A Proposal to Unify Some Concepts in the Theory of Fairness[1]

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So far, the theory of distributive justice has tried to single out a unique criterion of justice. However, different people hold conflicting ideas about justice. We propose a procedure for representing these individual opinions by means of "aspiration functions." We present three different ways of aggregating such opposing opinions into a socially acceptable judgement. Furthermore, we show that many well-known concepts are special cases of our approach. We study, under a restriction on the form of the aspiration functions, the conditions that are necessary and sufficient for a social choice correspondence to be generated from any of our concepts. *Journal of Economic Literature* Classification Numbers: D63, D71.

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1. INTRODUCTION

The theory of distributive justice studies "how a society or group should allocate its scarce resources or product among individuals with competing needs or claims" (Roemer [26, p. 1]). At the risk of being simplistic, there have been two main approaches to the theory of distributive justice (a detailed account of this theory is provided by Sen [28] and Roemer [26]). First the social welfare function (SWF) approach, in which the aim is to provide a complete social ranking of all feasible alternatives. The SWF is the representation of this social ranking in terms of a function. Examples of SWFs are the utilitarian (proposed by Bentham), the maximum (proposed by Rawls) or the product of utilities (proposed by Nash in the context of bargaining). The work of Arrow on how to derive a SWF from individual preferences is, according to Sen [28, p. 1074]; "... the big bang that characterized the beginning" of social choice theory. The second approach proposes the use of a social choice correspondence (SCC), which is a map from the set of economies into the set of feasible allocations. This branch of distributive justice aims "... no more than separate out a subset of the set of social states for special commendation" (Sen [28, p. 1106]). Some SCCs are characterized by properties that relate different economies such as consistency or population monotonicity (see, e.g., Aumann and Maschler [2] and Thomson and Lensberg [34]). For an overview of this literature, see Moulin [14] and Roemer [26]. Other SCCs are defined by selecting in each economy those allocations fulfilling certain properties: the envy-free correspondence (Foley [8]),[3] the proportional solution (Roemer and Silvestre [27]) and the egalitarian-equivalent correspondence (Pazner and Schmeidler [21]). We shall call this subapproach "Fairness" (see Thomson and Varian [35] and Arnsperger [1]).

In all of these contributions, however, the criterion of justice is either axiomatically justified or made plausible by appealing to intuition. The criterion of justice is never derived from people's opinions. Indeed, one interpretation of these contributions is that people do not have any opinions about justice and need to be enlightened by an impartial observer who is endowed with the appropriate tools to deal with the problems of distributive justice (another interpretation would be that the general opinion about justice is unanimous). It could be argued that people's preferences can incorporate judgements about justice. The standard approaches, however, do not provide a framework that enables us to distinguish between preferences "per se" and judgements about justice.

In this paper, we consider a situation in which people may have conflicting ideas about what they deserve, and we propose a framework in which we

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[3] See also Varian [37, 38], Daniel [5], and Piketty [22].

deal with this problem. Our framework allows us to reinterpret many well-known SCCs and concepts of fairness as special cases of our approach.

We first formalize the idea that an individual has about the fairness of a particular allocation, by assuming that for every economy and for every feasible allocation, each individual has what we call an "aspiration," which is a benchmark consumption bundle for her.[4] Consider, for instance, an Edgeworth box (two people, two commodities) and suppose that Mrs. 1 is a greedy person who can never get enough. Her aspiration, given that she consumes x_1, is $x_1 + \epsilon$, where ϵ is strictly positive. Mr. 2, on the contrary, is a conformist who always thinks that he is getting enough. His aspiration, given that he consumes x_2, is x_2. Alternatively, let us suppose that Mrs. 1's aspiration is x_2 and vice versa. The latter aspirations are those used by the envy-free correspondence. It is clear that many other aspirations are possible.

We assume that the aspiration of each individual contains her true ideas about what she thinks she deserves in a given allocation. Three remarks are in order here. First, there are no incentive considerations: we assume that people do not attempt to distort their true aspirations in order to obtain an advantage, so that the aspirations reflect each individual's true opinion about justice. Second, we assume that individuals are endowed with the maturity and knowledge necessary to form aspirations "correctly," i.e., we do not deal with the problem of the "tamed housewife" (see, for example, Sen [29]). Finally, we do not deal with how aspirations are formed. We recognize that these three points are indeed important, but this paper must be regarded as a first approach to the problem, concentrating on the effects of the concepts of justice on resource allocation, but without offering an explanation of how these ideas are (or should be) formed.

Given the aspirations, we need to decide on the fairness of a given allocation. This paper takes the view that the normative judgements of the society must be responsive to people's aspirations. We may well ask ourselves why social judgements should depend on people's aspirations which might easily be arbitrary. Our position on this point is that aspirations together with tastes are the building blocks of distributive justice. There is no reason to discard one without discarding the other. To discard people's aspirations when evaluating distributive justice, because some people might hold foolish aspirations, would be like discarding tastes because of the existence of foolish tastes.

We propose three different procedures to derive social judgements from individual aspirations. These procedures have been chosen because they are

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[4] Roemer [23] used the idea of a benchmark allocation against which to judge whether there is exploitation in a given allocation. He defined as the benchmark allocation the characteristic function of a cooperative game and proved that the set of nonexploitative allocations is the core of that game.

natural generalizations of notions that have already appeared in the literature on fairness. The first one leads us to the set of allocations that we call Adequate, where everybody gets exactly the same utility as they have in their aspirations. The second is the set of satisfactory allocations, where everybody gets at least the same utility as they have in their aspirations. Finally, we have the set of unbiased allocations, where either everybody is at least as well-off or everybody is worse than in their aspirations. The reason of this latter notion is that, if one individual happens to be better off than her reference point, and others are worse off, the solution is treating them *asymmetrically*. Note that if we use the same aspirations with our three previous concepts, the three sets will be related by inclusion.

In Section 2, we present a general model and the definitions of our three proposals. We investigate the possibility that the intersection of the set of allocations that fulfill any of our three definitions with the set of Pareto efficient allocations is nonempty for given aspirations. We prove that this is only the case for the set of unbiased allocations (Theorem 1).

In Section 3, we investigate whether the allocations selected by a given social choice correspondence coincide with the set of allocations that are adequate (resp. satisfactory) for some aspirations. In such a case, we say that the SCC is *attainable* in an adequate (resp., satisfactory) way. We restrict our analysis to those aspirations that are decentralized in the following way: the aspiration function of every individual is independent of the preferences of other individuals. We prove that there is a condition, which we call *nonoffsetting veto*, that is necessary and sufficient for any SCC to be attainable, either in an adequate or a satisfactory way (Theorem 2). Next, we use nonoffsetting veto to test whether well-known SCCs (such as Pareto correspondence, Walrasian correspondence, etc.) and properties like Maskin monotonicity, consistency, and population monotonicity can be generated by our notions of justice or not.

In Section 4 we study the concept of Unbiased allocations. We restrict our attention to those SCCs that are Pareto efficient and we propose a weaker notion of attainability. We say that a SCC is *weakly attainable* in an unbiased way, if the allocations selected by that SCC coincide with the intersection of the set of Pareto efficient allocations and the set of allocations that are unbiased for some aspirations. We present a condition called *selective offsetting veto* that any Pareto efficient SCC must satisfy in order to be weakly attainable in an unbiased way. This condition is sufficient in the case of two agents but not with three or more (Theorem 3). Again, we check if the correspondences and properties mentioned above satisfy selective offsetting veto.

Finally, in Section 5 we present some final remarks and outline paths for future research.

2. PRELIMINARIES

There are n ($\geqslant 2$) individuals. For all $i = 1, ..., n$ let X_i be her consumption set, $u_i \colon X_i \to R$ her utility function and U_i the set of all her admissible utility functions. Let $U \equiv U_1 \times \cdots \times U_n$. Let $A \subset X \equiv X_1 \times \cdots \times X_n$ be the set of feasible allocations, 2^A the power set of A, and $a = (a_1, ..., a_n) \in A$ an allocation. We assume that A is fixed and thus, an economy is an element of U denoted by u. A *social choice correspondence* is a mapping, denoted by S, from U into $2^A \backslash \{\varnothing\}$. The Pareto correspondence P is the SCC which maps every u in U into the allocations that are Pareto efficient for u.

We spell out three models that we will use to illustrate our ideas.

1. *Exchange Economies.* Let $e_i \in R^m_{++}$ be the initial endowments of individual i where $m \geqslant 2$ is the number of goods. Denote x_{ij} the amount of good j consumed by individual i. Then $a_i = (x_{i1}, ..., x_{im})$, $X_i \equiv R^m_+$, and $A = \{(a_1, ..., a_n) \in X \mid \sum_{i=1}^n x_{ij} \leqslant \sum_{i=1}^n e_{ij}, \forall j = 1, ..., m\}$. Utility functions are assumed to be continuously differentiable, strictly increasing and concave.

2. *Production Economies.* There are two goods, consumption and leisure. Every individual i has the same amount of time $\bar{l} \in R_+$ to divide between labor (l_i) and leisure ($\bar{l} - l_i$). The productive skill of i is denoted by $\delta_j \in R_+$. Individual i's bundle is $a_i = (x_i, l_i) \in X_i \equiv R_+ \times [0, \bar{l}]$ where x_i is her consumption. We assume that utility functions are strictly increasing on x_i and $\bar{l} - l_i$, continuously differentiable and concave. Technology is represented by a function $f \colon R_+ \to R_+$ that maps aggregate labor, measured in efficiency units ($\sum_{i=1}^n \delta_i l_i$), into output. We assume that f is strictly increasing, continuously differentiable and concave. Finally, $A = \{(x, l) \in X \mid \sum_{i=1}^n x_i \leqslant f(\sum_{i=1}^n \delta_i l_i)\}$.

3. *Economies with Public Goods.* There are two goods, the public good and a consumption good. Let $e_i \in R_+$ be the endowment of i of the private good. Denote by x_i (resp. y_i) the consumption of i of the private (resp., public) good. Utility functions are assumed to be strictly increasing, continuously differentiable and concave. The cost of producing the public good is cy_i with $c > 0$. Finally, $A = \{(x, y) \in X \mid y_1 = y_2 = \cdots = y_n, cy_i + \sum_{i=1}^n x_i \leqslant \sum_{i=1}^n e_i\}$.

