HANDBOOK OF MATHEMATICAL MODELS IN COMPUTER VISION

HANDBOOK OF MATHEMATICAL MODELS IN COMPUTER VISION

Edited by

Nikos Paragios
Ecole Nationale des Ponts et Chaussees

Yunmei Chen
University of Florida

Olivier Faugeras
INRIA

Springer

Library of Congress Cataloging-in-Publication Data

A C.I.P. Catalogue record for this book is available from the Library of Congress.

Handbook of Mathematical Models in Computer Vision, Edited by Nikos Paragios, Yunmei Chen and Olivier Faugeras

p.cm.

ISBN-10: (HB) 0-387-26371-3
ISBN-13: (HB) 978-0387-26371-7
ISBN-10: (eBook) 0-387-28831-7
ISBN-13: (eBook) 978-0387-28831-4
Printed on acid-free paper.

Printed in the United States of America.

9 8 7 6 5 4 3 2 1 SPIN 11055662 (HC) / 11552987 (eBook)
springeronline.com

Contents

Preface

Abstract

Biological vision is a rather fascinating domain of research. Scientists of various origins like biology, medicine, neurophysiology, engineering, mathematics, etc. aim to understand the processes leading to visual perception process and at reproducing such systems. Understanding the environment is most of the time done through visual perception which appears to be one of the most fundamental sensory abilities in humans and therefore a significant amount of research effort has been dedicated towards modelling and reproducing human visual abilities. Mathematical methods play a central role in this endeavour.

Introduction

David Marr's theory was a pioneering step towards understanding visual perception. In his view human vision was based on a complete surface reconstruction of the environment that was then used to address visual subtasks. This approach was proven to be insufficient by neuro-biologists and complementary ideas from statistical pattern recognition and artificial intelligence were introduced to better address the visual perception problem. In this framework visual perception is represented by a set of actions and rules connecting these actions. The emerging concept of active vision consists of a selective visual perception paradigm that is basically equivalent to recovering from the environment the minimal piece information required to address a particular task of interest.

Mathematical methods are an alternative to tackle visual perception. The central idea behind these methods is to reformulate the visual perception components as optimization problems where the minima of a specifically designed objective function "solve" the task under consideration. The definition of such functions is often an ill-posed problem since the number of variables to be recovered is much larger than the number of constraints. Furthermore, often the optimization process itself is ill-posed due the non-convexity of the designed function inducing the presence of local minima. Variational, statistical and combinatorial methods are three distinct and important categories of mathematical methods in computational vision.

Variational techniques are either based on the optimization of cost functions through the calculus of variations or on the design of partial differential equations whose steady state corresponds to the solution of the visual perception task. Such techniques have gained significant attention over the past decade and have been used to address image restoration and enhancement, image segmentation, tracking and stereo reconstruction among other problems. The possibility to use the calculus of variations in the optimization process is the most important strength of these methods combined with the fact that one can integrate many terms and build quite complicated objective functions at the expense of converging toward local minima.

Statistical methods often consist of two stages, a learning and an execution one. Complex conditional, multi-dimensional distributions are used to describe visual perception tasks that are learnt through a training procedure. Visual perception is then formulated as an inference problem, conditional to the observations (images). One can claim that such methods are suitable to address constrained optimization problems, in particular when the subset of solutions can be well described through a conditional parametric density function. They suffer from the curse of dimensionality, e.g. in the Bayesian case when very-high dimensional integrals have to be computed.

Discrete optimization is an alternative to the continuous case often addressed through statistical and variational methods. To this end, visual perception is often redefined as a labelling procedure at the image element level according to a predefined set of plausible classes. Such a simplification often reduces the dimensionality of the problem and makes possible the design of efficient optimization algorithms. On the other hand such methods can have limited performance because of the discretization of the solution space, in particular when the solution lives in a rather continuous in-homogeneous space. One can refer to graph-based methods for addressing such tasks.

