Jonathan Lawry

Modelling and Reasoning with Vague Concepts

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For a large class of cases - though not for all - in which we employ the word ‘meaning’ it can be defined thus: the meaning of a word is its use in language. - Ludwig Wittgenstein

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Preface

Vague concepts are intrinsic to human communication. Somehow it would seems that vagueness is central to the flexibility and robustness of natural language descriptions. If we were to insist on precise concept definitions then we would be able to assert very little with any degree of confidence. In many cases our perceptions simply do not provide sufficient information to allow us to verify that a set of formal conditions are met. Our decision to describe an individual as 'tall' is not generally based on any kind of accurate measurement of their height. Indeed it is part of the power of human concepts that they do not require us to make such fine judgements. They are robust to the imprecision of our perceptions, while still allowing us to convey useful, and sometimes vital, information. The study of vagueness in Artificial Intelligence (AI) is therefore motivated by the desire to incorporate this robustness and flexibility into intelligent computer systems. This goal, however, requires a formal model of vague concepts that will allow us to quantify and manipulate the uncertainty resulting from their use as a means of passing information between autonomous agents.

I first became interested in these issues while working with Jim Baldwin to develop a theory of the probability of fuzzy events based on mass assignments. Fuzzy set theory has been the dominant theory of vagueness in AI since its introduction by Lotfi Zadeh in 1965 and its subsequent successful application in the area of automatic control. Mass assignment theory provides an attractive model of fuzzy sets, but I became increasingly frustrated with a range of technical problems and unintuitive properties that seemed inherent to both theories. For example, it proved to be very difficult to devise a measure of conditional probability for fuzzy sets, that satisfied all of a minimal set of intuitive properties. Also, mass assignment theory provides no real justification for the truth-functionality assumption central to fuzzy set theory.

This volume is the result of my attempts to understand and resolve some of these fundamental issues and problems, in order to provide a coherent framework for modelling and reasoning with vague concepts. It is also an attempt to develop such a framework as can be applied in practical problems concerning automated reasoning, knowledge representation, learning and fusion. I do not believe AI research should be carried out in isolation from potential applications. In essence AI is an applied subject. Instead, I am committed to the idea that theoretical development should be informed by complex practical problems, through the direct application of theories as they are developed. Hence, I have dedicated a significant proportion of this book to presenting the application of the proposed framework in the areas of data analysis, data mining and information fusion, in the hope that this will give the reader at least some indication as to the utility of the more theoretical ideas.

Finally, I believe that much of the controversy in the AI community surrounding fuzzy set theory and its application arises from the lack of a clear operational semantics for fuzzy membership functions, consistent with their truth-functional calculus. Such an interpretation is important for any theory to ensure that its not based on an ad hoc, if internally consistent, set of inference processes. It is also vital in knowledge elicitation, to allow for the translation of uncertainty judgements into quantitative values. For this reason there will be a semantic focus throughout this volume, with the aim of identifying possible operational interpretations for the uncertainty measures discussed.

JONATHAN LAWRY

Acknowledgments

Time is becoming an increasingly rare commodity in this frenetic age. Yet time, time to organise one's thoughts and then to commit them to paper, is exactly what is required for writing a book. For this reason I would like to begin by thanking the Department of Engineering Mathematics at the University of Bristol for allowing me a six month sabbatical to work on this project. Without the freedom from other academic duties I simply would not have been able to complete this volume.

As well as time, any kind of creative endeavour requires a stimulating environment and I would like to thank my colleagues in Bristol for providing just such an environment. I was also very lucky to be able to spend three months during the summer of 2004 visiting the Secció Matemàtiques i Informàtica at the Universidad Politécnica de Cataluña. I would like to thank Jordi Recasens for his kindness during this visit and for many stimulating discussions on the nature of fuzziness and similarity. I am also grateful to the Spanish government for funding my stay at UPC under the scheme 'Ayudas para movilidad de profesores de universidad e investigadores Españoles y extranjeros'.

Over the last few years I have been very fortunate to have had a number of very talented PhD students working on projects relating to the label semantics framework. In particular, I would like to thank Nick Randon and Zengchang Qin who between them have developed and implemented many of the learning algorithms described in the later chapters of this book.

Finally a life with only work would be impoverished indeed and I would like to thank my wonderful family for everything else. To my mother, my wife Pepa, and daughters Ana and Julia - gracias por su amor y su apoyo.

Foreword

Fuzzy set theory, since its inception in 1965, has aroused many controversies, possibly because, for the first time, imprecision, especially linguistic imprecision, was considered as an object of investigation from an engineering point of view. Before this date, there had already been proposals and disputes around the issue of vagueness in philosophical circles, but never before had the vague nature of linguistic information been considered as an important issue in engineering sciences. It is to the merit of Lotfi Zadeh that he pushed this issue to the forefront of information engineering, claiming that imprecise verbal knowledge, suitably formalized, could be relevant in automating control or problem-solving tasks.