Going back to our general model, we define for all i a function $\psi_i \colon A \times U \to X_i$ which we call the *aspiration function* of individual i. Let $\psi = (\psi_1, ..., \psi_n)$. We call $\psi_i(a, u)$ the *aspiration* of i. We may interpret $\psi_i(a, u)$ as the bundle that i thinks is fair for her in the economy u if the allocation is a. For notational simplicity we will write $\psi_i(a)$ except when changes in the utility profile require the use of the initial notation. Note that $\psi(a) \in X_1 \times \cdots \times X_n$ but $\psi(a)$ may be unfeasible. We can think of

many possible aspiration functions. The following example illustrate one of these.

EXAMPLE 1. In a production economy u with $n = 2$, take a feasible allocation (x, l). Let $f(\delta_1 l_1 + \delta_2 l_2) = \delta_1 l_1 + \delta_2 l_2$. Let u^* be an economy where both individuals have the same utility functions as in u, but now they also have the same skill level δ^* satisfying $\delta^* l_1 + \delta^* l_2 = \delta_1 l_1 + \delta_2 l_2$. By our assumptions, any two Walrasian equilibria of u yield the same utility to the consumers. Let (x^*, l^*) be one of such equilibria of u^* (note that δ, u^*, and (x^*, l^*) depend on (l_1, l_2)). Take $\psi_i: A \times U \to X_i$ such that $\psi_i((x, l), u) = (x_i^*, l_i^*)$ for $i = 1, 2$. Thus, the aspiration in u is the allocation (x^*, l^*), that is "equitable" for an economy in which both individuals are equally skilled.

Next, we have to decide how individual comparisons between aspirations and bundles actually received are aggregated into a social judgement. We propose three different ways of doing so. The first is as follows.

DEFINITION 1. A feasible allocation a in u is adequate, given ψ, if

$$u_i(a_i) = u_i(\psi_i(a, u)) \qquad \text{for all } i.$$

This definition is related to the concept of egalitarian-equivalent allocations proposed by Pazner and Schmeidler [21]. Recall that $a \in A$ is egalitarian-equivalent if there is a z, such that $u_i(a_i) = u_i(z)$ for all i. An egalitarian-equivalent allocation is adequate by taking aspiration functions $\psi_i(a, u) = z$ for $i = 1, ..., n$. Another example of an adequate allocation is the *constant-returns-equivalent* correspondence proposed by Mas-Colell [12]. It selects an efficient allocation that all individuals deem indifferent with the bundle they would enjoy in some (hypothetical) economy where the technology is linear. See Roemer [26, Chap. 6] for a discussion on this and Moulin [16] where this correspondence is characterized.[5]

Our next proposal selects a wider set of allocations. For each agent, say i, it takes the utility level $u_i(\psi_i(a))$ as a lower bound.

DEFINITION 2. A feasible allocation a in u is satisfactory, given ψ, if

$$u_i(a_i) \geqslant u_i(\psi_i(a, u)) \qquad \text{for all } i.$$

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[5] Nash equilibrium can be seen as an adequate allocation. In this case, the aspiration of an agent is the list of strategies in which the strategies of other agents are fixed and her own strategy maximizes her utility. An aspiration, here, is what can be achieved effectively by an agent and not what she thinks she deserves.

An example of this is the concept of the envy-free correspondence proposed by Foley [8]. In this case $\psi_i(a, u)$ is constructed by taking the best bundle for i among the bundles of the rest of the agents. In exchange economies, by setting $\psi_i(a) = e_i$ for all i, we obtain the *individually rational* correspondence. If there is a commonly owned bundle Ω that has to be distributed among individuals, by allowing $\psi_i(a) = (1/n)\Omega$ for all i we obtain the *equal split lower bound* correspondence, a concept that traces back to Steinhaus [30]. In production economies we may construct aspiration functions as follows. Let us suppose that all individuals are identical to individual i. A fair allocation in such an economy would be that everybody works the same number of hours and receives the same amount of consumption goods. Moreover, we can select this allocation to be Pareto efficient. Call it $(a_1^i, ..., a_n^i)$. Then take $\psi_i(a) = a_1^i$. This is the so-called *unanimity lower bound* introduced by Gevers [9] (see also Moulin [16]). Other examples of aspiration functions that yield satisfactory allocations can be found in Kolm [10], Thomson [31], Diamantaras and Thomson [6], Moulin [17], Maniquet [11] and Fleurbaey and Maniquet [7].

In this paper, we wish to study the relationship between the above concepts and the SCCs. One way of doing so is to restrict aspiration functions by means of some conditions and observe the allocations that are adequate or satisfactory. These allocations vary with the economy and, thus, we have a correspondence mapping economies into allocations, i.e., a SCC. Unless the conditions imposed on aspiration functions are restrictive enough, however, this research strategy is unlikely to say anything precise about the SCCs generated in this way. Given that we know very little about aspirations, this research strategy seems a little hasty to us. Our approach here is quite the opposite: We fix a SCC and ask if there are aspiration functions such that, for each economy in the domain, the allocations in the range of this SCC are adequate or satisfactory (these aspirations belong to a large class called "decentralized aspirations," defined at the beginning of Section 3). The main advantage of our procedure is that with very few assumptions, we identify the SCCs that *cannot* be generated as either adequate or satisfactory. We also get some hints about the kind of aspirations that can be used "to support" allocations in the range of a SCC as adequate or as satisfactory. With this motivation in hand, we present the main definition of the paper.

DEFINITION 3. We say that a SCC S is attainable in an adequate way (resp., satisfactory way) if there is $\psi = (\psi_1, ..., \psi_n)$ such that, for all u in U, a belongs to $S(u)$ if and only if a is an adequate allocation (resp., a satisfactory allocation) for ψ.

Note that when a SCC is attainable in an adequate way, it will not be attainable, in general, in a satisfactory way, *with the same aspiration functions*. The reason for this is that, while all adequate allocations are also satisfactory, some allocations that are not adequate may well be satisfactory.

Our third proposal selects an even wider set of allocations.

DEFINITION 4. A feasible allocation a in u is unbiased, given ψ, if any of the two following statements hold:

(i) $u_i(a_i) \geqslant u_i(\psi_i(a, u))$ for all i, or

(ii) $u_i(a_i) < u_i(\psi_i(a, u))$ for all i.

If an allocation is not unbiased, some individuals feel disappointed when they compare their actual bundles with their aspirations, while others are satisfied with what they get. We feel that such an allocation can not pass any minimal fairness requirement and must therefore be discarded. Thus, the idea behind this concept is more on the negative: any allocation that is not unbiased can not be considered as a part of any reasonable solution, even though some unbiased allocations may not be reasonable according to some criteria.

The idea of unbiased allocations is related to the concept of *balanced* allocations proposed by Daniel [5]. If there are only two individuals, the set of balanced allocations contains all envy-free allocations, plus all allocations where both individuals prefer their opponent's bundle. This is a special case of an Unbiased allocation for specific aspiration functions, namely $\psi_1(a, u) = a_2$ and $\psi_2(a, u) = a_1$. Unbiased allocations are very close to the concepts of the axioms of *uniform preferences externality* and *uniform group externality* by Moulin [15]. In the first case $\psi_i(a, u)$ is what the SCC assigns to individual i in an economy where all utility functions are u_i. In the second one $\psi_i(a, u)$ is what i receives in an economy where she is the sole member of the society. The difference between these concepts and an unbiased allocation is that we do not allow for a situation in which some individuals get the same utility as in their aspiration bundles while others get less utility than in their aspiration bundles.

The existence of an adequate or a satisfactory allocation cannot be guaranteed for a given aspiration function. For instance, consider that for some i, u_i is strictly monotonic and let $\psi_i(a) = a_i + \epsilon$ with $\epsilon \gg 0$. The existence of an unbiased allocation, however, is guaranteed under mild assumptions. The problem is that it may contain allocations that are not very appealing. For instance, in an exchange economy, the allocation that gives zero goods to every individual is unbiased if each aspiration bundle contains a positive element. Many undesirable allocations of this sort are

discarded however by taking the intersection between the unbiased alloca-
tions and the set of Pareto efficient allocations. Our first result proves that
this intersection is nonempty under mild assumptions on the economy and
on aspiration functions ψ.

Assumption 1. The set of feasible allocations A is nonempty, compact,
and convex and utility functions are continuous, strictly increasing, and
admit a concave representation.

Let $\underline{a}_i$ be the "worst" bundle for individual i of those she can get from
A. By Assumption 1 it exists. Now consider the following definition.

DEFINITION 5. An aspiration ψ is bounded from below if for all $u \in U$
and $a \in A$.

$$u_i(\underline{a}_i) \leqslant u_i(\psi_i(a, u)) \qquad \text{for all } i.$$

This simply means that no aspiration can be worse than the worst
bundle in A.[6] Now we have the following.

THEOREM 1. *Suppose Assumption* 1 *holds and* ψ_i *is continuous on a and
bounded below for all i. Then, there is some allocation that is Pareto efficient
and unbiased.*

Proof. Let S^{n-1} be the $(n-1)$ dimensional simplex, that is, $S^{n-1} = \{\alpha \in R_+^n \mid \sum_{i=1}^n \alpha_i = 1\}$. Under our assumptions, an allocation a is Pareto
efficient if for a given $\alpha \in S^{n-1}$ it solves the following maximization
program:

$$\underset{a \in A}{\text{Max}} \sum_{i=1}^n \alpha_i u_i(a_i).$$

Under our assumptions the above program has a solution.

Let $\rho: S^{n-1} \to A$ be the result of the maximization problem. It is convex-
valued (due of the existence of a concave representation for the utility
functions and the convexity of A) and upper hemicontinuous (by Berge's
Maximum Theorem). Now, let us define for each i a function $D_i: A \to R$
(interpreted as the disappointment felt by i) as

$$D_i(a) = D_i(a_i, \psi_i(a)) = u_i(\psi_i(a)) - u_i(a_i).$$

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[6] Note that in exchange economies, production economies, and public goods economies,
Assumption 1 is satisfied and aspiration functions are bounded from below.