The choice of the most appropriate technique to address visual perception is rather task-driven and one cannot claim the existence of a universal solution to most of the visual perception problems. In this edited volume, our intention is to present the most promising and representative mathematical models to address visual perception through variational, statistical and combinatorial methods. In order to be faithful to the current state of the art in visual perception, a rather complete set of computational vision components has been considered starting from low level vision tasks like image enhancement and restoration and ending at complete reconstruction of scene's geometry from images.

The volume is organized in six thematic areas and thirty-three chapters presenting an overview of existing mathematical methodologies to address an important number of visual perception tasks.

Contributions & Contributors

Image reconstruction from either destroyed or incomplete data is a crucial low level task of visual perception. Local filter operators, diffusion methods as well as variational methods are among the most studied methods in the domain. The book starts with three tutorial chapters in this thematic area. The total variation method and diffusion filters as well as image decomposition in orthogonal bases, two of the most instrumental methods to address image reconstruction are presented in the first chapter. Image inpainting/completion is a more advanced problem consisting of restoring missing information in images ; it belongs to the same family and is covered in chapter 2. In the third chapter of this thematic area, an introduction to the problem as well as the most prominent techniques from the area of variational methods are presented.

Image segmentation and object extraction are of particular interest with applications in numerous domains. In its simplest instantiation the problem consists of creating an image partition with respect to some feature space, the regions being assumed to have uniform visual structure in this space. Such a problem can be solved in many ways. Labelling is an example where the objective is to assign to the local image element the most likely hypothesis given the observation. Two chapters explore such a concept in this thematic area, the watershed transformation is one of them and combinatorial optimization through the graph-cuts paradigm is another. Evolution of curves and surfaces is an alternative method to address the same problem. Classes are represented through moving interfaces that are deforming in order to capture image regions with consistent visual properties. The snake model - a pioneering framework - is the predecessor of the methods presented. First, an overview for finding multiple contours for contour completion from points or curves in 2D or 3D images is presented using the concept of minimal paths. Then in order a method that integrate region statistics is presented within deformable models leading to a new class of deformable shape and texture models. Use of prior knowledge is important within the segmentation process and therefore in the next chapter the design of shape priors for variational region-based segmentation is presented. Segmentation through the propagation of curves through the level set method is an established technique to grouping and object extraction Therefore, methods to address model-free as well as model-based segmentation are part of this thematic area. Last, but not least, a stochastic snake model based the theory of interacting particle systems and hydrodynamic limits is presented as a new way of evolving curves as a possible alternative to level set methods.

Representing and understanding structures is an essential component of biological vision, often used as a basis for high level vision tasks. Therefore, a thematic area dedicated to shape modelling and registration is present in this volume. Shape representations of various form are explored while at the same time the notions of establishing correspondences between different structures representing the same object are presented as well as methods recovering correspondences between shapes and images.

Motion analysis is a fundamental area of computational vision and mostly consists of two problems, estimating correspondences between images and being able to track objects of interest in a sequence of images. Optical flow estimation can be addressed in different ways. In this thematic area we explore the use of parametric motion models as well as the estimation of dense correspondences between images. Furthermore, we present a compendium of existing methods to detect and track objects in a consistent fashion within several frames as well as variational formulations to segment images and track objects in several frames. Understanding the real 3D motion is a far more complicated task of computational vision in particular when considering objects that do exhibit a number of articulations. Human motion capture is an example that is presented in this thematic area. We conclude with methods going beyond objects that are able to account, describe and reproduce the dynamics of structured scenes.

Stereo reconstruction is one of the best studied tasks in high level vision. Understanding and reproducing the 3D geometry of a scene is a fundamental component of biological vision. In this thematic area the shape from shading problem i.e. that of recovering the structure of the scene from one single image is first addressed. Different methods exploring the use of multiple cameras to recover 3D from images are then presented, based on differential geometry, variational formulations and combinatorial optimization. The notion of time and dynamic behaviour of scenes is also addressed where the objective is to create 3D temporal models of the evolving geometry.

Medical image analysis is one of the most prominent application domains of computer vision and in such a constrained solution space one can develop methods that can better capture the expected form of the structures of interest. Regularization, segmentation, object extraction and registration are the tasks presented in this thematic area. Model-free combinatorial methods that aim to recover organs of particular interest, statistical methods that aim to capture the variation of anatomical structures, and variational methods that aim to recover and segment smooth vectorial images are presented. Last, but not least a comprehensive review of statistical methods to image registration is presented, a problem that consists of recovering correspondences between different modalities measuring the same anatomical structure.

