opponent's commitment lasts for one more period when a player is making a move, the player will always commit for two periods. This condition is crucial because if the condition holds for an arbitrary level in the action space, then no player will ever find it beneficial to deviate by committing for only one period. As a result, if the nonsynchronization pattern is reached at some point of time, then it will last forever.

At the time when a player (labelled as player i) moves, suppose that the price of his opponent (player j) for the current period has been set previously at $x^j$. If player i only commits for one period, then it is easy to see that starting from the next period, the whole game starts again with both players moving simultaneously. It is obvious that player i's optimal choice for the current period is given by $R_i(x^j)$. As a result, player i's intertemporal payoffs is given by:

$$\max_{x^i}\left[U^i(x^i,x^j)+\beta V\right]=U^i\left(R_i(x^j),x^j\right)+\beta V, \tag{3.9}$$

where $V$ is the value of the timing-and-price game with predetermined prices, defined as the (expected) value of each player's intertemporal payoff at the beginning of the game. Note that the players' payoff functions and the assumptions on timing decisions are completely symmetric; thus, $V$ is the same for both players. On the other hand, if player i commits for two periods, then the two prices will be set at $R_i(x^j)$ and $x_L$ respectively, and his intertemporal payoff will be given by $V^i(x^j)$ of (3.7), with the expectation that neither player will deviate from choosing prices for two periods according to (3.3) and (3.2) in the future.

Combining the above results, if the current timing pattern is nonsynchronization and the opponent's current price is $x^j$, then this pattern will last forever when the following NDFN expression:

$$V^i(x^j)-\left[U^i\left(R_i(x^j),x^j\right)+\beta V\right]=\beta\left[W^i(x_L)-V\right] \tag{3.10}$$

is positive (i.e., the NDFN condition is satisfied). As the NDFN expression depends on the value of the game, whether this expression is positive or not can only be verified after the equilibrium of the game is obtained. This will be done in later sub-sections.

### *3.3. Pure-strategy equilibria*

At the beginning (period 0) of the game, both players choose the initial commitment length and the price(s) simultaneously, knowing that each of the players will again make such two-dimensional decision whenever the commitment expires. Consider first the optimal reaction function of player i if player j commits only for one period and sets price $x_0^j$ at period 0. It can be observed that player i's two-dimensional (length of the current commitment and the price(s)) problem here is exactly the same as the one associated with the NDFN condition. As a result, provided that the NDFN expression (3.10) is positive, player i's optimal response to player j's action of $x_0^j$ at period zero is to commit for two periods with prices at $R_i(x_0^j)$ and $x_L$ respectively.[9]

Next, consider the optimal reaction function of player i if player j chooses, at period 0, a two-period commitment with price $x_0^j$ for period 0 and price $x_1^j$ for period 1. If player i commits for two periods, then by similar arguments as before, player i's optimal choices of the prices are given by $R_i(x_0^j)$ and $R_i(x_1^j)$ respectively. As a result, his intertemporal payoff is:

$$\max_{x_0^i,\, x_1^i}\left[U^i(x_0^i,x_0^j) + \beta\, U^i(x_1^i,x_1^j) + \beta^2 V\right] = U^i\left(R_i(x_0^j),x_0^j\right) + \beta\, U^i\left(R_i(x_1^j),x_1^j\right) + \beta^2 V. \qquad (3.11)$$

If player i commits for one period, then he will face a nonsynchronized timing pattern (with player j's commitment not yet expired) in the next period when he moves again. On the anticipation that nonsynchronization will last forever, his intertemporal payoff starting from period 1 is given by

[9] A brief discussion is as follows. If player i commits for one period, then by arguments leading to (3.9), player i's optimal response is $R_i(x_0^j)$ and his intertemporal payoff is given by (3.9). If player i commits for two periods, then (on the assumption that the NDFN condition holds) the two prices are chosen according to (3.7).

$V^i(x_1^j)$. Therefore, player i's optimization problem at period zero is given by:

$$\max_{x_0^i}\left[U^i(x_0^i,x_0^j)+\beta V^i(x_1^j)\right]=U^i\left(R_i(x_0^j)\,,x_0^j\right)+\beta V^i(x_1^j). \tag{3.12}$$

It is easy to conclude from (3.11) and (3.12) that if the NDFN expression (3.10) is positive and player j chooses $x_0^j$ for period 0 and $x_1^j$ for period 1, then player i's optimal response at period 0 is to commit for one period and set his price at $R_i(x_0^j)$. An interesting observation from the above analysis is that because of the recursive nature of the problem, the NDFN condition (which guarantees no deviation from nonsynchronization if the players move alternatingly in a particular period) is also important for the determination of a player's optimal reaction function in the beginning of the game when both players move simultaneously.

Combining the above results, it can be shown that, if the NDFN condition is satisfied, there are two symmetric pure-strategy equilibria of this game: $\left((1,x_N),(2,x_N,x_L)\right)$ and $\left((2,x_N,x_L),(1,x_N)\right)$, where $(1,x_N)$ represents the commitment of one period and the corresponding period-0 price, and $(2,x_N,x_L)$ represents the commitment of two periods and the corresponding period-0 and period-1 prices. Moreover, if player i chooses $(1,x_N)$ at period 0 and player j chooses $(2,x_N,x_L)$, then player i's intertemporal payoff is:

$$V_1=U_{NN}+\beta V^i(x_L), \tag{3.13}$$

and that of his opponent is:

$$V_2=U_{NN}+\beta W^j(x_L). \tag{3.14}$$

*3.4. Mixed-strategy equilibrium and verification of the no-deviation-from-nonsynchronization condition*

It is shown above that there are two symmetric pure-strategy equilibria (provided that the NDFN condition is satisfied). In each equilibrium, one player commits for one period and the other commits for two periods at period 0. Moreover, it can be shown that $V_1 > V_2$, which means that in each of these two equilibria, the player who commits for one period in the beginning will have a higher payoff than the player committing for two periods.[10] Given the complete symmetry of the game, there is a coordination problem. While both players prefer nonsynchronization eventually (provided that the NDFN condition holds), each player also prefers choosing $(1,x_N)$ at the beginning while his opponent choosing $(2,x_N,x_L)$.