Since u_i and ψ_i are continuous, D_i is also continuous for all i. Consider now, for a fixed allocation $(\bar{a})$, the following maximization program:

$$\operatorname*{Max}_{(\alpha_1, \ldots, \alpha_n) \in S^{n-1}} \sum_{i=1}^{n} \alpha_i D_i(\bar{a}_i, \psi_i(\bar{a})),$$

and let us define the correspondence $\phi \colon A \to S^{n-1}$ assigning the solution to this problem to every feasible allocation. The correspondence ϕ is convex-valued and upper hemicontinuous (due to Berge's Maximum Theorem). Thus the mapping

$$\phi \circ \rho \colon S^{n-1} \to S^{n-1}$$

is convex-value and upper hemicontinuous (Border [3, Proposition 11.23]). Thus, the composite mapping $\phi \circ \rho$ has a fixed point that we call α^*. Let $a^* \in \rho(\alpha^*)$, and suppose that there are i, j such that $D_i(a^*) \geqslant 0$ and $D_j(a^*) < 0$. The mapping ϕ then implies $\alpha_j^* = 0$ and the mapping ρ will assign her worst bundle ω_j to j. Thus $a_j^* = \omega_j$ and $D_j(a^*) = D_j(\omega_j, \psi_j(a^*)) = u_j(\psi_j(a^*)) - u_j(\omega_j) < 0$. But this is impossible since ψ is bounded from below.[7] ∎

As an illustration of the theorem, let us consider the following example taken from Pazner and Schmeidler [20].

EXAMPLE 2. Let $u_1(x_1, l_1) = (\bar{l} - l_1) + \frac{11}{10}x_1$, $u_2(x_2, l_2) = (\bar{l} - l_2) + 2x_2$, $f(\delta_1 l_1 + \delta_2 l_2) = \delta_1 l_1 + \delta_2 l_2$, $\delta_1 = 1$, $\delta_2 = \frac{1}{10}$ and $\bar{l} = 1$. The reader can check that the set of Pareto efficient allocations is the union of two sets. First, all allocations where $l_1 = 1$, $l_2 = 0$ and $x_1 + x_2 = 1$. Second, all allocations where $l_1 = 1$, $l_2 > 0$, $x_2 = 0$ and $x_1 = 1 + \frac{1}{10}l_2$. Fix $\psi_1(x, l) = (x_2, l_2)$ and $\psi_2(x, l) = (x_1, l_1)$. These are the aspiration functions of the envy-free correspondence. From Pazner and Schmeidler [20] we know that no adequate or satisfactory allocation is Pareto efficient. We can check that the set of unbiased and Pareto efficient allocations is the following:

$$\{(x, l) \in A \mid l_1 = 1, l_2 = 0, x_1 + x_2 = 1 \text{ and } \tfrac{3}{4} \leqslant x_1 \leqslant \tfrac{21}{22}\}.$$

[7] Our Theorem 1 implies Theorem 1 in Moulin [17, p. 42]. He proves that in a particular cost-sharing problem, and for some aspiration functions ψ, there is some allocation a^* that is both Pareto efficient and $u_i(a_i^*) \leqslant u_i(\psi_i(a^*, u))$ for all i. But since, in his setup, no allocation satisfies $u_i(a_i^*) \geqslant u_i(\psi_i(a^*, u))$ for all i with $u_j(a_j^*) > u_j(\psi_j(a^*, u))$ for some j, the result follows.

Theorem 1 proves that the set of unbiased and Pareto efficient allocations is always non-empty.[8] This allows us to propose a different concept of attainability.

DEFINITION 6. We say that a SCC S is weakly attainable in an unbiased way if, for all u in U, any allocation a belongs to $S(u)$ if and only if a belongs to $P(u)$ and there is $\psi = (\psi_1, ..., \psi_n)$ such that a is an unbiased allocation for ψ.

3. ATTAINABILITY

If no restriction is imposed on aspirations, any SCC is attainable. For instance, let us suppose that we are in an exchange economy and we want to attain a given SCC $\hat{S}$ in an adequate way. Take an economy $\hat{u}$. For all $\hat{a} \in \hat{S}(\hat{u})$ let $\psi_i(\hat{a}, \hat{u}) = \hat{a}_i$ for all i and for all $\bar{a} \notin \hat{S}(\hat{u})$ let $\psi_i(\bar{a}, \hat{u}) = \bar{a} + \varepsilon$, with $\varepsilon > 0$, for all i. An identical construction can be used to attain any SCC in a satisfactory way. This observation calls for a restriction in the kind of aspiration functions that may be considered. We propose a restricted class of aspiration functions, namely those which are decentralized in the following sense.

DEFINITION 7. The aspiration functions ψ are decentralized if, for all i, ψ_i depends only on u_i and a. That is, the domain of ψ_i is $A \times U_i$.

The motivation for studying decentralized aspiration functions is twofold. On the one hand, we may argue that the information on the form of i's utility function is only known to i. On the other hand, from a normative point of view, we might argue that the information i has about j's preferences should not contaminate her judgement about what she deserves.[9] All the examples of aspiration functions that we presented in the previous section are decentralized, except two: the aspiration functions

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[8] Theorem 1 holds if we replace the assumption of continuity of utility and aspiration functions by the weaker assumption that the functions D_i's (defined in the proof of Theorem 1) are continuous. Thus in the envy-free solution with $n > 2$, it may happen that in allocation a individual i's aspiration is individual j's bundle, but with a small change in the allocation individual i's aspiration is now individual h's bundle. This change in ψ_i will not be continuous but D_i is continuous.

[9] Moulin [15] used the word "decentralization" to refer to those bounds on the utilities of the individuals involved in some distribution problem that do not depend on the preferences of other individuals. The same property has been named "the thin veil of ignorance" by Roemer [24].

corresponding to the egalitarian-equivalent correspondence and those of the constant-return-equivalent correspondence.[10]

The following definition presents a particular example of decentralized aspiration functions that we shall use extensively. Let $Z_i: A \to X_i$ be a correspondence. The set $Z_i(a)$ is interpreted as the set of bundles that i thinks she is entitled to, given the allocation a.

DEFINITION 8. The aspiration functions ψ are called rational if, for all i, $\psi_i = \arg\max u_i(a_i)$ with $a_i \in Z_i(a)$ (ties are broken by some arbitrary rule).

An example of rational aspirations are those implicit in the envy-free correspondence where $Z_i(a) = \{a_1, ..., a_n\}$. Another example would be the aspiration functions implicit in the concept of superequity proposed by Kolm [10] where $Z_i(a)$ is the convex hull of $\{a_1, ..., a_n\}$.

We should emphasize that the assumption that aspiration functions are decentralized plays an instrumental role: It serves to isolate an important but specific case that is analytically tractable and, thus, shows that the research program proposed in this paper is feasible. In future research however, other properties of aspiration functions should be considered.

We shall now consider the following property of a SCC. We shall use the notation (u_i', u_{-i}) to indicate that individual i has changed her utility function but the rest of the individuals have their same utility functions as in u.

DEFINITION 9. A SCC satisfies nonoffsetting veto if

$$\{a \in S(u) \text{ and } a \notin S(u_i', u_{-i}) \text{ for some } i\}$$

$$\to \{a \notin S(u_i', u_{-i}') \text{ for any admissible } u_{-i}'\}.$$

Nonoffsetting veto says that if a is selected at u but is not selected when i changes her utility function to u_i' (i.e., she vetoes a), a will never be selected in any economy where i's utility function is u_i'.[11] Theorem 2 below shows that nonoffsetting veto is necessary and sufficient for attainability in an adequate or satisfactory way. Following Theorem 2, we shall verify whether several well-known SCCs satisfy it or not. From an analytical point of view, however, the following property is sometimes easier to check:

[10] The aspiration functions constructed at the beginning of this section are not decentralized because they depend on $\hat{a}$, which in turn depends on the particular profile u.

[11] This property should not be confused with the property of "no veto power" introduced by Maskin [13]. In the latter, if all agents excepts possibly one, say i, agree that a is the top allocation in their utility functions u_{-i}, then $a \in S(u_i, u_{-i})$ for all u_i.

DEFINITION 10. A SCC satisfies the utility Cartesian product if

$$\{a \in S(\hat{u}) \text{ and } a \in S(\tilde{u})\} \rightarrow \{a \in S(\tilde{\tilde{u}}) \text{ where for all } i, \tilde{\tilde{u}}_i \text{ is either } \hat{u}_i \text{ or } \tilde{u}_i\}.$$

The property of utility Cartesian product says that if a given allocation is chosen in two different economies, it must also be chosen in any economy in which the profile of utility functions is a mixture of the utility functions of those two economies. In other words, given an allocation a, the set of utility profiles for which a is selected by a SCC has a Cartesian product structure. In the following lemma, we prove that nonoffsetting veto and utility Cartesian product are equivalent properties.

LEMMA 1. *A SCC satisfies nonoffsetting veto if and only if it satisfies the utility Cartesian product.*

Proof. (i) *Utility Cartesian product implies nonoffsetting veto.* Suppose a SCC satisfies the utility Cartesian product and consider $a \in S(u)$, $a \notin S(u'_i, u_{-i})$ for some i but $a \in S(u'_i, u'_{-i})$ for some u'_{-i}. As $a \in S(u_i, u_{-i})$ and $a \in S(u'_i, u'_{-i})$, the utility Cartesian product implies $a \in S(u'_i, u_{-i})$, a contradiction.

(ii) *Nonoffsetting veto implies utility Cartesian product.* Suppose a SCC satisfies nonoffsetting veto and consider $a \in S(\hat{u})$, $a \in S(\tilde{u})$, but $a \notin S(\tilde{\tilde{u}})$ for some profile $\tilde{\tilde{u}}$ where $\tilde{\tilde{u}}_i$ is either $\hat{u}_i$ or $\tilde{u}_i$. Then there is an individual j such that either $a \notin S(\hat{u}_j, \tilde{\tilde{u}}_{-j})$ or $a \notin S(\tilde{u}_j, \hat{u}_{-j})$. To see this, note that from the economy $\hat{u}$ (or $\tilde{u}$), to the economy $\tilde{\tilde{u}}$ some individuals have changed their utility functions. Then there must be some individual, say j, such that precisely when she changes her utility function from $\hat{u}_j$ (or $\tilde{u}_j$) to $\tilde{u}_j$ ($\hat{u}_j$), a is no longer selected by the SCC. Then by nonoffsetting veto either $a \notin S(\hat{u})$ or $a \notin S(\tilde{u})$, a contradiction. ∎

We prove next that, under decentralized aspiration functions, nonoffsetting veto (or equivalently, utility Cartesian product) is a necessary condition and, if preferences are not satiated (Assumption 2 below), a sufficient condition for a SCC to be attainable in an adequate or satisfactory way.

