Lower bounds for geometric diameter problems

H. Fournier, Antoine Vigneron

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H. Fournier, Antoine Vigneron. Lower bounds for geometric diameter problems. 7. Latin American Symposium on Theoretical Informatics (LATIN 2006), Mar 2006, Valdivia, Chile. hal-02753352

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Lower Bounds for Geometric Diameter Problems

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Outline

- Review of previous work on the 2D and 3D diameter problems.
- $\Omega(n \log n)$ lower bound for computing the diameter of a 3D convex polytope.
- Reduction from Hopcroft's problem to the diameter problem for point sets in $\mathbb{R}^7$.

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Previous work

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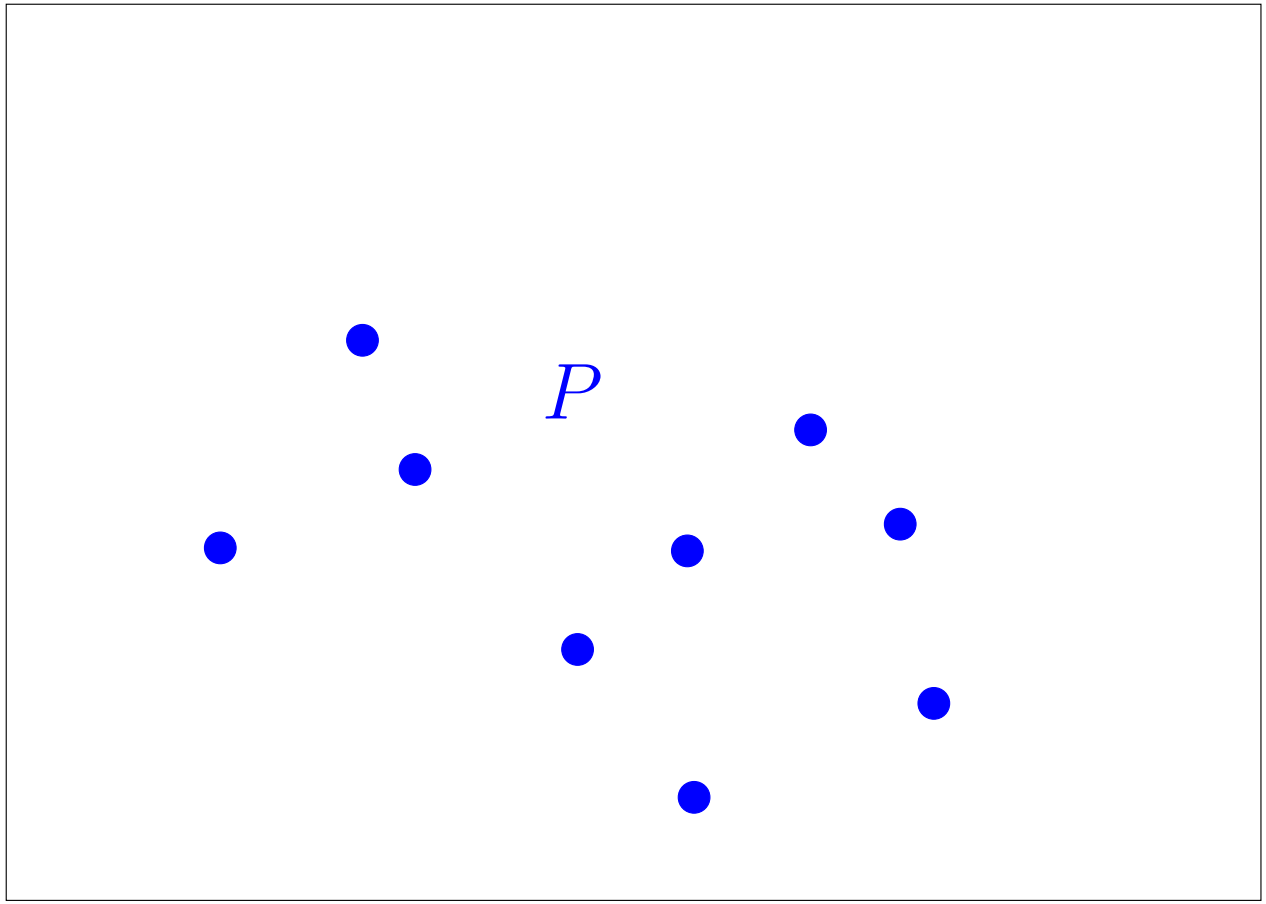
The diameter problem



- INPUT: a set P of n points in $\mathbb{R}^d$.
- OUTPUT: $\mathrm{diam}(P) := \max\{\mathrm{d}(x, y) \mid x, y \in P\}$.