In order to capture the spectrum of the different methods and present an overview of mathematical methodologies in computational vision a notable number of contributors was invited to complete such an effort. Eighty-three contributors from the academic and the industrial world, from nine different countries and thirty-eight institutions have participated in this effort. The final outcome consists of 6 thematic areas, 33 chapters, 625 pages and 929 references.

N. Paragios, Y. Chen & O. Faugeras

List of Contributors

Agrawal, Motilal
Artificial Intelligence Center
SRI International, Menlo Park, USA
mailto:agrawal@ai.sri.com
http://www.ai.sri.com/people/agrawal/

van Assen, Hans
Division of Image Processing, Department of Radiology
Leiden University, Leiden, Netherlands
mailto:H.C.van_ Assen@lumc.nl

Aubert, Gilles
Department of Mathematics
Universite de Nice/Sophia Antipolis, France
mailto:gaubert@math.unice.fr
http://math1.unice.fr/~gaubert/

Barlaud, Michel
Laboratoire I3S
CNRS-Universite de Nice/Sophia Antipolis, France
mailto:barlaud@i3s.unice.fr
http://www.i3s.unice.fr/~barlaud/

Barron, Carlos
Department of Computer Science
University of Houston, Houston, USA
mailto:cbarron@uh.edu

Bertalmío, Marcelo
Departament de Tecnologia
Universitat Pompeu Fabra, Barcelona, Spain
mailto:marcelo.bertalmio@upf.edu

http://www.tecn.upf.es/~mbertalmio

Bergtholdt, Martin
Department of Mathematics & Computer Science
University of Mannheim, Germany
mailto:bergtholdt@uni-mannheim.de
http://www.cvgpr.uni-mannheim.de/

Blake, Andrew
Microsoft Research Cambridge, UK
mailto:ablake@microsoft.com
http://www.research.microsoft.com/~ablake

Boykov, Yuri
Departament of Computer Science
University of Western Ontario, Canada
mailto:yuri@csd.uwo.ca
http://www.csd.uwo.ca/faculty/yuri/

Brox, Thomas
Faculty of Mathematics and Computer Science
Saarland University, 66041 Saarbrücken, Germany
mailto:brox@mia.uni-saarland.de
http://www.mia.uni-saarland.de/brox/

Bruckstein, Alfred M.
Computer Science Department
Technion, Haifa, Israel
mailto:freddy@cs.technion.ac.il

Caselles, Vicent
Departament de Tecnologia
Universitat Pompeu Fabra, Barcelona, Spain
mailto:vicent.caselles@upf.edu
http://www.iua.upf.es/~vcaselles/

Chan, Tony
Department of Mathematics
University of California at Los Angeles, USA
mailto:chan@math.ucla.edu
http://www.math.ucla.edu/~chan

Chen, Ting
Department of Radiology
NYU Medical School, New York, USA

mailto:ting.chen@med.nyu.edu

Chen, Yunmei
Department of Mathematics
University of Florida, Gainesville, USA
mailto:yun@math.ufl.edu
http://www.math.ufl.edu/~yun/

Cohen, Laurent
CEREMADE
Universite Paris IX Dauphine, Paris, France
mailto:cohen@ceremade.dauphine.fr
http://www.ceremade.dauphine.fr/~cohen

Cremers, Daniel
Imaging & Visualization Department
Siemens Corporate Research, Princeton, NJ, USA
mailto:daniel.cremers@scr.siemens.com
http://www.cs.ucla.edu/~cremers

Davis, Larry
Department of Computer Science
University of Maryland, College Park, USA
mailto:lsd@cs.umd.edu
http://cvl.umiacs.umd.edu/users/lsd/

Deriche, Rachid
I.N.R.I.A. Sophia Antipolis, France
mailto:Rachid.Deriche@inria.fr
http://www-sop.inria.fr/odyssee/team/Rachid.Deriche/