Assumption 2. For all u, for all $a = (a_1, ..., a_n) \in A$, and for all i, there is $a'_i \in X_i$ such that $u_i(a'_i) > u_i(a_i)$.

THEOREM 2. *Suppose aspiration functions are decentralized. Then, if a SCC is attainable in either an adequate or a satisfactory way, the SCC must satisfy nonoffsetting veto. Under Assumption 2, nonoffsetting veto is also sufficient.*

Proof. As the proof is identical for both notions, we present it for the adequate case only.

Necessity. Suppose some SCC is attainable in an adequate way and suppose the antecedent of nonoffsetting veto is satisfied. If $a \notin S(u'_i, u_{-i})$ for some i, it must be that $u'_i(a_i) \neq u'_i(\psi_i(a, u'_i))$. By decentralization, in the economy $u' = (u'_i, u'_{-i})$ the aspiration of i will be the same for all u'_{-i} as the one in the economy (u'_i, u_{-i}). Then $u'_i(a_i) \neq u'_i(\psi_i(a, u'_i))$ and $a \notin S(u'_i, u'_{-i})$.

Sufficiency. We arrive at our proof by constructing decentralized aspiration functions ψ such that for all $u \in U$, $a \in S(u)$ if and only if a is an adequate allocation. Fix an allocation $\hat{a} \in A$ and define $U(\hat{a}) = \{u \in U \mid \hat{a} \in S(u)\}$. This is the set of admissible economies is which $\hat{a}$ is selected by the SCC. Define also, for all i, the set $U_i(\hat{a}) = \{u_i \in U_i \mid (u_i, u_{-i}) \in U(\hat{a})$ for some $u_{-i} \in U_{-i}\}$. There are three cases:

(i) $U(\hat{a}) = \varnothing$. Take an arbitrary individual j and set $\psi_j(\hat{a}, u_j) = \hat{b}_j$ for all u_j where $\hat{b}_j$ is chosen such that $u_j(\hat{a}_j) < u_j(\hat{b}_j)$. This is always possible by Assumption 1.

(ii) $U(\hat{a}) = U$. For all i, take $\psi_i(\hat{a}, u_i) = \hat{a}_i$ for all u_i.

(iii) $U(\hat{a}) \neq \varnothing$, $U(\hat{a}) \neq U$. For all $u \in U$ and for all i fix ψ_i as

$$\psi_i(\hat{a}, u_i) = \hat{a}_i \quad \text{if} \quad u_i \in U_i(\hat{a})$$

$$\psi_i(\hat{a}, u_i) = \hat{b}_i \quad \text{if} \quad u_i \notin U_i(\hat{a}),$$

where we choose $\hat{b}_i$ as in (i). If $u \in U(\hat{a})$, $u_i(\hat{a}_i) = u_i(\psi_i(\hat{a}, u_i))$ for all i. Conversely, assuming that for some economy $\tilde{u} \in U$, $\tilde{u}_i(\hat{a}_i) = \tilde{u}_i(\psi_i(\hat{a}, \tilde{u}_i))$ for all i, $\psi_i(\hat{a}, u_i) = \hat{a}_i$ for all i, which implies $\tilde{u}_i \in U_i(\hat{a})$. Now utility Cartesian product implies $U(\hat{a}) = U_1(\hat{a}) \times \cdots \times U_n(\hat{a})$. Then $\tilde{u} = (\tilde{u}_1, ..., \tilde{u}_n) \in U(\hat{a})$, that is, $\hat{a} \in S(\tilde{u})$. ∎

We now present several well-known SCCs and study whether they fulfill nonoffsetting veto or not.

1. *Pareto Correspondence* $(P(u))$

(a) *Exchange Economies.* Nonoffsetting veto is not satisfied. In Fig. 1 we present an example with two individuals and two goods in which $a \in P(u_1, u_2)$, $a \notin P(u'_1, u_2)$ and $a \notin P(u_1, u'_2)$. However, $a \in P(u'_1, u'_2)$.

(b) *Production Economies.* Nonoffsetting veto is satisfied, since at any Pareto efficient allocation the marginal rates of substitution (MRS) must be equal to the marginal rate of transformation (MRT). Now if $a \in P(u)$ but $a \notin P(u'_i, u_{-i})$ for some i, it must be because the MRS of individual i has changes at a. But since the MRT remains unchanged at a, then
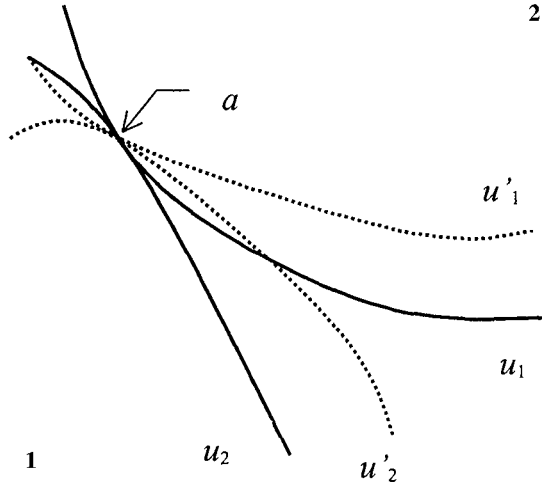
FIG. 1. The Pareto correspondence does not satisfy nonoffsetting veto in exchange economies.

$a \notin P(u'_i, u'_{-i})$ for all feasible u'_{-i}.[12] In fact this SCC can be attained with rational aspirations in an adequate way, if the set $Z_i(a)$ consists of allocations on or below a hyperplane passing through a and with a slope that is identical to the MRT evaluated at a.

(c) *Economies with Public Goods.* Nonoffsetting veto is not satisfied. The graphical intuition behind this is easily seen in a picture like Fig. 1, in which the Edgeworth box is substituted by the Kolm triangle. At any Pareto efficient allocation the sum of all marginal rates of substitution between the public and private good must be equal to the marginal cost of production of the public good. If $a \in P(u)$ but $a \notin P(u'_i, u_{-i})$ for some i, it must be because the MRS of individual i has changed for that particular allocation. But there are suitable changes in the utility functions of the rest of the agents to make a Pareto efficient again.

2. *Core Correspondence*

It behaves just like Pareto correspondence does.

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[12] If a is on the boundary, and $l_i^* = 0$ for some i, then $MRS_i \geqslant MRT$. If $a \notin P(u'_i, u_{-i})$, then $MRS'_i < MRT$ since the first-order conditions are necessary and sufficient. But then, $a \notin P(u'_i, u'_{-i})$ for all feasible u'_{-i}. Thus, nonoffsetting veto holds. The cases where $l_i^* = \bar{l}$ or $x_i^* = 0$ are dealt with similarly.

3. Walrasian Correspondence

(a) *Exchange Economies.* With only two goods, nonoffsetting veto is satisfied, unless the Walrasian equilibrium of some economy occurs at the initial endowments. The allocation a in Fig. 2 is a Walrasian equilibrium for economy (u_1, u_2) with initial endowments at e. If individual one changes her utility function to u_1', there is no utility function u_2' such that a is a Walrasian equilibrium for the economy (u_1', u_2'). However, with more than two goods, nonoffsetting veto is not satisfied, as the following example shows. Let us suppose that there are two agents and three goods. Good three is the numeraire. Utility functions and initial endowments are $u_1(x_{11}, x_{12}, x_{13}) = x_{11}^{1/3} x_{12}^{1/3} x_{13}^{1/3}$, $u_2(x_{21}, x_{22}, x_{23}) = x_{21}^{1/3} x_{22}^{1/3} x_{23}^{1/3}$, $e_1 = (2, 1, 0)$, and $e_2 = (0, 1, 2)$. For this economy the Walrasian equilibrium allocation is $((1, 1, 1), (1, 1, 1))$. Now let $u_1'(x_{11}, x_{12}, x_{13}) = x_{11}^{1/4} x_{12}^{1/2} x_{13}^{1/4}$ and $u_2'(x_{21}, x_{22}, x_{23}) = x_{21}^{1/4}, x_{22}^{1/2} x_{23}^{1/4}$. Clearly $((1, 1, 1), (1, 1, 1))$ is not a Walrasian equilibrium for (u_1', u_2) or (u_1, u_2') since it is not Pareto efficient. However this allocation is a Walrasian equilibrium for (u_1', u_2') since $(1, 1, 1)$ maximizes u_1' and u_2' at prices $(1, 2, 1)$.
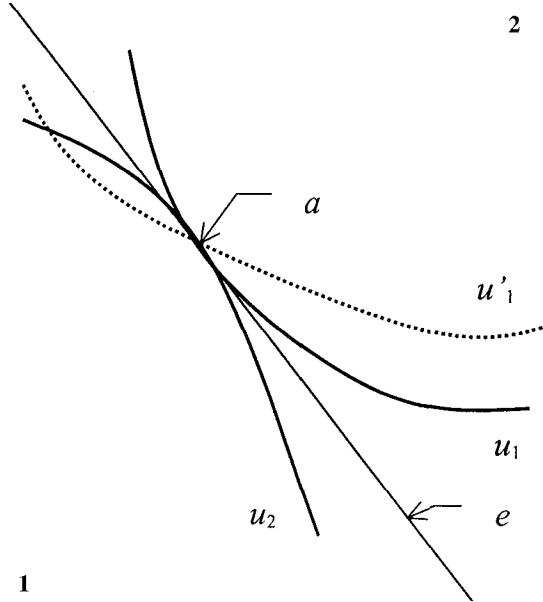

FIG. 2. With two goods, the Walrasian correspondence satisfies nonoffsetting veto in exchange economics.