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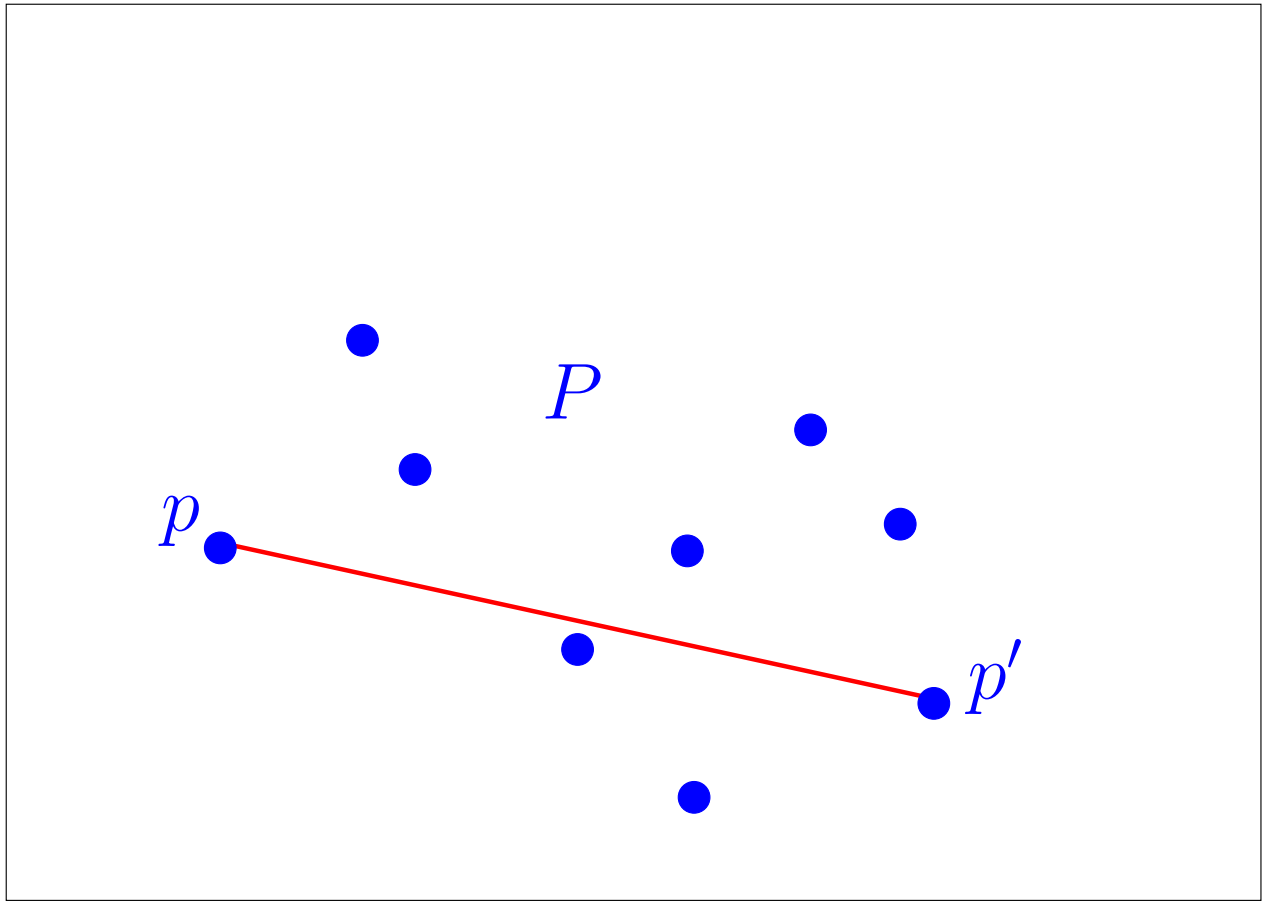

- $\mathrm{diam}(P) = \mathrm{d}(p, p')$.

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Decision problem

- We will give lower bounds for the *decision problem* associated with the diameter problem.
- INPUT: a set P of n points in $\mathbb{R}^d$.
- OUTPUT:
  - YES if $\mathrm{diam}(P) < 1$
  - NO if $\mathrm{diam}(P) \geqslant 1$

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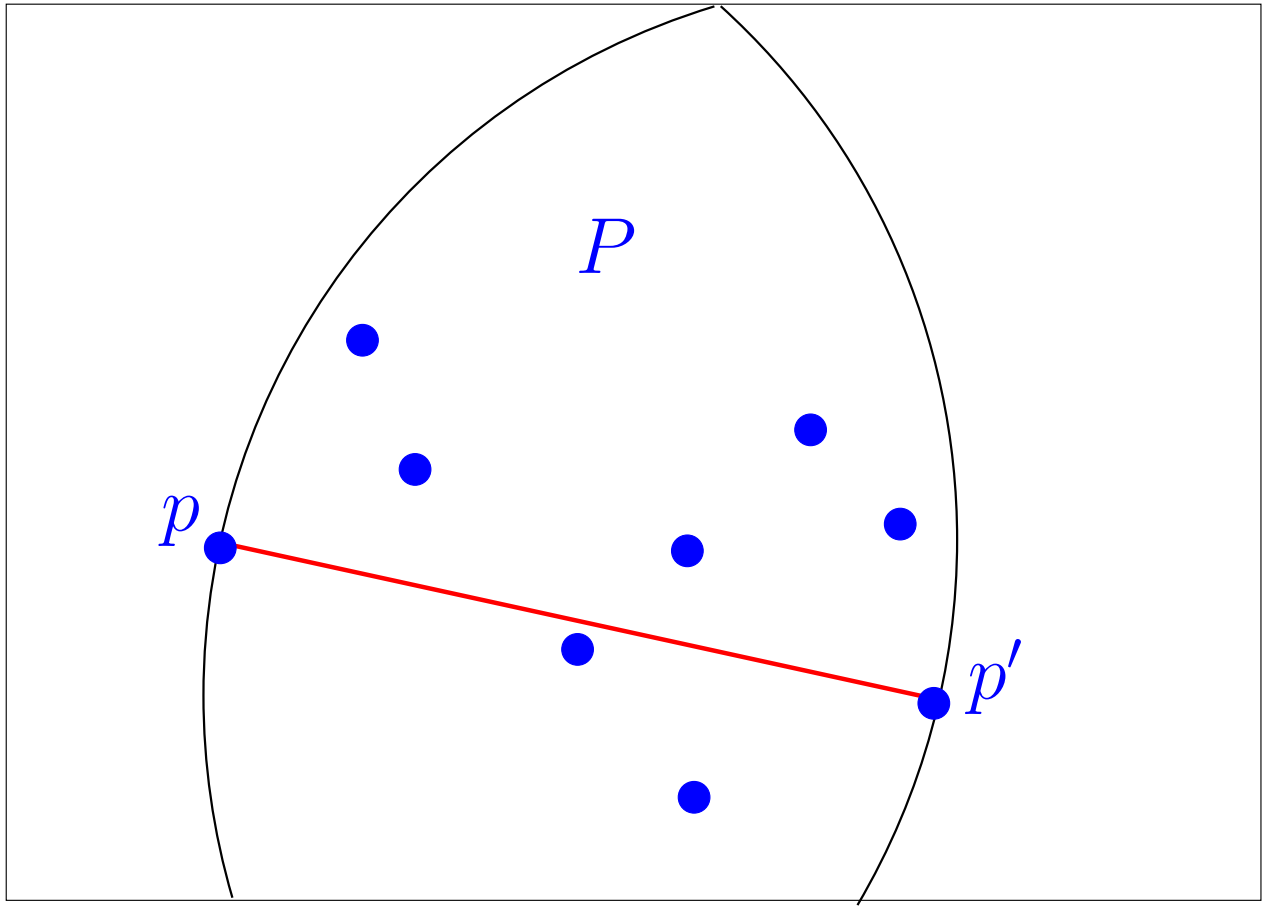
Observation