Doretto, Gianfranco
Computer Science Department
University of California at Los Angeles, USA
mailto:doretto@cs.ucla.edu
http://www.cs.ucla.edu/~doretto/

Esedoglu, Selim
Department of Mathematics
University of California at Los Angeles, USA
mailto:esedoglu@math.ucla.edu
http://www.math.ucla.edu/~esedoglu

Faugeras, Olivier
I.N.R.I.A. Sophia Antipolis, France

mailto:Olivier.Faugeras@inria.fr
http://www-sop.inria.fr/odyssee/team/Olivier.Faugeras

Fisher III, John
Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology, Cambridge, USA
mailto:fisher@ai.mit.edu
http://www.ai.mit.edu/people/fisher/

Fleet, David
Department of Computer Science
University of Toronto, Toronto, Canada
mailto:fleet@cs.toronto.edu
http://www.cs.toronto.edu/~fleet/

Frangi, Alejandro
Department of Technology
Pompeu Fabra University, Barcelona, Spain
mailto:alejandro.frangi@upf.edu
http://www.tecn.upf.es/~afrangi/

Grady, Leo
Imaging and Visualization Department
Siemens Corporate Research, Princeton, USA
mailto:leo.grady@siemens.com

Guo, Hongyu
Department of Computer, Information Science and Engineering
University of Florida, Gainesville, USA
mailto:hguo@cise.ufl.edu
http://www.cise.ufl.edu/~hguo

Haro, Gloria
Departament de Tecnologia
Universitat Pompeu Fabra, Barcelona, Spain
mailto:gloria.haro@upf.edu
http://www.tecn.upf.es/~gharo

Herbulot, Ariane
Laboratoire I3S
CNRS-Universite de Nice/Sophia Antipolis, France
mailto:herbulot@i3s.unice.fr
http://www.i3s.unice.fr/~herbulot/

Huang, Xiaolei

Division of Computer and Information Sciences
Rutgers, the State University of New Jersey, New Brunswick, USA
mailto:xiaolei@paul.rutgers.edu
http://www.research.rutgers.edu/~xiaolei/

Jehan-Besson, Stephanie
Laboratoire GREYC-Image
Ecole Nationale Supérieure d'Ingénieurs de Caen, France
mailto:stephanie.jehan@greyc.ensicaen.fr
http://www.greyc.ensicaen.fr/~jehan

Joshi, Sarang
Department of Radiation Oncology and Biomedical Engineering
University of North Carolina, Chapel Hill, USA
mailto:sjoshi@unc.edu
http://www.cs.unc.edu/~joshi

Joshi, Shantanu
Department of Electrical Engineering
Florida State University, Tallahassee, USA
mailto:joshi@eng.fsu.edu

Kakadiaris, Ioannis
Department of Computer Science
University of Houston, Houston, USA
mailto:ikakadia@central.uh.edu
http://www.vcl.uh.edu/~ioannisk/

Kaziska, Dave
Department of Statistics
Florida State University, Tallahassee, USA
mailto:kaziska@stat.fsu.edu

Keriven, Renaud
Département d'Informatique
Ecole Normale Supérieure, Paris, France
mailto:Renaud.Keriven@ens.fr
http://cermics.enpc.fr/~keriven/home.html

Kolmogorov, Vladimir
Microsoft Research Cambridge, UK
mailto:vnk@microsoft.com
http://www.research.microsoft.com/~vnk

Lenglet, Christophe

I.N.R.I.A. Sophia Antipolis, France
mailto:clenglet@sophia.inria.fr
http://www-sop.inria.fr/odyssee/team/Christophe.Lenglet/

Lelieveldt, Boudewijn
Division of Image Processing, Department of Radiology
Leiden University Medical Center, Leiden, Netherlands
mailto:B.Lelieveldt@lumc.nl

Markussen, Bo
Department of Computer Science
University of Copenhagen, Denmark
mailto:boma@diku.dk
http://www.bomar.dk/