(b) *Production Economies.* This correspondence satisfies nonoffsetting veto. The proof is similar to that for the case of the Pareto correspondence. Rational aspiration functions are constructed similarly.

(c) *Economies with Public Goods.* In this case, the concept that is parallel to Walrasian equilibrium is that of a Lindahl equilibrium. The same argument that is used to show that the Walrasian correspondence satisfies nonoffsetting veto in the case of two goods can be used here to show that the Lindahl equilibrium also satisfies nonoffsetting veto, provided that the Lindahl allocation is not at the initial endowments. If the Lindahl allocation occurs at the initial endowments, it is easy to show that nonoffsetting veto does not hold.[13]

4. *Envy-Free Correspondence.* It satisfies nonoffsetting veto in the case of exchange economies, production economies, and economies with public goods. Its intersection with the Pareto correspondence, however, only satisfies nonoffsetting veto in the case of production economies (provided it is nonempty; see Piketty [22]). In this case, the proof is identical to the one we used with the Pareto correspondence in production economies.

5. *Egalitarian-Equivalent Correspondence.* This correspondence does not satisfy nonoffsetting veto in any of the three models. Figure 3 illustrates this in the case of exchange economies (similar examples can be constructed for the case of production economies and economies with public goods). The initial economy is (u_1, u_2) and $a = (a_1, a_2)$ is an egalitarian-equivalent allocation with reference bundle z but is not egalitarian-equivalent either in (u'_1, u_2) or in (u_1, u'_2). In (u'_1, u'_2) however, the allocation a is egalitarian-equivalent with reference bundle z'. Note that the egalitarian-equivalent correspondence is attainable in an adequate way if aspirations are not decentralized (take the aspiration bundle as the bundle where the indifference curves of *all* individuals cross).

6. *Proportional Solution* (Roemer and Silvestre [27]). This correspondence is only defined in production economies and must fulfill two requirements. First, all chosen allocations must be Pareto efficient. Second, the consumption of each individual is proportional to her input contribution. This correspondence satisfies nonoffsetting veto. If $\hat{a}$ is a proportional

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[13] In the case in which the Lindahl equilibrium occurs at a boundary allocation, we have two cases. If $y^* = 0$, the Lindahl equilibrium occurs at the initial endowments, a case discussed above. If $x_i^* = 0$ for some i but $y^* > 0$, the budget constraint uniquely determines the Lindahl price p_i. Thus, if for some u'_i, the allocation is not a Lindahl equilibrium, this is because the bundle of i does not maximize u'_i at price p_i. Clearly, no change in the utility functions of the other agents can make the bundle to maximize u'_i. Thus, nonoffsetting veto holds.
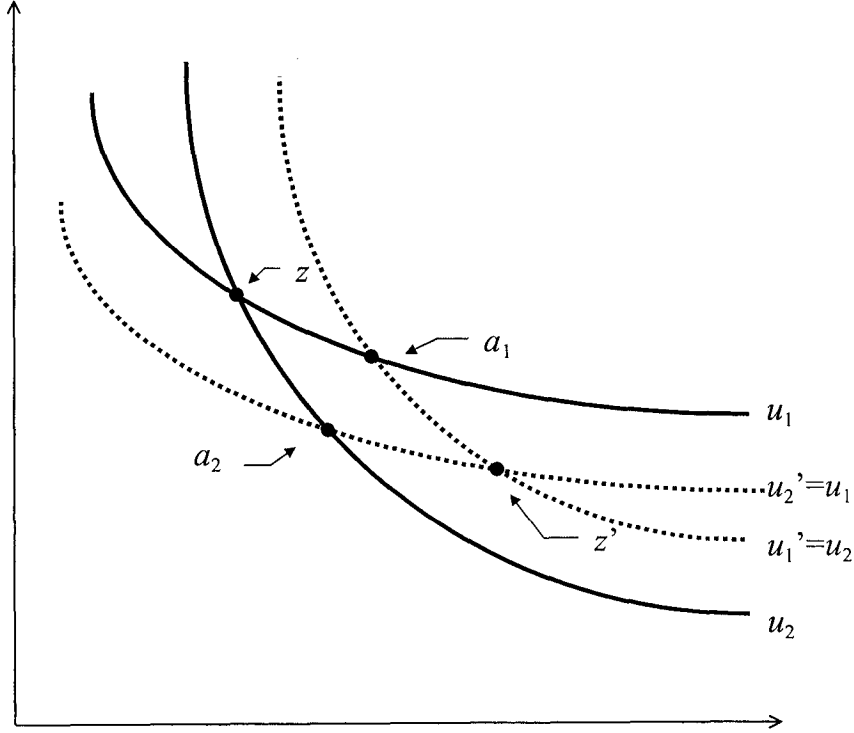
FIG. 3. The egalitarian-equivalent correspondence does not satisfy nonoffsetting veto in exchange economies.

allocation in the economy u but not in the economy (u_i', u_{-i}) it must be because it is not Pareto efficient in the latter, since the proportions have not changed. We can use the same argument as for the Pareto correspondence to show that $\hat{a}$ cannot be Pareto efficient for any economy in which u_i' is present.

7. All SCCs that select the set of allocations in which each individual is at least as well-off as in some *fixed* bundle, as in the equal split lower bound or the set of individually rational allocations, satisfy nonoffsetting veto.

If aspirations functions are rational, nonoffsetting veto is still a necessary condition of attainability but it is no longer sufficient, as shown in the following example:

EXAMPLE 3. Let us consider an exchange economy with two goods and two agents. Aggregate endowments are 2 of each good. $U_1 = \{u_1, u_1'\}$ where

u_1 is Cobb–Douglas and u'_1 is of the Leontieff type, with the kink at the 45° line. $U_2 = \{u_2\}$ where u_2 is Cobb–Douglas. Consider the following SCC: $S(u_1, u_2) = a = \{(1, 1), (1, 1)\}$ and $S(u'_1, u_2) = b = \{(0.5, 0.5), (1.5, 1.5)\}$. This SCC trivially satisfies nonoffsetting veto but cannot be attained in a satisfactory way with rational aspirations, since, if this were the case, $Z_1(a)$ would be included in the lower contour set of u_1 evaluated at a. But $Z_1(a)$ is also included in the lower contour set of u'_1 evaluated at a, and a is thus a Satisfactory allocation at (u'_1, u_2) as well.

We now focus on the study of the relationship between attainability and several different properties that have been proposed to characterize SCCs, Maskin monotonicity, consistency, and population monotonicity. The property of technological monotonicity requires an adaptation of the model presented in this paper and is relegated to an appendix.

The role of lower contour sets in Example 3 suggests that there is a connection between attainability with rational aspiration functions and the property of Maskin Monotonicity (see Masking [13]). We shall show that this is indeed the case.

DEFINITION 11. A SCC is Maskin monotonic (MM) if

$$\{a \in S(u), u_i(a_i) \geqslant u_i(a'_i) \text{ implies } u'_i(a_i) \geqslant u'_i(a'_i) \text{ for all } i\} \rightarrow \{a \in S(u')\}.$$

PROPOSITION 1. *Let S be a SCC that is attainable in a satisfactory way with rational aspirations. Then S is Maskin monotonic.*

Proof. Let $a \in S(u)$, where S is attainable in a satisfactory way with rational aspiration functions. Then

$$u_i(a_i) \geqslant u_i(z_i) \qquad \text{for all} \quad z_i \in Z_i(a).$$

Let us now consider a new economy u', which is a transformation of u of the type considered in the property of MM. By the antecedent of MM,

$$u'_i(a_i) \geqslant u'_i(z_i) \qquad \text{for all} \quad z_i \in Z_i(a).$$

But then, a is also a satisfactory allocation for u', and thus $a \in S(u)$, which implies that S is MM. $\blacksquare$

If S is an SCC that is attainable in an Adequate way with rational aspiration functions, S need not be MM. Constructing an example in exchange economies with 2 agents is quite simple. Let $a = S(u_1, u_2)$ with $u_i(a_i) = u_i(\psi_i(a, u_i))$ for $i = 1, 2$. Now choose u' such that $u'_i(a_i) \neq u'_i(\psi_i(a, u'_i))$ for some i.

In economic environments with $n > 2$, MM is a sufficient condition for Nash implementation. Thus, under the above conditions, attainability in a Satisfactory way implies Nash implementation.

We conclude our discussion on MM pointing out two important factors: First, that attainability with decentralized aspiration functions does not imply MM: In Example 3 we presented a SCC that satisfied nonoffsetting veto, and thus, by Theorem 2, was attainable. It did not satisfy MM, however. Secondly, the converse to Proposition 1 does not hold, i.e., MM does not imply attainability in a satisfactory way with rational aspiration functions: The Pareto correspondence in exchange economies, for instance, is MM but does not satisfy nonoffsetting veto, which is a necessary condition for a SCC to be attainable in a satisfactory way.

Consistency is another property that has been used to characterize SCCs.[14] It restricts the behavior of a SCC when dealing with economies with a variable number of agents. A SCC is consistent if whenever it chooses an allocation a for an economy with a set of agents N, then for any subgroup N' of N, it chooses the restriction of a to N' for the "reduced economy" with a set of agents N': this reduced economy is derived from the original one by attributing the corresponding components of a to the individuals in the complementary subgroup $N \backslash N'$. To present this property formally we need some additional notation. In particular, when the set of agents is N we shall use the notation A^N for the set of feasible allocations, a^N for a given allocation, U^N for the set of admissible economies and u^N for a given economy. We also need the following preliminary definition:

DEFINITION 12. The reduced economy of $u^N \in U^N$ relative to $N' \subset N$ and $a^N \in A^N$ is an economy, denoted by $r_{N'}^{a^N}(u^N)$, such that:

(i) The set of agents is N'.

(ii) The profile of utility functions is $u^{N'}$ where $u_i^N = u_i^{N'}$ for all $i \in N'$.

(iii) The set of feasible allocations $A^{N'}$ comprise all the alternatives at which all the individuals of the complementary subgroup $N \backslash N'$ receive their components of a^N.