- P lies in the intersection of the two balls with radius $\mathrm{d}(p, p')$ centered at p and p'.

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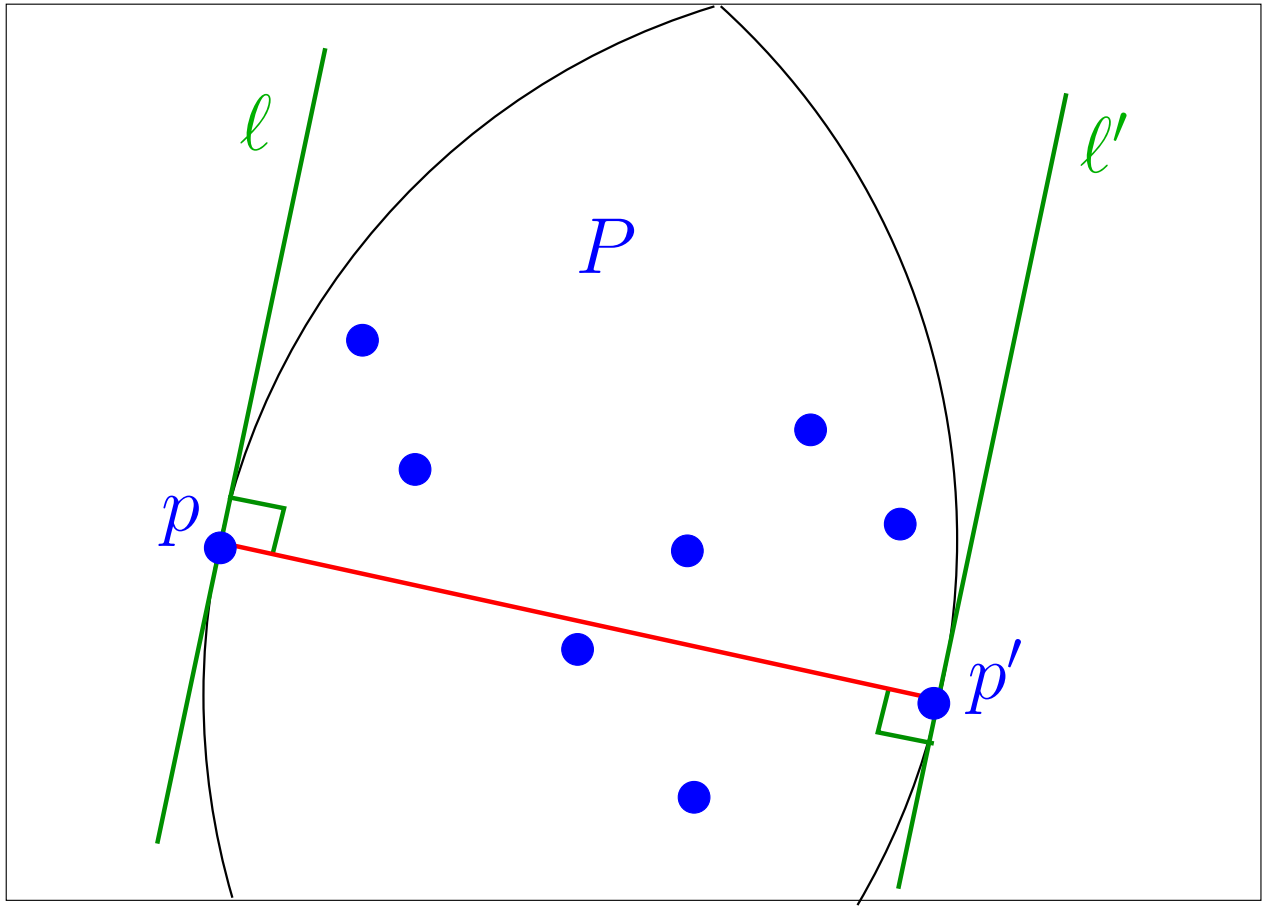
The diameter problem



- P lies between two parallel hyperplanes through p and p'. We say that (p, p') is an *antipodal pair*.