Metaxas, Dimitris
Division of Computer and Information Sciences
Rutgers, the State University of New Jersey, New Brunswick, USA
mailto:dnm@cs.rutgers.edu
http://www.cs.rutgers.edu/~dnm/

Meyer, Fernand
Centre de Morphologie Mathématique
Ecole des Mines de Paris, Paris, France
mailto:fernand.meyer@cmm.ensmp.fr
http://cmm.ensmp.fr

Mitchell, Steven
The University of Iowa, Iowa City, USA
mailto:steve@componica.com

Mittal, Anurag
Real-time Vision and Modeling Department
Siemens Corporate Research, Princeton, USA
mailto:anurag.mittal@siemens.com
http://www.umiacs.umd.edu/~anurag

Mrázek, Pavel
Upek, Husinecká 7, Praha 3, Czech Republic
mailto:pavel.mrazek@upek.com

Nain, Delphine
Departments of Electrical and Computer and Biomedical Engineering
Georgia Institute of Technology, Atlanta, USA
mailto:delfin@cc.gatech.edu

http://www.bme.gatech.edu/groups/bil/

Nielsen, Mads
Department of Innovation
IT University of Copenhagen, Denmark
mailto:malte@itu.dk
http://www.itu.dk/people/malte

Ordas, Sebastian
Department of Technology
Pompeu Fabra University, Barcelona, Spain
mailto:sebastian.ordas@upf.edu

Paragios, Nikos
C.E.R.T.I.S.
Ecole Nationale des Ponts et Chaussées, Champs sur Marne, France
mailto:nikos.paragios@certis.enpc.fr
http://www.enpc.fr/certis/people/paragios.html

Park, Frederick
Department of Mathematics
University of California at Los Angeles, USA
mailto:fpark@math.ucla.edu
http://www.math.ucla.edu/~fpark

Pollefeys, Marc
Department of Computer Science
University of North Carolina, Chapel Hill, USA
mailto:marc@cs.unc.edu
http://www.cs.unc.edu/~marc/

Pons, Jean-Philippe
C.E.R.T.I.S.
Ecole Nationale des Ponts et Chaussées, Champs sur Marne, France
mailto:Jean-Philippe.Pons@certis.enpc.fr
http://www.enpc.fr/certis/people/pons.html

Prados, Emmanuel
I.N.R.I.A. Sophia Antipolis, France
mailto:Emmanuel.Prados@sophia.inria.fr
http://www-sop.inria.fr/odyssee/team/Emmanuel.Prados/

Rangarajan, Anand
Department of Computer, Information Science and Engineering
University of Florida, Gainesville, USA

mailto:anand@cise.ufl.edu
http://www.cise.ufl.edu/~anand

Reiber, Johan H.C.
Department of Radiology
Leiden University Medical Center, Leiden, the Netherlands
mailto:J.H.C.Reiber@lumc.nl
http://www.lkeb.nl

Rousson, Mikael
Imaging and Visualization Department
Siemens Corporate Research, Princeton, USA
mailto:mikael.rousson@scr.siemens.com

Sapiro, Guillermo
Department of Electrical and Computer Engineering
University of Minnesota, Minneapolis, USA
mailto:guille@ece.umn.edu
http://www.ece.umn.edu/users/guille/

Schnörr, Christoph
Department of Mathematics & Computer Science
University of Mannheim, Germany
mailto:schnoerr@uni-mannheim.de
http://www.cvgpr.uni-mannheim.de/

Soatto, Stefano
Computer Science Department
University of California at Los Angeles, USA
mailto:soatto@cs.ucla.edu
http://www.cs.ucla.edu/~soatto/

Sonka, Milan
Dept. of Electrical and Computer Engineering
The University of Iowa, Iowa City, USA
mailto:milan-sonka@uiowa.edu
http://www.engineering.uiowa.edu/~sonka/

Srivastava, Anuj
Department of Statistics
Florida State University, Tallahassee, USA
mailto:anuj@stat.fsu.edu
http://stat.fsu.edu/~anuj

Steidl, Gabriele

Faculty of Mathematics and Computer Science
University of Mannheim, Mannheim, Germany
mailto:steidl@math.uni-mannheim.de
http://kiwi.math.uni-mannheim.de/