DEFINITION 13. A SCC is consistent (CONS) if

$$\{a^N \in S(u^N)\} \rightarrow \{a^{N'} \in S(r_{N'}^{a^N}(u^N)) \text{ where } a_i^N = a_i^{N'} \text{ for all } i \in N'\}.$$

[14] See Thomson [33] for an excellent introduction to the applications of this property.

Consider the following assumption on the correspondence $Z_i(a)$.

Assumption 3. Let $N' \subset N$, $a^N \in A^N$, and $a^{N'} \in A^{N'}$ such that for all $i \in N'$, $a_i^N = a_i^{N'}$. Then $Z_i(a^{N'}) \subseteq Z_i(a^N)$.

This assumption says that in the reduced economy no individual will enlarge the set of bundles she thinks she is entitled to. This seems a reasonable property: If i forms $Z_i(a)$ by looking at a and deciding what would be fair for her, it seems strange that when some agents leave the economy the set $Z_i(a)$ becomes larger.[15] However, consider the case in which an individual, say i, has a small bundle and the other individuals, from sympathy for i, have modest aspirations. If i leaves the economy with her bundle, the remaining agents might well revise their aspirations upwards in such a way that the above assumption is violated.

PROPOSITION 2. *Let S be a SCC which is attainable in a satisfactory way with rational aspirations satisfying Assumption* 3. *Then S is consistent.*

Proof. Let $u^N \in U^N$ and $a^N \in S(u^N)$. If S is attainable in a satisfactory way, there are aspiration functions ψ_i such that

$$u_i(a_i^N) \geqslant u_i(\psi_i(a^N, u_i^N)) \geqslant u_i(z_i) \qquad \text{for all} \quad z_i \in Z_i(a).$$

Let $N' \subset N$ and consider the reduced economy $r_{N'}^{a^N}(u^N)$. Since $a_i^N = a_i^{N'}$ for all $i \in N'$, by Assumption 3 we have that

$$u_i(a_i^{N'}) = u_i(a_i^N) \geqslant u_i(\psi_i(a^N, u_i^N)) \geqslant u_i(\psi_i(a^{N'}, u_i^N)) \qquad \text{for all} \quad i \in N'.$$

Thus, $a^{N'} \in S(r_{N'}^{a^N}(u^N))$ and S is consistent. ∎

Remark 1. If, in Assumption 3, we write $Z_i(a^{N'}) = Z_i(a^N)$, Proposition 3 also holds for any SCC that is attainable in an adequate way.

Note that the converse of Proposition 2 does not hold: the Pareto SCC in exchange economies for instance is not attainable but it is consistent.

The property of population monotonicity also deals with changes in the number of agents. It requires that "when additional agents arrive, and the profile of welfare levels chosen by the solution for the initial group remains feasible only by "ignoring the newcomers," then none of the agents initially present gain" (Thomson [32]). Conversely, if some individuals leave the economy carrying no resources at all, none of the remaining agents loses. We state this property formally.

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[15] The aspiration functions implicit in the envy-free correspondence and in the concept of super-equity (see below Definition 8) satisfy Assumption 3.

DEFINITION 14. Let $N' \subset N$ and $A^{N'} = A^N$. A SCC is population monotonic (POPMON) if for all $i \in N'$ and for all $a^{N'} \in S(u^{N'})$ and $a^N \in S(u^N)$, $u_i(a_i^{N'}) \geqslant u_i(a_i^N)$.

Assumption 4. Let $N' \subset N$, $a^N \in A^N$, and $\hat{a}^{N'} \in A^{N'}$. Then, $Z_i(a^N) \subseteq Z_i(\hat{a}^{N'})$ for all $i \in N'$.

Assumption 4 states two things. First, that the bundles to which each individual thinks she is entitled, do not depend on the particular allocation, but rather on the number of individuals included in the economy. Second, that no individual becomes greedier when more individuals arrive and the resources do no change. We must point out that this is a less reasonable property than the one stated in Assumption 3, since in Assumption 4 the inclusion has to hold for every allocation.

PROPOSITION 3. *If S is a SCC that is attainable in an adequate way with rational aspirations satisfying Assumption 4, then S is POPMON.*

Proof. Let $a^N \in S(u^N)$. As S is attainable in an adequate way, there are aspiration functions ψ_i such that for all i, $u_i(a_i^N) = u_i(\psi_i(a^N, u_i^N))$ where $\psi_i(a^N, u_i^N) = \arg\max u_i^N(z_i)$ for all $z_i \in Z_i(a^N)$. Let $\hat{a}^{N'} \in S(u^{N'})$. Again there are aspiration functions ψ_i such that for all i, $u_i(\hat{a}_i^{N'}) = u_i(\psi_i(\hat{a}^{N'}, u_i^{N'}))$ where $\psi_i(\hat{a}^{N'}, u_i^{N'}) = \arg\max u_i^{N'}(z_i)$ for all $z_i \in Z_i(a^{N'})$. By Assumption 4 we know that $Z_i(a^N) \subseteq Z_i(\hat{a}^{N'})$ which in turn implies $u_i(\psi_i(a^N, u_i^N)) \leqslant u_i(\psi_i(\hat{a}^{N'}, u_i^{N'}))$ and thus $u_i(a_i^N) \leqslant u_i(\hat{a}_i^{N'})$ for all $i \in N'$. Then S is POPMON. ∎

Assumption 4 is strong but, without it, Proposition 3 does not hold. In such a case the set $Z(\cdot)$ of some individual shrinks when other individuals leave the economy without any resources at all. If the SCC is adequate attainable it might be that the maximum utility attainable within a smaller set is less than it was initially.

We end our discussion of POPMON by noting two things. First, the converse of Proposition 3 does not hold. For instance, the egalitarian-equivalent correspondence is POPMON, but is not attainable in an adequate way. Second, even if Assumption 4 holds, a satisfactory attainable SCC is not necessarily POPMON, as shown by the following example.

EXAMPLE 4. Consider a pure exchange economy with aggregate endowment ω. Let $N = \{1, 2, 3\}$ and assume all individuals have Cobb–Douglas utility functions. For all $a^N \in A^N$, let $Z_1(a^N) = \{a_1^N \mid a_1^N \leqslant \frac{1}{5}\omega\}$, $Z_2(a^N) = \{a_2^N \mid a_2^N \leqslant \frac{1}{3}\omega\}$, and $Z_3(a^N)$ be exactly the same as $Z_2(a^N)$. Assume that the only allocation chosen by the SCC is the one that gives $\frac{1}{3}$ of ω to each individual. Now let $N' = \{1, 2\}$ and $Z_1(a^{N'}) = Z_1(a^N)$ but $Z_2(a^{N'}) = \{a_2^{N'} \mid a_2^{N'} \leqslant \frac{3}{4}\omega\}$. Because the SCC is attainable in a satisfactory way, it

must assign to individual 2 a bundle such that her utility is at least as high as with $\frac{3}{4}$ of the aggregate endowment. But this implies that individual one loses with the departure of individual 3.

Finally, we must point out that if we strengthen Assumption 4 by writing $Z_i(a^N) = Z_i(\hat{a}^{N'})$, any SCC that is attainable in a satisfactory way is also POPMON.

4. WEAK ATTAINABILITY

We now turn our attention to weak attainability. The following condition is necessary for a SCC to be weakly attainable in an unbiased way and, in the two-agent case, it is also sufficient. Unfortunately sufficiency fails with more than two individuals.

DEFINITION 15. A SCC satisfies selective offsetting veto if for all $a \in S(u)$ and $u' = (u'_1, ..., u'_n)$ such that either $a \in S(u'_i, u_{-i})$ for all i or $a \notin S(u'_i, u_{-i})$, but $a \in P(u'_i, u_{-i})$ for all i, then $a \in S(u')$.

Selective offsetting veto states two things. First, if nobody vetoes an allocation when utility functions are changed one by one, they will not veto when all the utility functions change simultaneously. Second, if every individual vetoes an allocation when they change their utility functions one by one, but this allocation remains Pareto efficient, this veto is offset when all individuals make such a change simultaneously.

Consider the following assumption:

Assumption 5. (a) Let $a \in S(u)$. Then $u_i(a_i) > u_i(\underline{a}_i)$ for all i and $a_i \in int\ X_i$.

(b) Utility functions are continuously differentiable and concave.

THEOREM 3. *Suppose the aspiration functions are decentralized and Assumption 5 holds. Then*

(a) *Selective offsetting veto is necessary for any SCC to be weakly attainable in an unbiased way;*

(b) *under Assumption 2 and $n = 2$ it is also sufficient;*

(c) *with more than two agents it is not a sufficient condition.*

Proof. Under Assumption 5 we have

(i) For all profiles u, $u' \in U$ such that $a \in P(u)$ and $a \in P(u'_i, u_{-i})$ for all i, it must be that $a \in P(u')$.

TABLE I

The Case $a \in S(u)$

	u_2	u_2'		
u_1	$a \in S(u_1, u_2)$ ($\gg$)	$a \in S(u_1, u_2')$ ($\gg$)	$a \notin S(u_1, u_2')$ ($> <$)	$a \notin P(u_1, u_2')$ ($> <$ or $\gg$)
u_1'	$a \in S(u_1', u_2)$ ($\gg$)	$a \in S(u_1', u_2')$ ($\gg$)	$a \notin S(u_1', u_2')$ ($> <$)	$a \notin P(u_1', u_2')$ ($\gg$ or $> <$)
	$a \notin S(u_1', u_2)$ ($< >$)	$a \notin S(u_1', u_2')$ ($< >$)	$a \in S(u_1', u_2')$ ($\ll$)	$a \notin P(u_1', u_2')$ ($< >$ or $> <$ or $\gg$) $a \in S(u_1', u_2')$ ($\gg$)
	$a \notin P(u_1', u_2)$ ($\gg$ or $< >$)	$a \notin P(u_1', u_2')$ ($\gg$ or $< >$)	$a \notin P(u_1', u_2')$ ($\gg$ or$< >$ or $> <$)	$a \notin S(u_1', u_2')$ ($< >$ or $> <$) $a \notin P(u_1', u_2')$ ($\gg$ or $< >$ or $> <$)

Note. For the sake of brevity, in Tables I and II $a \notin S(u)$ means $a \notin S(u)$ but $a \in P(u)$.