Fuzzy sets are simple mathematical tools for modelling linguistic information. Indeed they operate a simple shift from Boolean logic, assuming that there is more to "truth-values" than being true or being false. Intermediate cases, like "half-true" make sense as well, just like a bottle can be half-full. So, a fuzzy set is just a set with blurred boundaries and with a gradual notion of membership. Moreover, the truth-functionality of Boolean logic was kept, yielding a wealth of formal aggregation functions for the representation of conjunction, disjunction and other connectives. This proposal also grounds fuzzy set theory in the tradition of many-valued logics. This approach seems to have generated misunderstandings in view of several critiques faced by the theory of fuzzy sets. A basic reason for the reluctance in established scientific circles to accept fuzzy set theory is probably the fact that while this very abstract theory had an immediate intuitive appeal which prompted the development of many practical applications, the notion of membership functions had not yet been equipped with clear operational semantics. Namely, it is hard to understand the meaning of the number 0.7 on the unit interval, in a statement like "Mr. Smith is tall to degree 0.7", even if it clearly suggests that this person is not tall to the largest extent.

This lack of operational semantics, and of measurement paradigms for membership degrees was compensated for by ad hoc techniques like triangular fuzzy sets, and fuzzy partitions of the reals, that proved instrumental for addressing practical problems. Nevertheless, degrees of membership were confused with degrees of probability, and orthodox probabilists sometimes accused the fuzzy set community of using a mistaken surrogate probability calculus, the main argument being the truth-functionality assumption, which is mathematically inconsistent in probability theory. Besides, there are still very few measurement-theoretic works in fuzzy set theory, while this would be a very natural way of addressing the issue of the meaning of membership grades. Apparently, most measurement-theory specialists did not bother giving it a try.

Important progress in the understanding of membership functions was made by relating fuzzy sets and random sets: while membership functions are not probability distributions, they can be viewed as one-point coverage functions of random sets, and, as such, can be seen as upper probability bounds. This is the right connection, if any, between fuzzy sets and probability. But the price paid is the lack of universal truth-functionality.

The elegant and deep monograph written by Jon Lawry adopts this point of view on membership functions, for the purpose of modelling linguistic scales, with timely applications to data-mining and decision-tree learning. However it adopts a very original point of view. While the traditional random set approach to fuzzy sets considers realisations as subsets of some numerical reference scale (like a scale of heights for "short and tall"), the author assumes they are subsets of the set of labels, obtained from answering yes/no questions about how to label objects. This approach has the merit of not requiring an underlying numerical universe for label semantics. Another highlight of this book is the lucid discussion concerning the truth-functionality assumption, and the proposal of a weaker, yet tractable, "functionality" assumption, where independent atomic labels play a major role. In this framework, many fuzzy connectives can be given an operational meaning. This book offers an unusually coherent and comprehensive, mathematically sound, intuitively plausible, potentially useful, approach to linguistic variables in the scope of knowledge engineering.

Of course, one may object to the author's view of linguistic variables. The proposed framework is certainly just one among many possible other views of membership functions. Especially, one may argue that founding the measurement of gradual entities on yes-no responses to labelling questions may sound like a paradox, and does not properly account for the non-Boolean nature of gradual notions. The underlying issue is whether fuzzy predicates are fuzzy because their crisp extension is partially unknown, or because they are intrinsically gradual in the mind of individuals (so that there just does not exist such a thing as "the unknown crisp extension of a fuzzy predicate"). Although it sounds like splitting hairs, answering this question one way or another has drastic impact on the modelling of connectives and the overall structure of the underlying logic. For instance if "tall" means a certain interval of heights I cannot precisely describe, then "not tall" just means the complement of this interval. So, even though I cannot precisely spot the boundary of the extension of "tall", I can claim that being "tall and not tall" is an outright contradiction, and "being tall or not tall" expresses a tautology. This view enforces the laws of contradiction and excluded-middle, thus forbidding truth-functionality of connectives acting on numerical membership functions. However, if fuzzy predicates are seen as intrinsically gradual, then "tall" and "not tall" are allowed to overlap, then the underlying structure is no longer Boolean and there is room for truth-functionality. Fine, would say the author, but what is the measurement setting that yields such a non-Boolean structure and provides for a clear intuition of membership grades? Such a setting does not exist yet and its discovery remains as an open challenge.

Indeed, while the claim for intrinsically gradual categories is legitimate, most interpretative settings for membership grades proposed so far (random sets, similarity relations, utility ...) seem to be at odds with the truth-functionality assumption, although the latter is perfectly self-consistent from a mathematical point of view (despite what some researchers mistakenly claimed in the past). It is the merit of this book that it addresses the apparent conflict between truth-functionality and operational semantics of fuzzy sets in an upfront way, and that it provides one fully-fledged elegant solution to the debate. No doubt this somewhat provocative but scientifically solid book will prompt useful debates on the nature of fuzziness, and that new alternative proposals will be triggered by its in-depth study. The author must be commended for an extensive work that highlights an important issue in fuzzy set theory, that was perhaps too cautiously neglected by its followers, and too aggressively, sometimes misleadingly, argued about, by its opponents from more established fields.

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