There is a simple and intuitive mixed-strategy equilibrium to solve the coordination problem in reaching nonsynchronization. In the mixed-strategy equilibrium, each player chooses $(1,x_N)$ with probability $p$ and $(2,x_N,x_L)$ with probability $1-p$. It is easy to see from Table I that at the mixed-strategy equilibrium, the value of the game, $V$, is given by:

$$V = p(U_{NN}+\beta V) + (1-p)V_1 = pV_2 + (1-p)(U_{NN}+\beta U_{LL}+\beta^2 V), \tag{3.15}$$

where $U_{LL}$ is defined in (2.3) with $x_r = x_L$. Obviously, there is an interdependence between $p$ and $V$ in the two equalities of (3.15). Combining these two equalities, it can be shown that only one root of $p$ is between zero and one. Once $p$ is determined, $V$ can be obtained as:

$$V = \frac{U_{NN} + (1-p)\beta V^i(x_L)}{1-p\beta}. \tag{3.16}$$

The above analysis derives the mixed-strategy equilibrium of the timing-and-price game *provided that the NDFN condition holds*. Moreover, the value of the game is expressed in terms of the underlying parameters of the model. The next step is to verify that the NDFN condition is in fact satisfied for every period along the equilibrium time path. Combining (3.4), (3.8), (3.10) and (3.16),

[10]Combining (3.4), (3.5), (3.7) and (3.8), it can be shown that $V^i(x_L) = U_F + \beta W^i(x_L)$ and $W^i(x_L) = U_L + \beta V^i(x_L)$. Solving these two equations simultaneously, one obtains:

$$V^i(x_L) = (1-\beta^2)^{-1}(U_F+\beta U_L), \tag{3.7a}$$

$$W^i(x_L) = (1-\beta^2)^{-1}(U_L+\beta U_F). \tag{3.8a}$$

As $U_F > U_L$ according to (3.6), it is easy to conclude that $V^i(x_L) > W^i(x_L)$. Combining this result with (3.13) and (3.14), $V_1 > V_2$ is concluded. The crucial aspect of the above proof is that $U_F > U_L$, i.e., the payoff of the follower is higher than that of the leader for the 'Stackelberg game' with payoff function (2.2). As a result of this 'second mover advantage' (Gal-Or [21]), each player prefers his opponent choosing $(2,x_N,x_L)$ at period 0 so that he can be the follower at period 1, the earliest possible period.

it can be shown that

$$NDFN = \beta\left[W^i(x_L) - V\right] = \frac{\beta\left\{(U_L - U_{NN}) + p\beta\left[(1-\beta)V^i(x_L) - U_L\right]\right\}}{1 - p\beta} \tag{3.17}$$

is positive, as $U_L > U_{NN}$ according to the first inequality of (3.6) and $(1-\beta)V^i(x_L) > U_L$ according to (3.7a) in footnote 10 and the second inequality of (3.6).

### *3.5. Equilibrium outcome and the intuition*

From the analysis of the previous sub-sections, it can be concluded that in the mixed-strategy equilibrium, each player chooses $(1, x_N)$ with probability $p$ and $(2, x_N, x_L)$ with probability $1-p$ at the beginning of the game. If both players' choices turn out to be $(1, x_N)$ (resp. $(2, x_N, x_L)$), then the game re-starts again at period 1 (resp. period 2) with both players repeating the previous mixed strategies. On the other hand, if it turns out that one player chooses $(1, x_N)$ and the other chooses $(2, x_N, x_L)$, then the nonsynchronized timing pattern emerges. Moreover, once the nonsynchronized pattern emerges, it will last forever with each player committing for two periods and choosing the prices according to (3.3) and (3.2). This is because the NDFN expression (3.17) is positive for each time period. Nonsynchronization is the absorbing state of this infinite-horizon timing-and-price game. As a result, the timing pattern of the game will eventually go to nonsynchronization with probability one.

The intuition of the above results is that in a differentiated-product price competition model, it is beneficial for the players to move sequentially rather than simultaneously, according to (3.6). With both players choosing predetermined prices, alternating moves ensure that each player acts as either a Stackelberg leader or a Stackelberg follower every period. Even though there is a coordination problem in reaching this outcome (as each player prefers choosing the one-period commitment at period 0 ~~and~~ while his opponent choosing the two-period commitment), this section shows that ~~this~~ the coordination problem can be solved. The crucial factor is that there is no incentive to deviate from nonsynchronization once it is reached. Knowing that the NDFN condition holds, each player chooses a mixed strategy in the beginning of the game. With 'trial and error', the coordination problem is solved automatically and nonsynchronization will emerge eventually.

Moving beyond the specific model of differentiated-product price competition, Section 5 will give a more general explanation in terms of the structural factors (the slope of the reaction function and the sign of the externality effect) and examine the importance of strategic complementarity in determining the equilibrium timing pattern. Before that, the equilibrium outcome of the timing-and-price game with fixed prices will be considered in the next section.

## 4. EQUILIBRIUM TIMING PATTERN WITH FIXED PRICES

In the timing-and-price game with fixed prices, a player's price is constant throughout the commitment length, whether it is one or two periods. This specification of constant action (wage or price) throughout the commitment length has been used in Taylor [32] in the macroeconomics literature, and Cyert and DeGroot [13] and Maskin and Tirole [27, 28, 29] in the industrial organization literature. There are interesting differences between this game and that with predetermined prices, and they will be discussed in Sub-section 6.1. On the other hand, some of the analysis in this section is similar to those in Section 3, and these aspects will only be discussed briefly.

First of all, it is assumed that after a particular time period, each player commits his action for two periods and the players move alternatingly. Without loss of generality, it is assumed that player A (resp. B) chooses the action at every even (resp. odd) period, which lasts for two periods. Therefore, $x^A_{2k+1} = x^A_{2k}$ and $x^B_{2k} = x^B_{2k-1}$. This structure is the simplest form of an alternating-move game, which has been used in Cyert and DeGroot [13] who apply the finite-horizon version to understand Cournot competition, and in Maskin and Tirole [27, 28, 29] who use the infinite-horizon version to examine various kinds of oligopolistic competition.

Each player is assumed to choose a Markov (or feedback) strategy, i.e., an action dependent on payoff-relevant state variables only.[11] If nonsynchronization lasts forever with each player choosing his action for two periods, then the opponent's most recent action is the only payoff-relevant variable. The restriction to Markov strategies leads to a search for dynamic reaction functions of the following form:

$$x_t^i = D_i(x_{t-1}^j), \tag{4.1}$$

where i = A (if t=2k) or B (if t=2k+1), $i \neq j$, and $D_A(.)$ and $D_B(.)$ are time-invariant functions. A Markov reaction function pair $\{D_A(x_{-1}^B),\ D_B(x_{-1}^A)\}$ constitutes a Markov perfect equilibrium (MPE) if the reaction function of player i (i=A,B) maximizes his intertemporal payoff given his opponent's reaction function.[12]

The problem is solved by applying the game-theoretic analogue of dynamic programming. Define two value functions for each player, with one function at the period when he chooses his action and the other when his opponent makes decision. The value function $V^i(x_{-1}^j)$ (resp. $W^i(x_{-1}^i)$), $i,j=A,B$ and $i \neq j$, is defined as player i's intertemporal payoff at the beginning of a period in which he (resp. his opponent) moves, given that $x_{-1}^j$ (resp. $x_{-1}^i$) has been chosen by his opponent (resp. him) in the last period and with the expectation that the optimal dynamic reaction functions $\{D_A, D_B\}$ will be chosen forever subsequently.