(ii)   For all profiles, u, $u' \in U$ such that $a \in P(u)$, $a \in P(u_i', u_{-i})$ for some i and $a \notin P(u_j', u_{-j})$ for some j, it must be that $a \notin P(u')$.[16]

   (a)   Suppose S is weakly attainable in an Unbiased way via some decentralized aspiration functions ψ, $a \in S(u)$ and there is $u' = (u_1', ..., u_n')$ such that $a \in S(u_i', u_{-i})$ for all i. By definition of weak attainability $a \in P(u_i', u_{-i})$ for all i and thus, $a \in P(u')$. Suppose that $u_i(a_i) \geqslant u_i(\psi_i(a, u_i))$ for all i. If $a \in S(u_i', u_{-i})$ it must be that $u_i'(a_i) \geqslant u_i'(\psi_i(a, u_i'))$. As this is the case for all i and $a \in P(u')$, it holds that $a \in S(u')$. If $a \notin S(u_i', u_{-i})$ but $a \in P(u_i', u_{-i})$ it must be that $u_i'(a_i) < u_i'(\psi_i(a, u_i'))$. As this is the case for all i, and again $a \in P(u')$, then $a \in S(u')$. The case in which $u_i(a_i) < u_i(\psi_i(a, u_i))$ for all i is dealt with similarly.

   (b)   Let $a \in S(u_1, u_2)$ for some (u_1, u_2). Consider the economies (u_1', u_2), (u_1, u_2') and (u_1', u_2'). We illustrate this in Table I. In brackets we write how aspiration functions should be constructed. For example, ($\gg$) means we choose aspiration functions ψ such that $u_i(a_i) > u_i(\psi_i(a, u_i))$ for all i. We start the first cell with ($\gg$) but we could start with ($\ll$) instead. Table II illustrates the case in which $a \notin S(u)$ but $a \in P(u)$. The case where $a \notin P(u)$ and, thus, $a \notin S(u)$, is similar to the one dealt with in Table I.

[16] It is easy to present examples where, by dropping the conditions on Assumption 5, either (i) or (ii) fails.

TABLE II

The Case of $a \notin S(u)$ but $a \in P(u)$

	u_2	u_2'		
u_1	$a \notin S(u_1, u_2)$ $(<>)$	$a \in S(u_1, u_2')$ $(\ll)$	$a \notin S(u_1, u_2')$ $(<>)$	$a \notin P(u_1, u_2')$ $(<>$ or $\ll)$
	$a \in S(u_1', u_2)$ $(\gg)$	$a \notin S(u_1', u_2')$ $(><)$	$a \in S(u_1', u_2')$ $(\gg)$	$a \notin P(u_1', u_2')$ $(\gg$ or $><)$
	$a \notin S(u_1', u_2)$ $(<>)$	$a \in S(u_1', u_2')$ $(\ll)$	$a \notin S(u_1', u_2')$ $(<>)$	$a \notin P(u_1', u_2')$ $(><$ or $\gg)$
u_1'				$a \in S(u_1', u_2')$ $(\gg$ or $\ll)$
	$a \notin P(u_1', u_2)$ $(<>$ or $\gg)$	$a \notin P(u_1', u_2')$ $(\ll$ or $><)$	$a \notin P(u_1', u_2')$ $(<>$ or $\gg)$	$a \notin S(u_1', u_2')$ $(<>$ or $><)$
				$a \notin P(u_1', u_2')$ $(<>$ or $\gg$ or $\ll)$

(c) We present an example with three agents: Suppose $a \in S(u_1', u_2, u_3)$, $a \in S(u_1, u_2', u_3)$, $a \in S(u_1, u_2, u_3')$, and $a \notin S(u_1, u_2, u_3)$ but $a \in P(u_1, u_2, u_3)$. Note that this example does not contradict selective offsetting veto. To get rid off a in the economy (u_1, u_2, u_3) we need aspiration functions such that $u_i(a_i) \geq u_i(\psi_i(a, u_i))$ and $u_j(a_j) < u_j(\psi_j(a, u_j))$ for some pair i, j with $j \neq i$. Without loss of generality, suppose that $i = 1$ and $j = 2$. But then a cannot be weakly attained in the economy (u_1, u_2, u_3'). ∎

We now verify whether the SCCs considered in the case of attainability satisfy selective offsetting veto under Assumption 5.

1. *Pareto Correspondence.* It trivially satisfies selective offsetting veto.

2. *The Core Correspondence.* It satisfies selective offsetting veto in exchange economies when $n = 2$. When the first antecedent applies, the consequent is always true. The second antecedent can never occur. We leave whether or not it is satisfies for $n > 2$ as an open problem.

3. *Walrasian Correspondence.* This correspondence satisfies selective offsetting veto. As in the previous case, only the first antecedent can occur.

4. *Envy-Free Correspondence.* Since the envy-free correspondence is not a subset of the Pareto correspondence it is not weakly attainable. Thus, let us consider the intersection of the envy-free and the Pareto correspondence (assuming that it is nonempty). It is obvious that in this case, selective offsetting veto is satisfied.

5. *Egalitarian Equivalent Correspondence.* Again, the egalitarian-equivalent correspondence is not a subset of the Pareto correspondence and, thus, it is not weakly attainable. Moreover, the intersection between the egalitarian-equivalent correspondence and the Pareto correspondence does not satisfy selective offsetting veto. Figure 4 illustrates this in the case of exchange economies. The allocation $a = (a_1, a_2)$ is egalitarian-equivalent in the economy (u_1, u_2) and it is also Pareto efficient. In (u'_1, u_2) and (u_1, u'_2) it is not egalitarian-equivalent but it is indeed Pareto efficient. In (u'_1, u'_2), however, although it is Pareto efficient it is not egalitarian-equivalent.

6. *The Proportional Solution.* It satisfies trivially selective offsetting veto.

7. All SCCs that select the set of allocations in which each individual is at least as well off as in some *fixed* bundle, as in the equal split lower
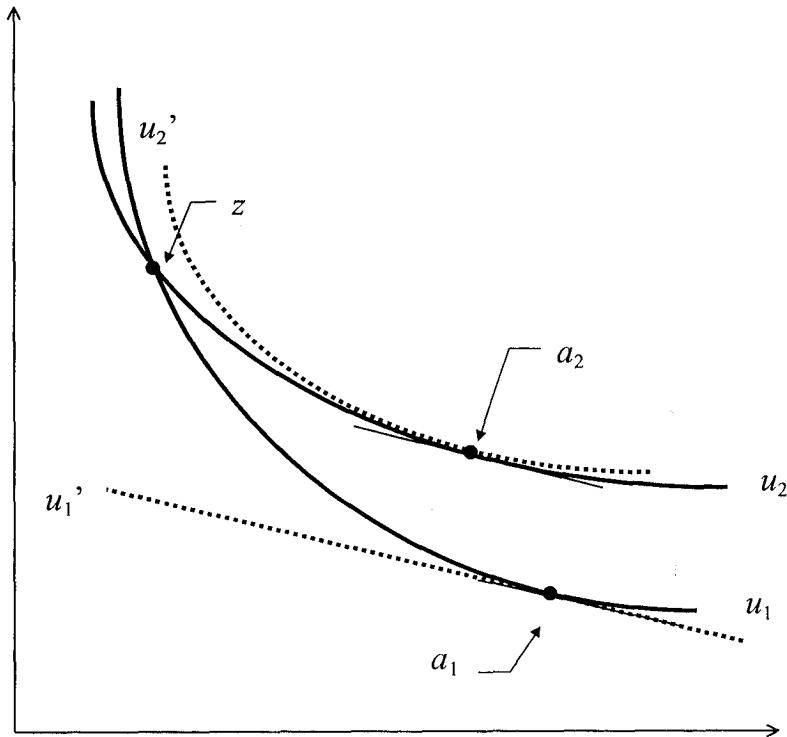

FIG. 4. The intersection between the Pareto correspondence and the egalitarian-equivalent correspondence does not satisfy selective offsetting veto in exchange economies.

bound or the set of individually rational allocations which satisfy selective offsetting veto.

As in the previous section we study the relationship between weakly attainability and some standard properties from the literature on SCCs.

We consider first MM. We find that MM neither implies Weak Attainability nor the converse. Figure 5 illustrates the first assertion in an example with two individuals and two goods. In both the economy (u_1, u_2) and the economy (u'_1, u_2) the unique allocation chosen by some SCC is a. Note that the second economy is a monotonic transformation of the first one. However, a is an Unbiased allocation for economy (u_1, u_2) although it is not for economy (u'_1, u_2). Aspiration functions are constant at the points drawn in the figure.

Figure 6 illustrates the second assertion. Again we assume that aspiration functions are constant at the points represented in the figure. The SCC selects a in the economy (u_1, u_2) and b in the economy (u'_1, u_2) where u'_1 is of the Leontieff type. Both allocations are unbiased in their respective economies. MM is violated however, since, according to this property, a must be chosen in the economy (u'_1, u_2).