Stewart, Charles
Department of Computer Science
Rensselaer Polytechnic Institute, Troy, USA
mailto:stewart@cs.rpi.edu
http://www.cs.rpi.edu/~stewart

Sun, Yiyong
Imaging and Visualization Department
Siemens Corporate Research, Princeton, USA
mailto:yiyong.sun@siemens.com

Szeliski, Richard
Microsoft Research, Redmond, USA
mailto:szeliski@microsoft.com
http://www.research.microsoft.com/~szeliski/

Tannenbaum, Allen
Departments of Electrical and Computer and Biomedical Engineering
Georgia Institute of Technology, Atlanta, USA
mailto:tannenba@bme.gatech.edu
http://www.bme.gatech.edu/groups/bil/

Tschumperlé, David
GREYC - UMR CNRS 6072
Centre National de la Recherche Scientifique (CNRS), Caen, France
mailto:David.Tschumperle@greyc.ensicaen.fr
http://www.greyc.ensicaen.fr/~dtschump/

Unal, Gozde
Intelligent Vision and Reasoning
Siemens Corporate Research, Princeton, USA
mailto:gozde.unal@siemens.com

Veksler, Olga
Departament of Computer Science
University of Western Ontario, Canada
mailto:olga@csd.uwo.ca
http://www.csd.uwo.ca/faculty/olga/

Vemuri, Baba

Department of Computer, Information Science and Engineering
Univiversity of Florida, Gainesville, USA
mailto:vemuri@cise.ufl.edu
http://www.cise.ufl.edu/~vemuri

Wang, Zhizhou
Imaging and Visualization Department
Siemens Corporate Research, Princeton, USA
mailto:zhizhou.wang@siemens.com

Weickert, Joachim
Faculty of Mathematics and Computer Science
Saarland University, Saarbrücken, Germany
mailto:weickert@mia.uni-saarland.de
http://www.mia.uni-saarland.de/weickert/

Welk, Martin
Faculty of Mathematics and Computer Science
Saarland University, Saarbrücken, Germany
mailto:welk@mia.uni-saarland.de
http://www.mia.uni-saarland.de/welk/

Weiss, Yair
School of Computer Science and Engineering
The Hebrew University of Jerusalem, Jerusalem, Israel
mailto:yweiss@cs.huji.ac.il
http://www.cs.huji.ac.il/~yweiss/

Wells III, William
Department of Radiology
Harvard Medical School and Brigham and Women's Hospital, Boston, USA
mailto:sw@bwh.harvard.edu
http://splweb.bwh.harvard.edu:8000/pages/ppl/sw/homepage.html

Williams, James
Imaging and Visualization Department
Siemens Corporate Research, Princeton, USA
mailto:jimwilliams@siemens.com

Wilson, Dave
Department of Mathematics
University of Florida, Gainesville, USA
mailto:dcw@math.ufl.edu

http://www.math.ufl.edu/~dcw/

Yezzi, Anthony
Departments of Electrical and Computer and Biomedical Engineering
Georgia Institute of Technology, Atlanta, USA
mailto:ayezzi@ece.gatech.edu
http://www.ece.gatech.edu/profiles/ayezzi/

Yip, Andy
Department of Mathematics
University of California at Los Angeles, USA
mailto:mhyip@math.ucla.edu
http://www.math.ucla.edu/~mhyip

Zabih, Ramin
Department of Computer Science
Cornell University, Ithaca, USA
mailto:rdz@cs.cornell.edu
http://www.cs.cornell.edu/~rdz

Zeitouni, Ofer
School of Mathematics
University of Minnesota, Minneapolis, USA
mailto:zeitouni@math.umn.edu

Zöllei, Lilla
Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology, Cambridge, USA
mailto:lzollei@csail.mit.edu
http://people.csail.mit.edu/people/lzollei/

Zucker, Steven
Department of Computer Science and Program in Applied Mathematics
Yale University, New Haven, USA
mailto:steven.zucker@yale.edu
http://www.cs.yale.edu/homes/vision/zucker/steve.html

Part I

Image Reconstruction