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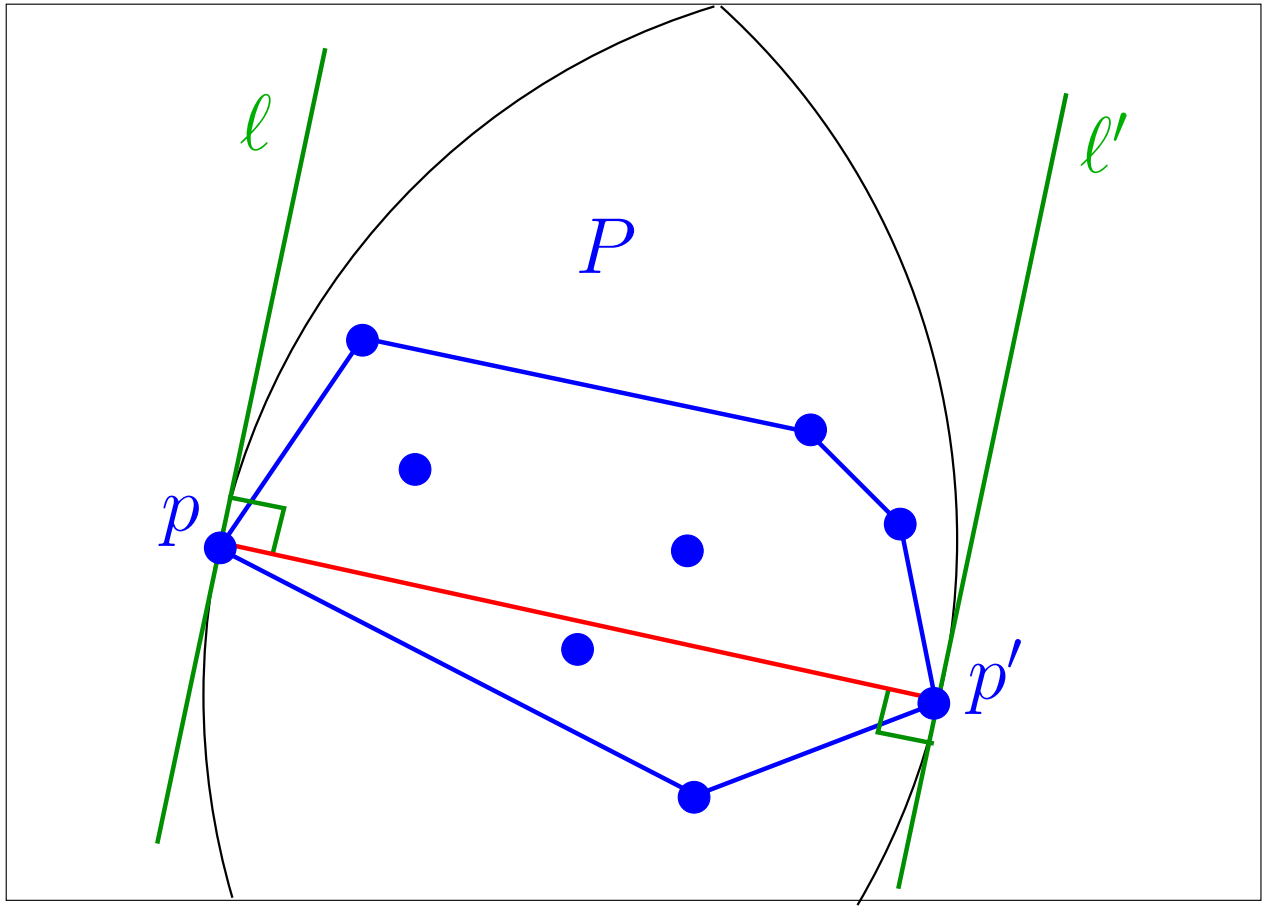

- Any antipodal pair (and therefore any diametral pair) lies on the convex hull $\mathrm{CH}(P)$ of P.

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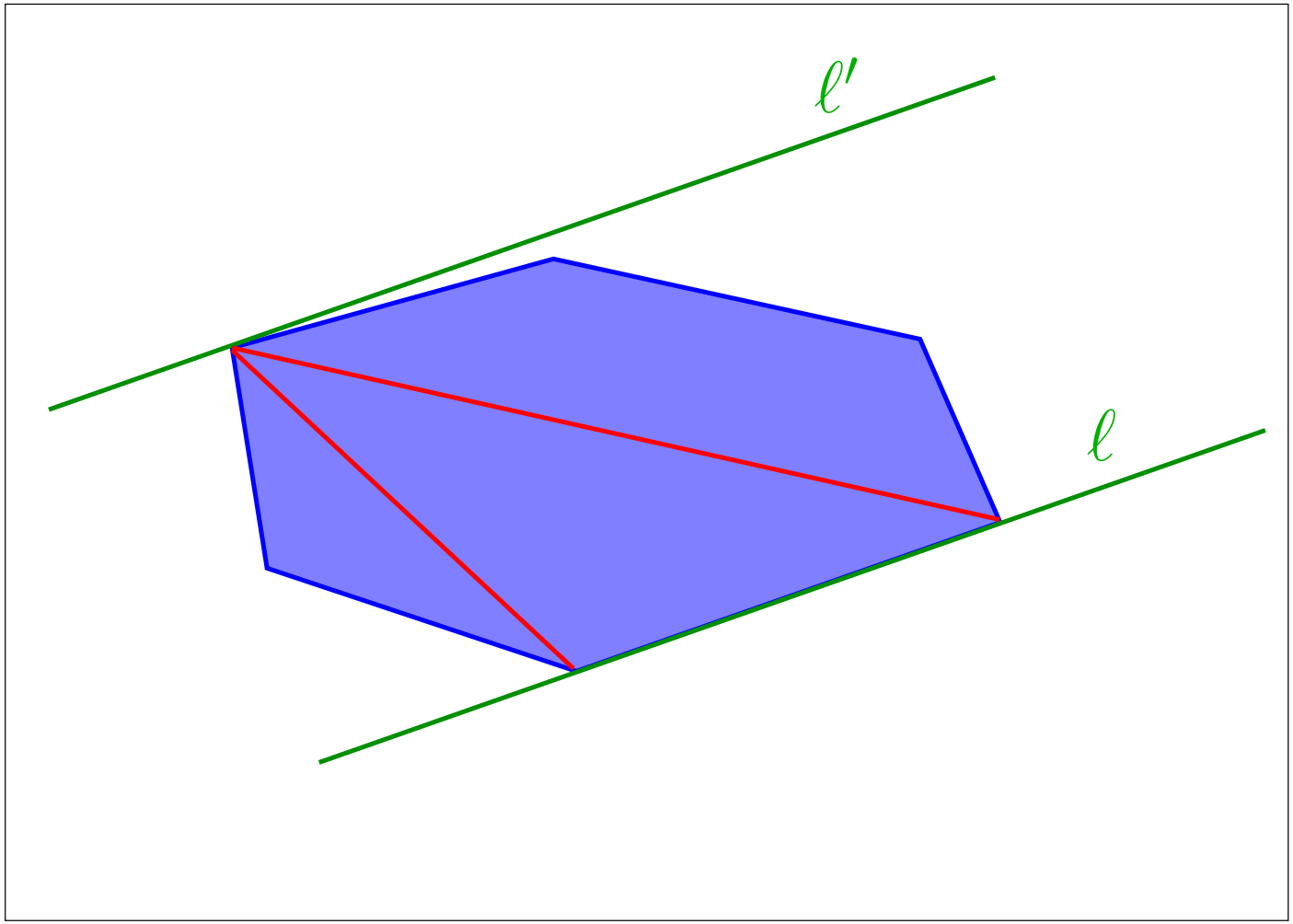
Finding the antipodal pairs