According to the results of dynamic programming, the reaction function pair $\{D_A, D_B\}$ forms

---

[11]By restricting to Markov strategies, a player reacts only to the opponent's current action (such as a price cut) and not to earlier history or to one's own past actions. This specification seems "to accord better with the customary conception of a reaction in the informal industrial organization literature than do, say, the reactions emphasized in the repeated game tradition" (Maskin and Tirole [28, p. 553]). Moreover, it can be shown that the Markov reaction functions in this section (as well as (3.2) and (3.3) in Section 3) are optimal in the corresponding finite-horizon model. Thus the equilibrium is not sensitive to whether the horizon is infinite or arbitrarily long but finite. These advantages (over the repeated game approach) make it particularly appropriate to examine the roles of synchronization versus nonsynchronization in solving the strategic conflicts. On the other hand, if the repeated game approach is used, then its well-known problems (such as large number of equilibria and non-robustness) are also present in the timing-and-price games of this paper.

[12]A player has no incentive to choose non-Markov strategies if his opponent's strategies are Markov (Maskin and Tirole [28, Proposition 1]). Note also that the terminology of MPE follows the various papers of Maskin and Tirole, whereas Kydland [24] and Basar and Olsder [5] use the term 'feedback Nash equilibrium' for the same concept.

a MPE if and only if there exists value functions $\{(V^A, W^A),\ (V^B, W^B)\}$ such that the reaction functions and the value functions are mutually consistent in the following manner:

$$V^i(x^j_{-1}) = \max_{x^i}\left[U^i(x^i, x^j_{-i}) + \beta W^i(x^i)\right] = U^i\left(D_i(x^j_{-1}),\, x^j_{-1}\right) + \beta W^i\left(D_i(x^j_{-1})\right), \tag{4.2}$$

$$W^i(x^i_{-1}) = U^i\left(x^i_{-1},\, D_j(x^i_{-1})\right) + \beta V^i\left(D_j(x^i_{-1})\right). \tag{4.3}$$

With single-period payoff function (2.2), the equilibrium dynamic reaction functions are given by:

$$x^i_t = D_i(x^j_{t-1}) = \frac{(1-b)h}{(2-g-\beta gb)} + bx^j_{t-1}, \tag{4.4}$$

where i = A (if t=2k) or B (if t=2k+1), and $b$ $(0<b<1)$ satisfies

$$\beta^2 gb^4 + 2\beta gb^2 - 2(1+\beta)b + g = 0. \tag{4.5}$$

Moreover, it can be shown that[13]

$$V^i(x^j_{-1}) = \frac{gb}{2}(x^j_{-1})^2 + \frac{g(1-b)h}{(2-g-\beta gb)}x^j_{-1} + \frac{g(1-b)^2(1+2\beta^2 b+\beta^2 b^2)}{2b(1-\beta^2)(2-g-\beta gb)^2}h^2. \tag{4.6}$$

It is easy to show that the price set by each player at the steady state, $x_{STAG}$, is given by:

$$x_{STAG} = \frac{h}{2-g-\beta gb}. \tag{4.7}$$

Moreover, it can be shown from (4.4) and (4.7) that

$$D_i(x^j) - x^j = (1-b)(x_{STAG} - x^j), \tag{4.8}$$

which implies that whenever a price is less than $x_{STAG}$, the price in the next period (set by the other player) is higher than the current price provided that nonsynchronization continues. As a result, the sequence of prices (set by the two players alternatingly) is monotonically increasing over time to approach the steady state value.

The next step is to obtain the NDFN condition. At the time when player i moves, suppose

[13]For completeness, the other value function of player i is given by:

$$W^i(x^i_{-1}) = \frac{(2b-g)}{2\beta b}(x^i_{-1})^2 + \frac{(g-2b+\beta gb^2)h}{\beta b(2-g-\beta gb)}x^i_{-1} + \frac{\beta g(1-b^2)^2}{2b(1-\beta^2)(2-g-\beta gb)^2}h^2. \tag{4.6a}$$

that his opponent's price has been set at $x^j$ and it will last until the end of the current period. If player i commits for one period, then it is easy to see that his intertemporal payoff is the same as (3.9), where $V$ is now the value of the timing-and-price game with fixed prices.[14] On the other hand, if player i commits for two periods, then his price is set optimally at $D_i(x^j)$ and his intertemporal payoff is given by $V^i(x^j)$ of (4.2), with the expectation that in the future neither player will deviate from committing for two periods and choosing (4.4). Thus, if the current timing pattern is nonsynchronization and the opponent's current price is $x^j$, then this timing pattern will continue if a positive value appears in the following NDFN expression:

$$\begin{aligned} & V^i(x^j) - \left[U^i\left(R_i(x^j),x^j\right) + \beta V\right] \\ &= \left[U^i\left(D_i(x^j),x^j\right) - U^i\left(R_i(x^j),x^j\right)\right] + \left[\beta W^i\left(D_i(x^j)\right) - \beta V\right] \\ &= \frac{g(2b-g)}{4}(x^j)^2 + \frac{(\beta gb+g-2b)gh}{2(2-g-\beta gb)}x^j + \left[\frac{g(1-b)^2(1+2\beta^2 b+\beta^2 b^2)}{2b(1-\beta^2)(2-g-\beta gb)^2}h^2 - \frac{h^2}{4} - \beta V\right]. \end{aligned} \tag{4.9}$$

At the beginning (period 0) of the game, both players make their choices (commitment length and price) simultaneously, knowing that such two-dimensional decision will again be made whenever the current commitment expires. For ease of notation, the choice of player j can be represented by $(1,x_0^j)$ or $(2,x_0^j)$ where the first and second arguments represent respectively player j's commitment length and price at period 0. Following similar analysis as in Section 3, it can be shown that if the NDFN expression (4.9) is positive, then (a) player i's optimal response to player j's choice of $(1,x_0^j)$ is to choose $\left(2,D_i(x_0^j)\right)$; and (b) player i's optimal response to player j's choice of $(2,x_0^j)$ is to choose $\left(1,R_i(x_0^j)\right)$. With the single-period payoff function (2.2), solving these two optimal timing-and-price reaction functions simultaneously gives two symmetric pure-strategy equilibria of this game as

[14]To economize on the use of notation and to facilitate comparison, some of the symbols (such as $V$, $V_1$, $V_2$ and $p$) in this section are the same as those in Section 3, even though they refer to different games.