The second property is consistency. We can provide a result similar to the one in Remark 1: If in Assumption 3 we write $Z_i(a^{N'}) = Z_i(a^N)$, Proposition 2 also holds for a SCC attainable in an unbiased way. Once
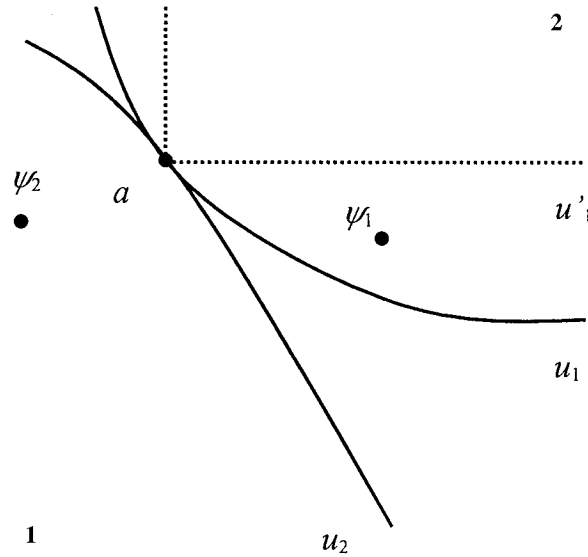

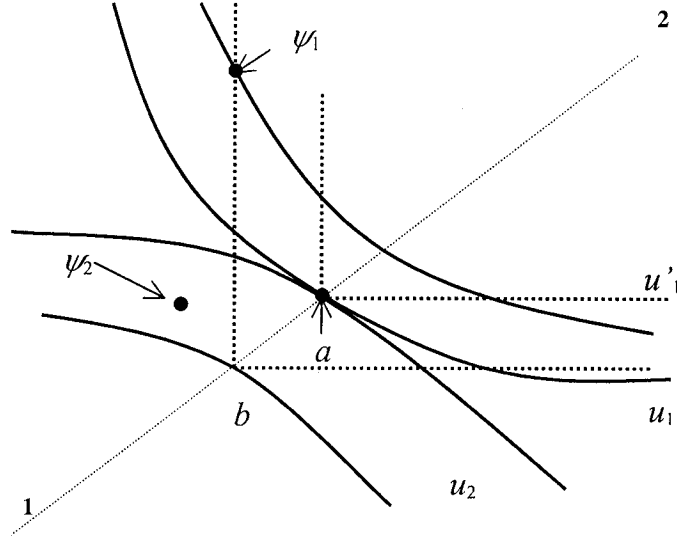FIG. 5. Maskin monotonicity does not imply weak attainability.

FIG. 6. Weak attainability does not imply Maskin monotonicity.

again, the converse is not true: The intersection of the Pareto correspondence and the egalitarian-equivalent correspondence is consistent but it is not weakly attainable in an unbiased way since it fails selective offsetting veto.

The third property is population monotonicity. We obtain a result that is similar to the case of satisfactory attainability. If we strength Assumption 4 by requiring $Z_i(a^N) = Z_i(\hat{a}^{N'})$ it is easy to show that any unbiased attainable SCC is POPMON. Even in this case, the converse is not true, as shown in Example 4.

5. FINAL COMMENTS

In this paper we have proposed a general method for representing the aspirations of individuals. We have argued that many concepts of justice can be understood in terms of aspiration fulfillment. Moreover, properties used to characterize social choice correspondences, like consistency and population monotonicity, can be obtained by imposing certain properties to aspirations. This suggests that imposing properties to aspirations may be a fruitful way to identify new solutions or, perhaps, to obtain impossibility results.

We end this paper suggesting two possible connections of our approach. On the one hand, the idea that concepts of justice should be derived from individuals may be connected with notions like Harsanyi's veil of

ignorance, Arrow's extended sympathy or Sen's theory of capabilities and related axiomatics of economic opportunities (see Ok and Kranich [18] and the papers cited there). On the other hand, our notion of an aspiration is close to that of an objection in the theory of cooperative games (see Osborne and Rubinstein [19], chapter 14) and to individual bargaining solution functions (see Van Damme [36]).

APPENDIX: RESOURCE/TECHNOLOGICAL MONOTONICITY (RMON)

In this appendix we study the connection between the properties of resource and technological monotonicity and our concepts of attainability. Resource monotonicity states that when the aggregate endowment in the economy grows, no agent must be adversely affected. Technological monotonicity states that when the technology improves, no agent must be adversely affected.[17] In this appendix, we offer a unified treatment of technological and resource monotonicity.

Throughout this appendix, we fix the profile of utility functions $u = (u_1, ..., u_n)$. Let ω define a parameter representing the aggregate endowment or the technology of the economy. The set of economies is Ω and ω a particular economy. The feasible set for a given ω is denoted by $A(\omega)$. Define $\mathscr{A} = \bigcup_{\omega \in \Omega} A(\omega)$. The set of Pareto efficient allocations for given ω is denoted by $P(\omega)$. A SCC S is defined as $S: \Omega \to \mathscr{A}$, where $S(\omega) \subseteq A(\omega)$.

DEFINITION 16. A SCC S satisfies resource/technological monotonicity (RESMON) if

$$\{a \in S(\omega), A(\omega) \subseteq A(\omega')\} \to \{\forall a' \in S(\omega'), u_i(a_i) \leqslant u_i(a_i') \text{ for all } i\}.$$

We will use the notation $Z_i(a, \omega)$ to stress the dependent of ω.

ASSUMPTION 6. Let $\omega, \omega' \in \Omega$ such that $A(\omega) \subseteq A(\omega')$. Then $Z_i(a, \omega) \subseteq Z_i(a, \omega')$ for all a and for all i.

This assumption precludes the possibility that the set of bundles that i thinks she is entitled to, shrinks when there is an improvement in the aggregate resources or in the technology of the economy. We now have the following result:

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[17] See Roemer [25] and Chun and Thomson [4].

PROPOSITION 4. *If S is an adequate attainable SCC with rational aspirations satisfying Assumption* 6, *then S is RESMON.*

Proof. The proof of this proposition runs parallel to that of Proposition 3.

As in the case of POPMON, a SCC that is either attainable in a satisfactory way or weakly attainable in an unbiased way is not necessarily RESMON (we can use an example similar to Example 4 to prove it).

We now study the converse of the above proposition under the following assumption.

Assumption 7. (i) For all i, X_i is convex and $0 \notin X_i$.

(ii) $A(\omega)$ is convex.

(iii) Utility functions are strictly increasing, continuous, strictly quasi-concave, and such that, for all a_i, $u_i(\alpha a_i) \to \infty$ when $\alpha \to \infty$.

THEOREM 4. *Suppose Assumption* 7 *holds. Let S be a social choice function with* $S(\omega) \in P(\omega)$. *Then S is adequate and satisfactory attainable and weakly unbiased attainable with aspiration functions that are rational, increasing and continuous in a. Moreover, if S is RESMON, the sets* $Z_i(a, \omega)$ *satisfy Assumption* 6 *for all i.*

Proof. Let $\omega = \hat{\omega}$ and $\hat{a} = S(\hat{\omega})$. For all i and for all $a \in A(\hat{\omega})$ define $\lambda_i(a, \hat{a}) \in R$ such that $u_i(\lambda_i(a, \hat{a}) \hat{a}_i) = u_i(a_i)$. For any feasible allocation a, $\lambda_i(a, \hat{a})$ exists (because $u_i(\alpha a_i) \to \infty$ when $\alpha \to \infty$) and is unique (because u_i is strictly increasing). Moreover, if $u_i(a_i) > u_i(\hat{a}_i)$ (resp. $u_i(a_i) < u_i(\hat{a}_i)$) then $\lambda_i(a, \hat{a}) > 1$ (resp. $\lambda_i(a, \hat{a}) < 1$). Now define for all i aspiration functions as

$$\psi_i(a, \hat{\omega}) = (\alpha + (1 - \alpha) \lambda_i(a, \hat{a})) \hat{a}_i \qquad \text{with} \quad 0 < \alpha < 1.$$

These aspiration functions attach to every allocation a point that lies in the path that connects $\hat{a}_i$ and the origin. Specifically, it is located between $\hat{a}_i$ and $\lambda_i(a, \hat{a}) \hat{a}_i$. Since $\hat{a}_i$ and $\lambda_i(a, \hat{a}) \hat{a}_i$ belong to X_i, convexity of X_i implies that $\psi_i(a, \hat{\omega})$ belongs to X_i. The numbers $\lambda_i(a, \hat{a})$ are continuous in a and $\hat{a}$ (by continuity of u_i) and thus $\psi_i(a, \hat{\omega})$ is continuous in a. Since $\lambda_i(a, \hat{a})$ is increasing in $a, \psi_i(a, \hat{\omega})$ is also increasing in a. These aspirations are rational by taking $Z_i(a, \omega) = \{a_i \mid u_i(a_i) \leq u_i(\psi_i(a, \hat{\omega}))\}$ for all i (a tie-breaking rule must be chosen to select $\psi_i(a, \hat{\omega})$ when indifference occurs).

Let $\hat{a} = S(\hat{\omega})$. Then $\lambda_i(\hat{a}, \hat{a}) = 1$ for all i and thus $\psi_i(\hat{a}) = \hat{a}_i$ and $u_i(\hat{a}_i) = u_i(\psi_i(\hat{a}, \omega))$ for all i. Therefore, $\hat{a}$ is an adequate, satisfactory, and unbiased allocation.

Let $a \neq \hat{a} = S(\hat{\omega})$. There are three cases:

(i) $u_i(a_i) \geqslant u_i(\hat{a}_i)$ for all i, with at least one strict inequality. This is not possible by Pareto efficiency of S.

(ii) $u_i(a_i) = u_i(\hat{a}_i)$ for all i. Consider the allocation $\beta a + (1 - \beta) \hat{a}$, $0 < \beta < 1$. Because $A(\hat{\omega})$ and X_i are convex for all i, $\beta a + (1 - \beta) \hat{a}$ is individually and socially feasible. Since all individuals have strictly quasi-concave preferences, the allocation $\beta a + (1 - \beta) \hat{a}$ Pareto dominates the allocation $\hat{a}$ contradicting the Pareto efficiency of S.

(iii) $u_i(a_i) < u_i(\hat{a}_i)$ for some individual i. Then $\lambda_i(a, \hat{a}) < 1$ and $(1 - \alpha (1 - \lambda_i(a, \hat{a}))) < 1$. Thus, the bundle $\psi_i(a, \hat{\omega})$ is greater in every component than $\lambda_i(a, \hat{a}) a_i$ and by strict monotonicity, $u_i(a_i) = u_i(\lambda_i(a, \hat{a}) a_i) < u_i(\psi_i(a, \hat{\omega}))$. To sum up, $\hat{a}$ is the only Adequate and Satisfactory allocation. Note that a is not either Pareto efficient or unbiased and thus it can not be weakly Unbiased.

Finally, if S is RESMON, the fact that $\hat{a} = S(\hat{\omega})$ and $a' = S(\omega')$, with $A(\hat{\omega}) \subseteq A(\omega')$ implies $u_i(\hat{a}) \leqslant u_i(a_i')$ for all i. But in this case, from the definition of $Z_i(\cdot)$ above, it follows that $Z_i(a, \hat{\omega}) \subseteq Z_i(a, \omega')$. Assumption 6 is therefore satisfied. ∎

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