- The rotating calipers technique.



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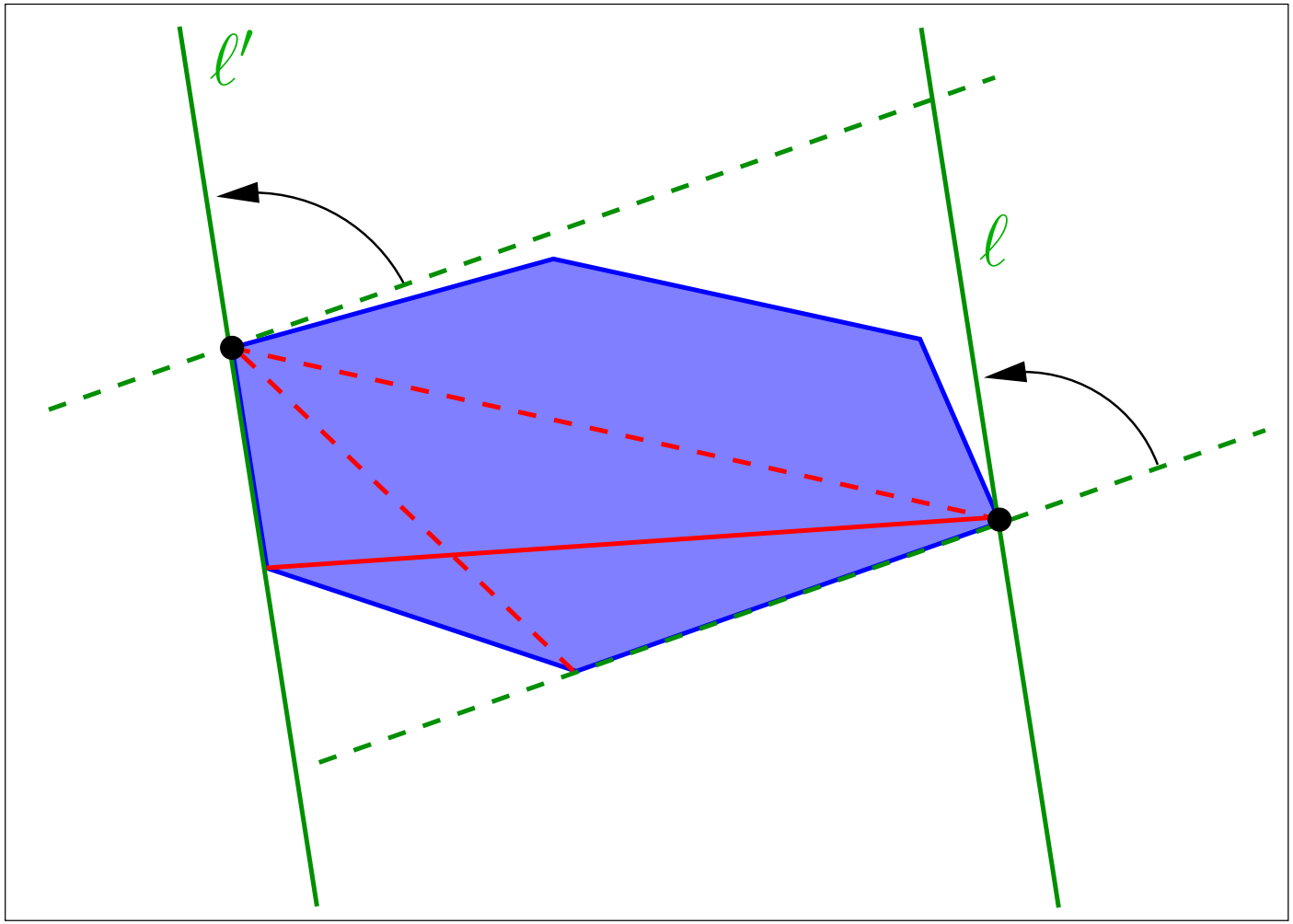