$((1, x_1), (2, x_2))$ and $((2, x_2), (1, x_1))$,[15] where

$$x_1 = \frac{(2 - gb - \beta gb)h}{(2 - gb)(2 - g - \beta gb)}, \tag{4.10}$$

$$x_2 = \frac{(2 - gb - \beta gb^2)h}{(2 - gb)(2 - g - \beta gb)}. \tag{4.11}$$

Moreover, if player i chooses $(1, x_1)$ at period 0 and player j chooses $(2, x_2)$, then player i's intertemporal payoff is given by $V_1$ and that of his opponent is given by $V_2$, where

$$V_1 = U^i\big(R_i(x_2), x_2\big) + \beta V^i(x_2), \tag{4.12}$$

$$V_2 = V^j(x_1). \tag{4.13}$$

There is also a coordination problem at the beginning of the timing-and-price game for fixed prices because $V_1 > V_2$. As in the previous game, the coordination problem can be solved if both players choose mixed strategies. In the mixed-strategy equilibrium, each player chooses $(1, x_1)$ with probability $p$ and $(2, x_2)$ with probability $1 - p$. It can be deduced from Table II that at the mixed-strategy equilibrium, $p$ is defined by the following quadratic equation:

$$\begin{aligned} \beta\big[\beta U_{11} - (1+\beta) U_{22} - \beta V_1 + V_2\big]p^2 + \big[(1-\beta^2) U_{11} + (1+\beta)^2 U_{22} - (1 - 2\beta^2) V_1 - V_2\big]p \\ + (1+\beta)\big[(1-\beta) V_1 - U_{22}\big] = 0, \end{aligned} \tag{4.14}$$

where $U_{11}$ and $U_{22}$ are defined in (2.3) with $x_r = x_1$ and $x_r = x_2$ respectively. It can be shown that only one root of $p$ in (4.14) is between zero and one. Once $p$ is determined, $V$ is obtained as:

$$V = \frac{p U_{11} + (1-p) V_1}{1 - p\beta}. \tag{4.15}$$

The last step is to verify that the NDFN condition is satisfied for every period along the equilibrium time path, and there are two main ingredients in the proof. First, it can be shown from

---

[15]Note that $x_1$ and $x_2$ are related by $x_1 = R_i(x_2)$ and $x_2 = D_j(x_1)$, and the subscript in $x_1$ or $x_2$ does not represent the time period. Moreover, it is necessary to verify (later) that the NDFN condition holds for $x_N$ in order to show that $((1, x_N), (1, x_N))$ and $((2, x_N), (2, x_N))$ are not equilibria of this game.

(2.5), (4.7), (4.10) and (4.11) that

$$x_N < x_1 < x_2 < x_{STAG}. \tag{4.16}$$

Combining (4.8) and (4.16), it can be concluded that the prices chosen by both players at different time periods will always lie between $x_1$ and $x_{STAG}$. The second ingredient in showing the satisfaction of the NDFN condition makes use of the functional form of (4.9). The NDFN expression (4.9) is a quadratic function in $x^j$ with an ever-decreasing slope (i.e. a negative second derivative).[16] As it can further be shown that the interval $[x_N, x_{STAG}]$ lies on the increasing branch of the convex curve, the NDFN expression is increasing in this interval.[17]

Combining the above two ingredients, it is sufficient to just check whether the NDFN condition is satisfied at $x_N$. While it is difficult to obtain a closed-form expression due to the complicated relationships of the various parameters in (4.9), (4.14) and (4.15), an extensive computational check suggests that for $h > 0$,

$$NDFN(x_N) = V^i(x_N) - \left(U_{NN} + \beta V\right) \tag{4.17}$$

is positive for all parameter combinations of $0 < \beta < 1$ and $0 < g < 1$. (The computation includes 9801 cases, with each of $\beta$ and $g$ running from 0.01 to 0.99 with an increment of 0.01, as well as other cases with either parameter very close to zero or one.) The NDFN condition is satisfied at all time periods. Some calculated values of (4.17) are given in Table III. An interesting observation is that while the NDFN expression is increasing in the discount factor $\beta$, it is always positive for $0 < \beta < 1$. The players will not deviate from the Pareto superior equilibrium of nonsynchronization for all

[16]It can be shown that the stable root of (4.5) lies between 0 and $g/2$. As a result, $2b - g < 0$.

[17]Note that it is necessary to check whether the NDFN condition holds at $x_N$ or not (see footnote 15), even though (4.8) and (4.16) imply that the price set by either player is always at least as high as $x_1$ (which is greater than $x_N$) at the equilibrium path (if the NDFN condition holds).

possible values of the discount factor between 0 and 1.[18]

To conclude, both players choose mixed strategies at the beginning of the game. Since the NDFN expression (4.9) is positive for each time period, nonsynchronization is the absorbing state of this timing-and-price game with fixed prices, and it will eventually emerge.

With the players choosing fixed prices, nonsynchronized moves ensure that whenever a player moves, the price of the other player remains unchanged temporarily. The player choosing his current price needs not worry about his action being undercut by his opponent. Therefore, it is optimal for him to set a higher price (in order to maximize his intertemporal payoff, on the anticipation that (4.4) will be chosen in the future). Furthermore, such action induces the other player to follow suit in the future when she sets her price, according to (4.8) and (4.16). As a result, the prices increase monotonically over time, and both players benefit. As in the game in Section 3, the satisfaction of the NDFN condition means that the coordination problem in reaching nonsynchronization can be solved automatically when the players choose mixed strategies at the beginning.

## 5. UNDERSTANDING THE ROLE OF STRATEGIC COMPLEMENTARITY FOR ENDOGENOUS NONSYNCHRONIZATION

The analysis in previous sections shows that in the presence of strategic complementarity and positive externality, nonsynchronization emerges as the equilibrium timing pattern, whether predetermined or fixed prices are chosen by the players. This section attempts to provide a more general explanation to the above results by interpreting a synchronized-move dynamic game as a series of short-term simultaneous-move games, and a nonsynchronized-move dynamic game as a series of Stackelberg games in which each player is taking the roles of Stackelberg leader and follower

[18]This result differs from the typical result of the repeated game literature (in which actions contingent on payoff-irrelevant history are commonly used) that the Pareto superior equilibrium can be sustained only when the discount factor is close to 1.

alternatingly and symmetrically.[19] Therefore, the payoff of a player under synchronization is the present discounted value of the stream of payoffs for the series of simultaneous-move games, whereas his payoff under nonsynchronization depends on the payoff of a leader and that of a follower for the corresponding Stackelberg games. It is well-known that the Stackelberg leader's payoff is higher than or equal to that in the simultaneous-move game since the leader can always achieve the Nash payoff in the Stackelberg game, if he wishes. On the other hand, the analysis of the Stackelberg *follower*'s payoff is more interesting and relevant regarding the aggregate pattern of time-dependent adjustment rules. This is examined in Sub-section 5.1. The relevance of these results for the equilibrium timing pattern in repeated interaction is examined in Sub-section 5.2. Note that the analysis of this section is conducted for a general payoff function $U^i(x^i,x^j)$ with $U^i_{ii} < 0$, and is therefore not only applicable to the specific payoff function (2.2) used in previous sections.

*5.1. Effect of the Stackelberg leader's action on the follower*

To examine the effect of the Stackelberg leader's action on the follower, compare the Nash equilibrium of a simultaneous-move game and the subgame perfect equilibrium of a Stackelberg game with the same payoff functions for the players.[20] The Nash equilibrium of the simultaneous-move game, $x^A = x^B \equiv x_N$, is obtained by solving simultaneously the two reaction functions defined implicitly in (2.4), where $i$ = A and B (and $j \neq i$) respectively.

The subgame perfect equilibrium for the Stackelberg game with player j (where j may be A or B) as the leader is derived as follows. Taking the choice of leader j as given, follower i chooses

---

[19]Tirole [33, p. 343] has suggested this interpretation of an alternating-move game in discussing issues about strategic overinvestment. See also Maskin and Tirole [27, p. 956].

[20]Note that this paper does not focus on the comparison of the payoffs of the Stackelberg leader and follower (as done in Gal-Or [21] or Dowrick [16]), even though there are similar structural factors for this question and the one in this sub-section. In the comparison of synchronized versus nonsynchronized patterns, it does not matter whether the payoff of the Stackelberg leader or that of the follower is higher. What is important is how the payoff of either player in a Stackelberg game compared with the equilibrium payoff of the corresponding simultaneous-move game.

her strategic variable to maximize her payoff. The optimal reaction function of follower i, $R_i(x^j)$, is defined implicitly in (2.4). Anticipating the action of follower i, leader j chooses his action to maximize (3.1).

The next step is to examine whether follower i obtains a higher payoff in the simultaneous-move game or in the Stackelberg game. Differentiating (3.1) with respect to $x^j$ gives:

$$\frac{dU^j_{RF}(x^j)}{dx^j} = U^j_j\left(x^j,\ R_i(x^j)\right) + U^j_i\left(x^j,\ R_i(x^j)\right)R_i'(x^j), \tag{5.1}$$

where

$$R_i'(x^j) \equiv \frac{dR_i(x^j)}{dx^j} = -\frac{U^i_{ij}\left(R_i(x^j), x^j\right)}{U^i_{ii}\left(R_i(x^j), x^j\right)}, \tag{5.2}$$

which is obtained by differentiating (2.4) with respect to $x^j$. The sign of $R_i'(x^j)$ is the same as that of $U^i_{ij}$ since $U^i_{ii} < 0$ is assumed. When leader j changes his choice, this action has direct and strategic effects on his own payoff through the first and second terms of the right-hand side of (5.1). At the Nash equilibrium level $x_N$, there is no direct effect (to the first order) on his payoff. On the other hand, because of interdependence, his action affects follower i's choice and thus his own payoff indirectly.

Similarly, define the 'reduced-form' payoff function of follower i as:

$$U^i_{RF}(x^j) \equiv U^i\left(R_i(x^j), x^j\right). \tag{5.3}$$

Differentiating (5.3) with respect to $x^j$ gives:

$$\frac{dU^i_{RF}(x^j)}{dx^j} = U^i_i\left(R_i(x^j), x^j\right)R_i'(x^j) + U^i_j\left(R_i(x_j), x^j\right) = U^i_j\left(R_i(x^j), x^j\right), \tag{5.4}$$

where the second equality holds as follower i reacts optimally to any choice of $x^j$ according to

(2.4).[21]

To see how a change of the strategic variable of leader j, from the initial level at the Nash equilibrium, would affect the payoff of follower i, one can write the change in the payoff of follower i as a product of the change in $x^j$ and the derivative of the payoff of follower i with respect to $x^j$. Therefore, the sign of the change in follower i's payoff is given by:

$$sign(\Delta U^i) = sign(\Delta x^j)\, sign\left[\frac{dU^i_{RF}(x_N)}{dx^j}\right] = sign\left[\frac{dU^j_{RF}(x_N)}{dx^j}\right]\, sign\left[\frac{dU^i_{RF}(x_N)}{dx^j}\right], \qquad (5.5)$$

where the second equality arises because leader j will increase $x^j$ if the derivative of his reduced-form payoff function (3.1) is positive and vice versa.

Substituting (5.1) and (5.4), both evaluated at the Nash equilibrium (such that the first term in the right-hand side of (5.1) drops), into (5.5) gives:

$$sign(\Delta U^i) = sign\big(R_i'(x_N)\big)\, sign\big(U_i^j(x_N,x_N)\big)\, sign\big(U_j^i(x_N,x_N)\big). \qquad (5.6)$$

Under the symmetry assumption, $U_i^j(x_N,x_N)$ and $U_j^i(x_N,x_N)$ have the same sign and therefore the two externality effects 'cancel' out (in sign). It can then be concluded from (5.6) that whether the follower's payoff increases or decreases is solely related to whether strategic complementarity or substitutability is present. Follower i will benefit in the presence of strategic complementarity and vice versa. The above results are represented in the two-by-two classification according to the sign of the externality effect and that of the slope of the optimal reaction function in Table IV.

For the price competition model in previous sections which exhibits strategic complementarity and positive externality, leader j knows that his profit will be increased if follower i's price is raised (because of positive externality, $U_i^j > 0$); to induce follower i to do that, leader j increases his price since strategic complementarity is present, i.e., $R_i' > 0$. This explains why leader j in this Stackelberg

[21]The analysis leading to (5.4), which is an application of the envelope theorem, is very similar to the proof of Proposition 4 in Cooper and John [12].

game would like to increase his price from the Nash equilibrium level. On the other hand, since $U_j^i > 0$ (the externality is positive), an increase in the price of leader j (with the intention of raising his own payoff) will have the effect of benefiting follower i as well. Sequential moves provide a better outcome for both players as their prices are set higher than their Nash counterparts. Similar argument applies to the game with strategic complementarity and negative externality (such as the nominal-wage setting game considered in Lau [26]). In that case, both players of the Stackelberg game benefit from the lower level of the strategic variables (as compared to the Nash equilibrium); see Table IV.

*5.2. Implications to the equilibrium timing pattern in repeated interaction*

How does the analysis of the effect of the leader's action on the follower's payoff in a Stackelberg game relate to the equilibrium timing pattern in repeated interaction? The link lies in the interpretation of a nonsynchronized-move dynamic game as a series of Stackelberg games with symmetric Stackelberg leadership, whereas a synchronized-move dynamic game as a series of short-term simultaneous-move games. Since the payoff of the leader in a Stackelberg game is at least as high as his payoff in a simultaneous-move game, the effect of the leader's action on the follower is crucial to the equilibrium timing pattern. In the presence of strategic complementarity, the above analysis shows that the follower in a Stackelberg game also gains. Therefore, in a nonsynchronized-move dynamic game with strategic complementarity, each player has a higher payoff than its Nash equilibrium counterpart, both in the first and second periods of a commitment when he is a 'follower' and 'leader' respectively.[22] It is, therefore, not surprising that nonsynchronization is the equilibrium timing pattern in the presence of strategic complementarity, irrespective of whether positive or

[22]On the other hand, the follower will be hurt when strategic substitutability is present. In terms of the alternating-move dynamic game, each player will have a higher (again, when compared with the payoff at the Nash equilibrium) payoff in one period of the commitment but a lower payoff in another. In general, the equilibrium timing pattern in the presence of strategic substitutability is ambiguous and may be sensitive to the functional form of the players' payoffs.

negative externality is present.

To summarize, the analysis of this section shows that two factors influencing the benefit or disadvantage to the follower in a Stackelberg game are the externality effect and the slope of the reaction function. Moreover, by interpreting a nonsynchronized-move dynamic game as a series of Stackelberg games, the insight in the analysis of the Stackelberg game provides the explanation for nonsynchronization in repeated interaction when strategic complementarity exists, which is the main focus of this paper.