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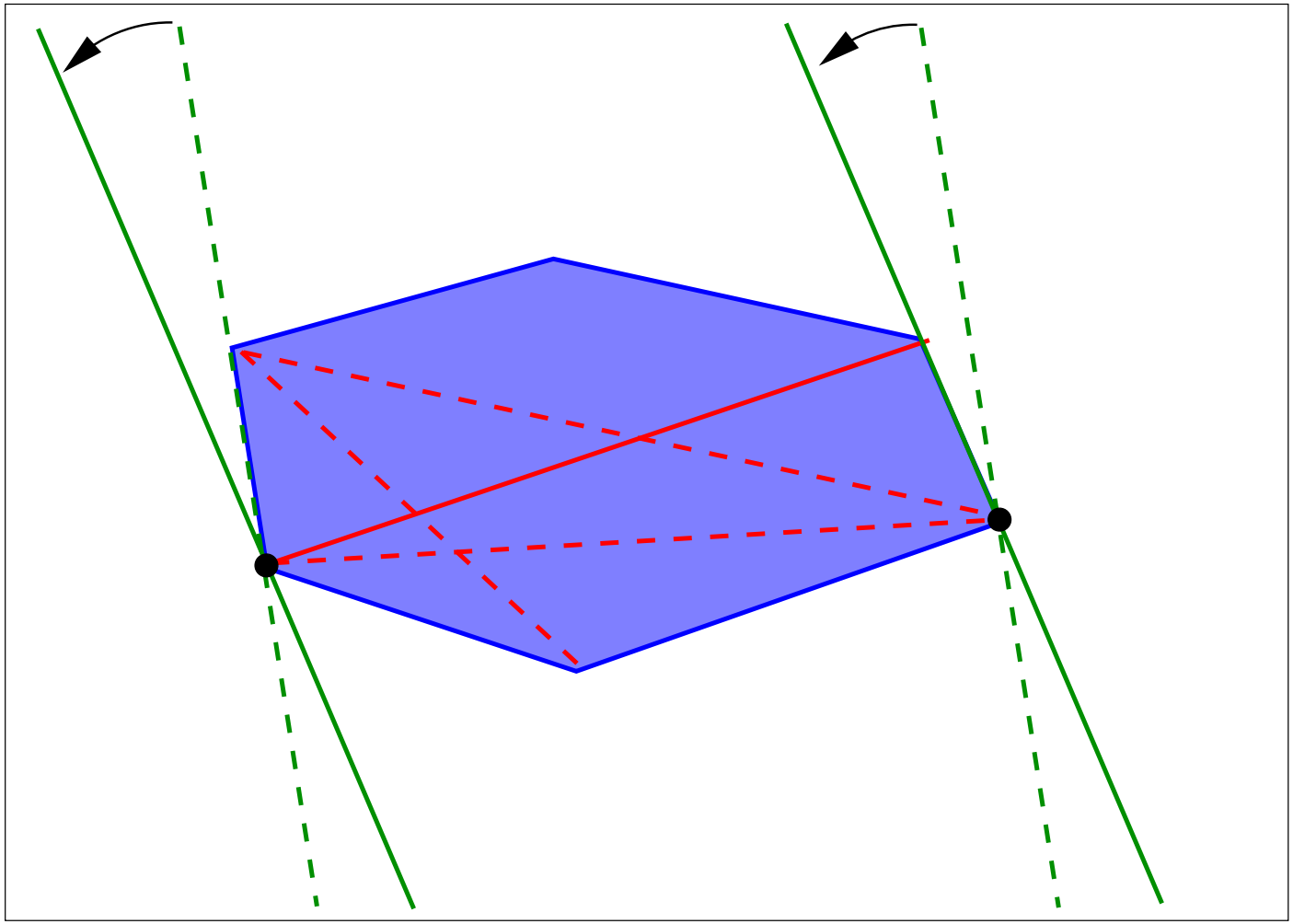

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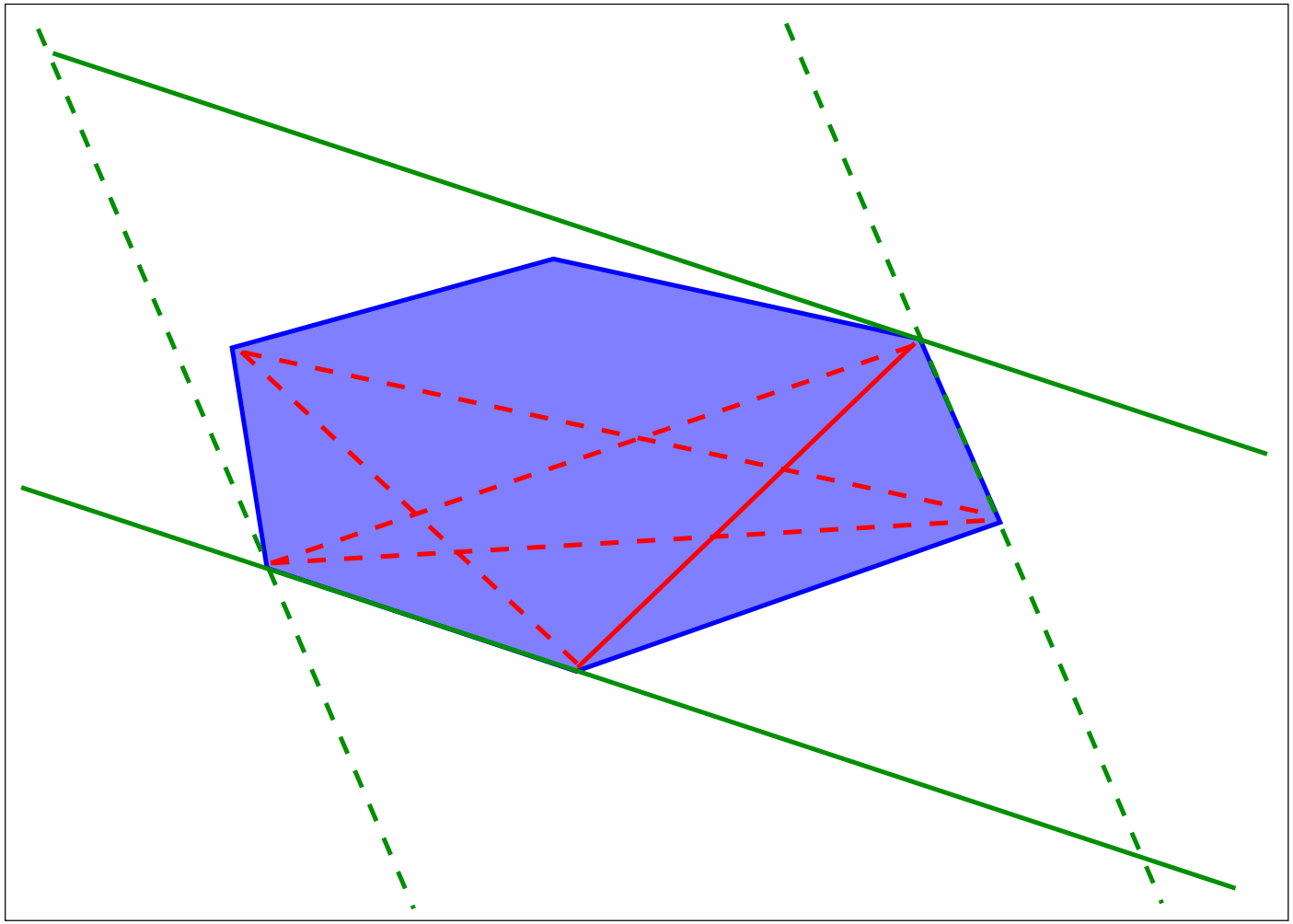