## 6. DISCUSSION AND EXTENSION

This section provides further discussion and extension. Specifically, Sub-section 6.1 compares and contrasts the games with predetermined and fixed actions, Sub-section 6.2 considers the use of correlated strategies to solve the coordination problem in the game with predetermined prices, and Sub-section 6.3 discusses briefly the equilibrium timing pattern in games with more than two players.

### *6.1. A comparison of the games with predetermined and fixed prices*

The timing-and-price games with predetermined and fixed prices respectively are analyzed in previous sections. While these two games share a lot of similarities in the derivation (in Sections 3 and 4) as well as the underlying intuition (as explained in Section 5), there are also some interesting differences between them. With fixed prices, the intertemporal link is strong. The price set by a player at period t will not only affect his payoff at periods t and t+1 (at the equilibrium of nonsynchronization, with each player's commitment length being two periods) but also the payoffs at t+2 and afterwards, through the dynamic reaction functions in (4.4). As a result of this intertemporal link, the sequence of prices converges to the steady state level, as shown in (4.8). On the other hand, each player's prices alternate between (3.2) and (3.3) during a commitment length of two periods in the game with predetermined prices. This lack of intertemporal link in models with predetermined actions is observed in both the version without strategic interaction (Fischer [19], Fethke and Policano [17, 18]) and the version with strategic interaction used in this paper.

The difference of predetermined and fixed prices also has implications on the solution of the game, especially on the NDFN condition. With predetermined prices, the same static reaction function $R_i(x^j)$ is used for the current period whether the player commits for one or two periods; see (3.7) and (3.9). As a result, the NDFN expression (3.10) only involves the comparison of future payoffs. With fixed prices, a player chooses $R_i(x^j)$ if committing for one period but he chooses $D_i(x^j)$ if committing for two periods. There is a current gain (as $U^i(R_i^*(x^j),x^j) > U^i(D_i^*(x^j),x^j)$), but there is greater future loss (given by $\beta W^i(D_i^*(x^j)) - \beta V$ in (4.9)) if a player deviates by committing for one period.

An alternative way to look at the above difference is that the NDFN expression (4.9) with fixed prices depends on the current price of his opponent, but the NDFN expression (3.10) with predetermined prices does not. As a result, the verification of the NDFN condition for the game with fixed prices is more difficult than the game with predetermined prices. In particular, while it is feasible to verify the NDFN condition analytically for the game with predetermined prices, it is only possible to verify the NDFN condition computationally for the game with fixed prices.

### *6.2. Correlated equilibrium*

In either Section 3 or Section 4, a mixed strategy equilibrium is considered to solve the coordination problem in reaching nonsynchronization. The mixed strategy equilibrium corresponds to the players choosing their strategies independently. Alternatively, if the players are able to correlate their strategies, then the idea in Aumann [1] suggests that they may gain higher payoffs.

Consider the use of correlated strategies to solve the coordination problem in the game with predetermined prices (Section 3).[23] Suppose the players can engage in preplay discussion and choose

[23]A similar correlated equilibrium can be obtained for the game with fixed prices. As the underlying idea is similar, only the game with predetermined prices (in which an analytical solution is available) is presented so as to conserve space.

their strategies based on the realization of a publicly observable random variable such as a 'coin flip'. One simple way to obtain a better outcome by correlated strategies is that in the beginning of the game, player i chooses $(1,x_N)$ if heads and $(2,x_N,x_L)$ if tails, and player j chooses $(2,x_N,x_L)$ if heads and $(1,x_N)$ if tails. Each player's intertemporal payoff at the correlated equilibrium is given by:

$$V_{CORR} = \frac{V_1 + V_2}{2} = U_{NN} + \frac{\beta}{2}\left[V^i(x_L) + W^i(x_L)\right], \tag{6.1}$$

where the various components in the middle and right-hand side terms have been defined in Section 3. While each player's action is still random (as in the mixed strategy equilibrium) at period zero, there is a perfect coordination of their choices and nonsynchronization is achieved immediately without 'trial and error'.

As the value of the game corresponding to the correlated equilibrium differs from $V$ in (3.16), it is necessary to check whether the NDFN condition in this case still holds or not. Combining (3.4), (3.8), (3.10) and (6.1), it can be shown that

$$NDFN_{CORR} = \beta\left[W^i(x_L) - V_{CORR}\right] = \beta\left\{(U_L - U_{NN}) + \frac{\beta}{2}\left[V^i(x_L) - W^i(x_L)\right]\right\} > 0. \tag{6.2}$$

Therefore, the players would not deviate from nonsynchronization even when they are able to correlate their strategies. At this correlated equilibrium, nonsynchronization is reached starting from period one and then perpetuates forever.

While correlated strategies may solve the coordination problem and give the players higher payoffs by reaching nonsynchronization faster (in an expected sense), one may question that if preplay communication is possible, why would they not further consider sustaining an even better outcome (such as the cooperative outcome) by more sophisticated mechanisms. This is possible when the discount factor is close to one for the usual 'Folk Theorem' reasons. Comparatively, the concept of mixed strategy equilibrium used in earlier sections is closer in spirit to the focus of this paper, which examines the roles of different timing patterns (rather than other possible mechanisms) in solving the strategic conflicts of the players.

*6.3. Equilibrium timing pattern in games with more than two players: Some preliminary thoughts*

This paper considers games with two players and shows that nonsynchronization is the equilibrium timing pattern in the presence of strategic complementarity, whether the players choose predetermined or fixed actions. It would be helpful to see whether the above results can be extended to games with more interacting players.

As briefly mentioned in the Introduction, Lau [26] and this paper can be regarded as two relating steps to address the question of equilibrium pattern under time-dependent adjustment rules. Specifically, Lau [26] considers, in a two-player game with fixed actions, the relative ability of synchronization and nonsynchronization in overcoming macroeconomic externality of wage adjustment. That game is simpler to analyze since the commitment length is restricted to be two periods and therefore, the choice of each player, when he moves, is only one-dimensional. Building on some of the results of that paper, this paper further considers whether the Pareto superior timing pattern will arise as the equilibrium outcome by allowing the players to choose both the commitment length and the strategic variable. It is found that nonsynchronization is the equilibrium timing pattern when strategic complementarity is present.

Applying the above two-step approach to a game with more than two players, it is easier to first consider whether synchronization or nonsynchronization is the Pareto superior timing pattern, before examining the equilibrium pattern. In the two-player game considered in Lau [26], the strategic benefit of staggered wage adjustment lies in its ability in overcoming the externalities in decentralized wage setting. When the number of wage setters increases, the effect of a change in the wage of each sector on the aggregate price level becomes smaller and smaller. (In examining whether the results of their earlier paper are robust with respect to the number of sectors, Fethke and Policano [18, pp. 871-872] mention a similar point: "If the economy consists of a larger number of small sectors, ... the effect on the aggregate price level of the actions by any one sector is negligible.") As a result, the externality problem becomes more severe. It is logical to conjecture that the benefit of a nonsynchronized timing pattern in overcoming macroeconomic externalities becomes more important when the number of wage setters increases. In fact, some preliminary results, based on computational methods, suggest that the conclusion of Pareto superior nonsynchronized wage adjustment for (the deterministic version of) the game in Lau [26] can be extended to a game with three players.