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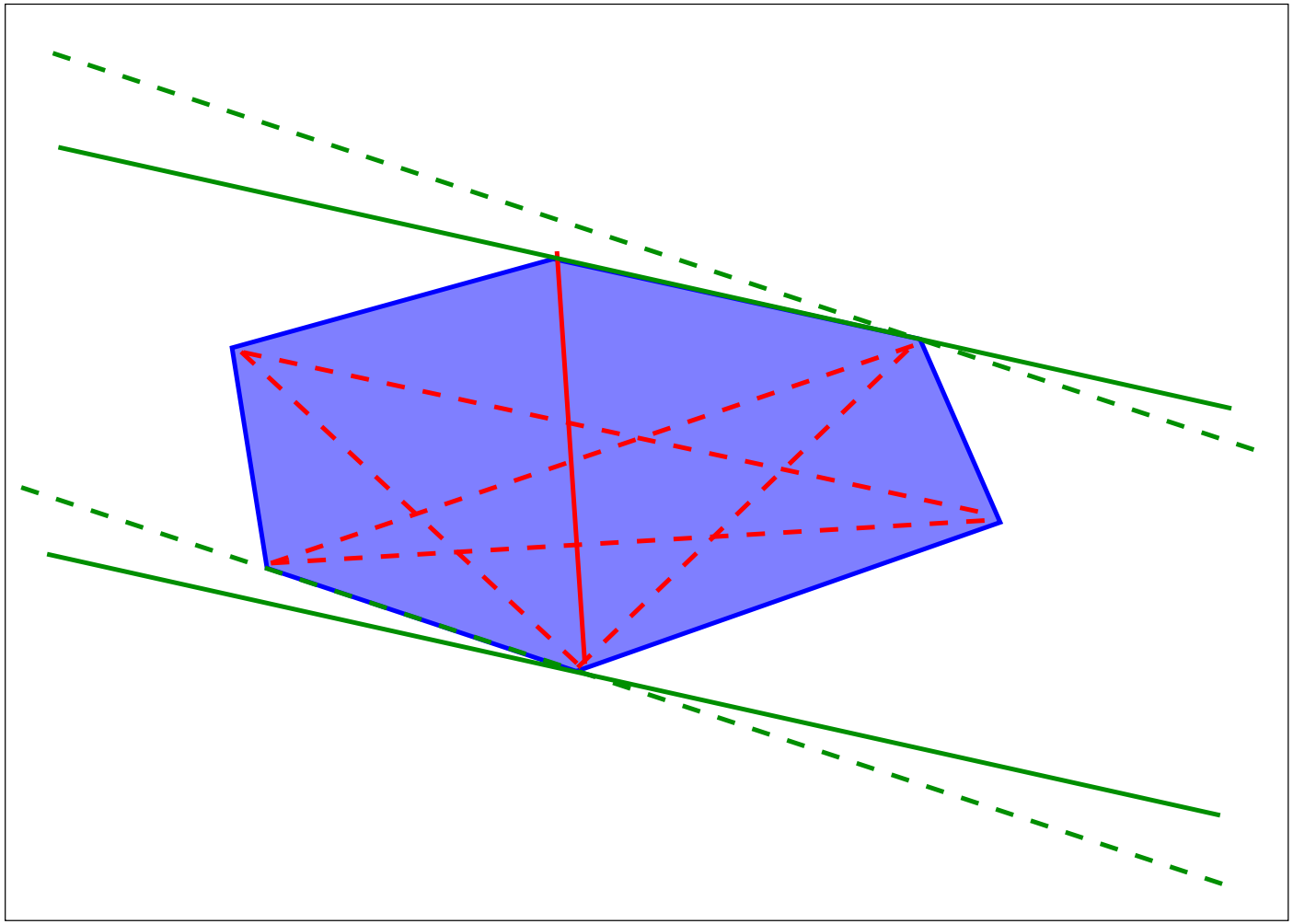

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Computing the diameter of a 2D-point set

- Compute the convex hull $\mathrm{CH}(P)$ of P.
  - $O(n \log n)$ time.
- Find all the antipodal pairs on $\mathrm{CH}(P)$.
  - There are at most n such pairs in non–degenerate cases.
  - $O(n)$ time using the rotating calipers technique.
- Find the diametral pairs among the antipodal pairs.
  - $O(n)$ time by brute force.
- Conclusion:
  - The diameter of a 2D-point set can be found in $O(n \log n)$ time
  - The diameter of a convex polygon can be found in $O(n)$ time.

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3D-diameter problem

- Randomized $O(n \log n)$ time algorithm (Clarkson and Shor, 1988).
  - Randomized incremental construction of an intersection of balls and decimation.
- Deterministic $O(n \log n)$ time algorithm (E. Ramos, 2000).
- These two algorithms compute the diameter of an n-point set in $\mathbb{R}^3$.
- Can we compute the diameter of a convex $3D$-polytope in linear time?
  - No, we give an $\Omega(n \log n)$ lower bound.