To summarize, two results may be useful to the question about the equilibrium timing pattern in games with more than two players. First, it is found that nonsynchronization is the Pareto superior timing pattern for a game with three players when strategic complementarity is present. Second, it is found that with two players, the preferred timing pattern of nonsynchronization (in a simpler game) will arise as the equilibrium timing pattern (of a more complicated game) in the presence of strategic complementarity. Based on these two points, it seems likely that with three or more players, the Pareto superior pattern of nonsynchronization will arise as the equilibrium timing pattern of a game allowing for two-dimensional choices of both the commitment length and the strategic variable.

## 7. CONCLUSION

This paper aims to explain an important institutional feature of staggered time-dependent adjustment rules in a number of macroeconomic models. Arguing that the presence of interdependence among economic agents is a necessary ingredient for the issue of aggregate timing pattern to be relevant and interesting, this paper seeks a game-theoretic explanation of nonsynchronization by pursuing the idea in Maskin and Tirole [27, 28, 29] that the nature of a game may differ with respect to the timing pattern. This paper derives explicitly the equilibrium timing pattern in a model with strategic complementarity and positive externality when the players interact strategically by choosing either predetermined prices (in Section 3) or fixed prices (in Section 4).[24]

[24]One reason for the success in obtaining the aggregate timing pattern endogenously is the tractability provided by the use of a two-player model, which is quite consistent with this paper's emphasis on strategic interaction. Obviously, whether small or large number of agents is assumed in a model depends on the purpose of the analysis. For example, problems associated with information imperfection are likely to be more serious in an economy with a large number of agents. In examining whether information extraction is an important factor determining the aggregate timing pattern, it is more reasonable to use a specification with a large number of agents, as done in Ball and Cecchetti [2]. In general, the specification of a large number (approaching

These results contribute to the literature of the microfoundations of staggered adjustment rules.[25] In particular, this paper provides a game-theoretic explanation of nonsynchronization for both predetermined and fixed actions, whereas in previous microfoundation works emphasizing other explanations, Fethke and Policano [17, 18] focus on predetermined (wage) adjustment only, and Ball and Cecchetti [2] and Ball and Romer [4] focus on fixed (price) adjustment only. Moreover, it shows that the slope of the reaction functions and the sign of the externality effect are two important factors in determining the outcome of time-dependent adjustment rules. When the agents interact strategically and dynamically, nonsynchronization is the equilibrium timing pattern in the presence of strategic complementarity. On the other hand, the sign of the externality effect determines whether the players' strategic variables are higher or lower under nonsynchronization. In the presence of strategic complementarity and positive (resp. negative) externality, the players moving alternatingly benefit from the higher (resp. lower) level of the strategic variables.

In deriving the various results, this paper enriches the applicability of the framework used in Fudenberg and Tirole [20] and Bulow et al. [10]. The slope of the reaction function and the sign of the externality effect have been found to be important in the analysis of business strategies (Fudenberg and Tirole [20]), multiproduct oligopoly (Bulow et al. [10]), the relative payoffs of the Stackelberg leader and follower (Gal-Or [21]), macroeconomic coordination failures (Cooper and John [12]) and adjustment cost games (Lapham and Ware [25]). More examples can be found in Fudenberg and Tirole [20], Bulow et al. [10], and Milgrom and Roberts [30]. The results of this paper add to the above lists the effect on the aggregate pattern of time-dependent adjustment rules in repeated

---

infinity) of agents and that of a small number of agents (two for models with strategic interaction) can be regarded as two extreme but convenient simplifying assumptions which make the analysis tractable.

[25]Maskin and Tirole [28, pp. 565-566; 29, pp. 590-591] discuss briefly a model with fixed actions in which nonsynchronized moves are derived (rather than imposed) by allowing each player to have a choice of being out of market for a period so as to move from one cohort to another. Besides making a different (and perhaps more realistic) specification of allowing the players to commit for either one or two periods (rather than to wait for a period and obtain no current payoff), Section 4 of this paper provides a more detailed analysis of how nonsynchronization arises endogenously as a result of the players choosing mixed strategies at the beginning of the game.

interaction.

Finally, this paper reveals a link between strategic complementarity and nonsynchronized decision dates. In Taylor [32] and Blanchard [6], nonsynchronized adjustment is suggested as an important ingredient in the explanation of persistent real effect of nominal shocks, which is a crucial feature of non-Walrasian explanations of economic fluctuations. This paper further explains, in a game theoretic framework, nonsynchronization in terms of a property of the agents' objective functions: strategic complementarity. An important paper by Cooper and John [12] shows that strategic complementarity is crucial for the presence of two other important macroeconomic phenomena: multiple equilibria and multiplier effects. It appears that strategic complementarity is an important element in non-Walrasian explanations of business cycles. Further studies on the importance of strategic complementarity in other issues related to economic fluctuations, such as whether strategic complementarity is a determinant of nominal rigidity in state-dependent adjustment rules, would also be useful to enhance our understanding.

**Table I: Strategies and payoffs at the beginning of the game with predetermined prices**

| A \ B | $(1, x_N)$ | $(2, x_N, x_L)$ |
|---|---|---|
| $(1, x_N)$ | $U_{NN}+\beta V,\ U_{NN}+\beta V$ | $V_1,\ V_2$ |
| $(2, x_N, x_L)$ | $V_2,\ V_1$ | $U_{NN}+\beta U_{LL}+\beta^2 V,\ U_{NN}+\beta U_{LL}+\beta^2 V$ |

**Table II: Strategies and payoffs at the beginning of the game with fixed prices**

| A \ B | $(1, x_1)$ | $(2, x_2)$ |
|---|---|---|
| $(1, x_1)$ | $U_{11}+\beta V,\ U_{11}+\beta V$ | $V_1,\ V_2$ |
| $(2, x_2)$ | $V_2,\ V_1$ | $(1+\beta)U_{22}+\beta^2 V,\ (1+\beta)U_{22}+\beta^2 V$ |

**Table III: Selected calculations of the no-deviation-from-nonsynchronization expression**

| $\beta$\g | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|
| 0.1 | $1.2\times10^{-7}$ | $5.8\times10^{-6}$ | $3.8\times10^{-5}$ | 0.0001 | 0.0003 |
| 0.3 | $1.5\times10^{-6}$ | $6.2\times10^{-5}$ | 0.0004 | 0.002 | 0.004 |
| 0.5 | $5.2\times10^{-6}$ | 0.0002 | 0.001 | 0.005 | 0.017 |
| 0.7 | $1.2\times10^{-5}$ | 0.0005 | 0.003 | 0.01 | 0.05 |
| 0.9 | $2.5\times10^{-5}$ | 0.0009 | 0.006 | 0.027 | 0.10 |

Notes:

(a) Each entry represents the magnitude of (4.17) divided by $h^2$.

(b) As each of $V_1$, $V_2$, $U_{11}$, $U_{22}$ and $U_{NN}$ is a product of $h^2$ and a term involving only on parameters $\beta$ and $g$, it is easy to observe from (4.14) that $p$ depends only on $\beta$ and $g$. Consequently, each of $V$ in (4.15) and $NDFN(x_N)$ in (4.17) is also a product of $h^2$ and a term depending only on $\beta$ and $g$.

**Table IV: Factors determining the benefit or disadvantage to the Stackelberg follower**

| | Positive externality<br>($U^i_j > 0$; $U^j_i > 0$) | Negative externality<br>($U^i_j < 0$, $U^j_i < 0$) |
|---|---|---|
| Strategic Complementarity<br>($U^i_{ij} > 0$) | $x_j \uparrow$<br>$(U^i_{ij}) \downarrow \quad \searrow (U^i_j)$<br>$x_i \uparrow \quad U_i \uparrow$<br>$\searrow (U^j_i)$<br>$U_j \uparrow$<br>Case 1: 'Fat Cat' | $x_j \downarrow$<br>$\downarrow \quad \searrow$<br>$x_i \downarrow \quad U_i \uparrow$<br>$\searrow$<br>$U_j \uparrow$<br>Case 2: 'Puppy Dog' |
| Strategic Substitutability<br>($U^i_{ij} < 0$) | $x_j \downarrow$<br>$\downarrow \quad \searrow$<br>$x_i \uparrow \quad U_i \downarrow$<br>$\searrow$<br>$U_j \uparrow$<br>Case 4: 'Lean and Hungry' | $x_j \uparrow$<br>$\downarrow \quad \searrow$<br>$x_i \downarrow \quad U_i \downarrow$<br>$\searrow$<br>$U_j \uparrow$<br>Case 3: 'Top Dog' |

Notes:

(a) The terminology (of the Stackelberg leader j) inside the quotation marks follows Fudenberg and Tirole [20].

(b) The payoff of follower i is higher (than the Nash payoff) in Case 1 or 2, but is lower in Case 3 or 4.

## REFERENCES


1. R. J. Aumann, Subjectivity and correlation in randomized strategies, *Journal of Mathematical Economics* 1 (1974), 67-96.

2. L. Ball and S. G. Cecchetti, Imperfect information and staggered price setting, *American Economic Review* 78 (1988), 999-1018.

3. L. Ball and N. G. Mankiw, Price adjustment and economic fluctuations, *Economic Journal* 104 (1994), 247-261.

4. L. Ball and D. Romer, The equilibrium and optimal timing of price changes, *Review of Economic Studies* 56 (1989), 179-198.

5. T. Basar and G. J. Olsder, "Dynamic Non-cooperative Game Theory", Academic Press, New York, 1982.

6. O. J. Blanchard, The wage price spiral, *Quarterly Journal of Economics* 101 (1986), 543-565.

7. O. J. Blanchard and S. Fischer, "Lectures on Macroeconomics", MIT Press, Cambridge, 1989.

8. O. J. Blanchard and N. Kiyotaki, Monopolistic competition and the effects of aggregate demand, *American Economic Review* 77 (1987), 647-666.

9. J. Bryant, A simple rational expectations Keynes-type model, *Quarterly Journal of Economics* 98 (1983), 525-528.

10. J. I. Bulow, J. D. Geanakoplos and P. D. Klemperer, Multimarket oligopoly: Strategic substitutes and complements, *Journal of Political Economy* 93 (1985), 488-511.

11. A. Caplin and J. Leahy, State-dependent pricing and the dynamics of money and output, *Quarterly Journal of Economics* 106 (1991), 683-708.

12. R. Cooper and A. John, Coordinating coordination failures in Keynesian models, *Quarterly Journal of Economics* 103 (1988), 441-463.

13. R. M. Cyert and M. H. DeGroot, Multiperiod decision models with alternating choice as the solution to the duopoly problem, *Quarterly Journal of Economics* 84 (1970), 410-429.

14. G. De Fraja, Staggered vs. synchronised wage setting in oligopoly, *European Economic Review* 37 (1993), 1507-1522.

15. P. A. Diamond, Aggregate demand management in search equilibrium, *Journal of Political Economy* 90 (1982), 881-894.

16. S. Dowrick, Von Stackelberg and Cournot duopoly: Choosing roles, *Rand Journal of Economics* 17 (1986), 251-260.

17. G. Fethke and A. Policano, Wage contingencies, the pattern of negotiation and aggregate implications of alternative contract structures, *Journal of Monetary Economics* 14 (1984), 151-170.

18. G. Fethke and A. Policano, Will wage setters ever stagger decisions? *Quarterly Journal of Economics* 101 (1986), 867-877.

19. S. Fischer, Long term contracts, rational expectations, and the optimal money supply rule, *Journal of Political Economy* 85 (1977), 163-190.

20. D. Fudenberg and J. Tirole, The fat-cat effect, the puppy-dog ploy, and the lean and hungry look, *American Economic Review Papers and Proceedings* 74 (1984), May, 361-366.

21. E. Gal-Or, First mover and second mover advantages, *International Economic Review* 26 (1985), 649-653.

22. J. A. Gray, On indexation and contract length, *Journal of Political Economy* 86 (1978), 1-18.

23. A. K. Kashyap, Sticky prices: New evidence from retail catalogs, *Quarterly Journal of Economics* 110 (1995), 245-274.

24. F. Kydland, Noncooperative and dominant player solutions in discrete dynamic games, *International Economic Review* 16 (1975), 321-335.

25. B. Lapham and R. Ware, Markov puppy dogs and related animals, *International Journal of Industrial Organization* 12 (1994), 569-593.

26. S.-H. P. Lau, Aggregate pattern of time-dependent adjustment rules, I: A game-theoretic analysis of staggered versus synchronised wage setting, *Economic Journal* 106 (1996), 1645-1658.

27. E. Maskin and J. Tirole, A theory of dynamic oligopoly, III: Cournot competition, *European Economic Review* 31 (1987), 947-968.

28. E. Maskin and J. Tirole, A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed costs, *Econometrica* 56 (1988), 549-569.

29. E. Maskin and J. Tirole, A theory of dynamic oligopoly, II: Price competition, kinked demand curves, and Edgeworth cycles, *Econometrica* 56 (1988), 571-599.

30. P. Milgrom and J. Roberts, Rationalizability, learning, and equilibrium in games with strategic complementarities, *Econometrica* 58 (1990), 1255-1277.

31. D. Romer, "Advanced Macroeconomics", McGraw-Hill, New York, 1996.

32. J. B. Taylor, Aggregate dynamics and staggered contracts, *Journal of Political Economy* 88 (1980), 1-23.

33. J. Tirole, "The Theory of Industrial Organization", MIT Press, Cambridge, 1988.