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Model of computation

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Real-RAM

- Real Random Access Machine.
- Each registers stores a *real* number.
- Access to registers in unit time.
- Arithmetic operation $(+, -, \times, /)$ in unit time.

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Algebraic computation tree: definition

- Input: $\bar{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$.
- Output: YES or NO
- It is a binary tree with 3 types of nodes
  - Leaves: YES or NO
  - Degree-1 nodes: computation nodes. Perform $\{+, -, \times, /, \sqrt{\cdot}\}$ on two operands. An operand is
    - a real constant,
    - some input number x_i, or
    - the value obtained by computation nodes that are ancestors of the current node.
  - Degree-2 nodes: branching nodes. Compares with 0 the value obtained at a computation node that is an ancestor of the current node.

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Algebraic computation tree (ACT)

- We say that an ACT *decides* $S \subset \mathbb{R}^n$ if
  - $\forall (x_1, \ldots, x_n) \in S$, it reaches a leaf labeled YES, and
  - $\forall (x_1, \ldots, x_n) \notin S$, it reaches a leaf labeled NO.
- The ACT model is stronger than the real–RAM model.
- To get a lower bound on the worst-case running time of a real-RAM that decides S, it suffices to have a lower bound on the *depth* of all the ACTs that decide S

Theorem 1 (Ben-Or) *Any ACT that decides S has depth*

$$\Omega(\log(\textit{number of connected components of } S)).$$

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Lower bound for 3D convex polytopes

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Problem statement

- We are given a convex 3-polytope P with n vertices.
- P is given by the coordinates of its vertices and its *combinatorial structure*:
  - All the inclusion relations between its vertices, edges and faces.
  - The cyclic ordering of the edges of each face.
- Example: P is given in a doubly-connected edge list.
- Problem: we want to decide whether $\mathrm{diam}(P) < 1$.
- We show an $\Omega(n \log n)$ lower bound. Our approach:
  - We define a family of convex polytopes.
  - We show that the sub-family with diameter < 1 has $n^{\Omega(n)}$ connected components.
  - We apply Ben-Or's bound.

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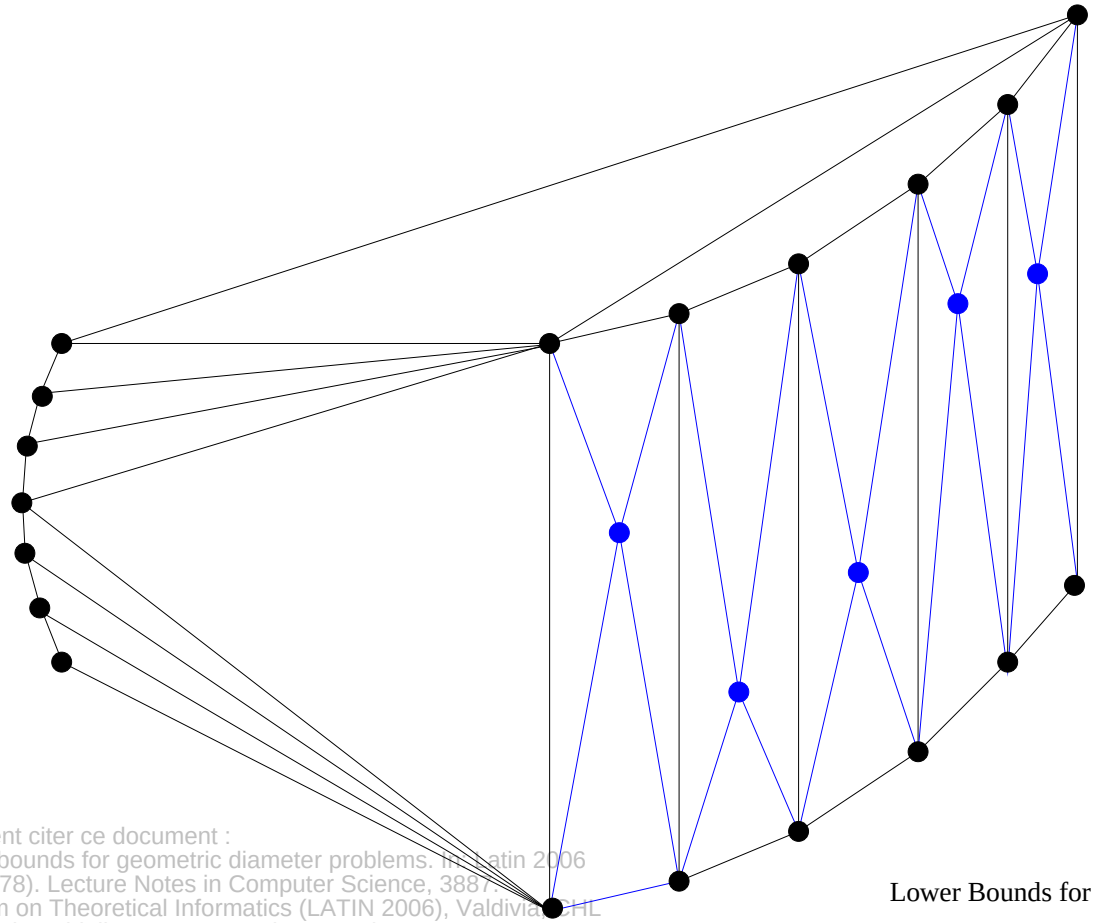
Polytopes $P(\bar{\beta})$

- The family of polytopes is parametrized by $\bar{\beta} \in \mathbb{R}^{2n-1}$.
- When n is fixed, only the $2n-1$ blue points change with $\bar{\beta}$.



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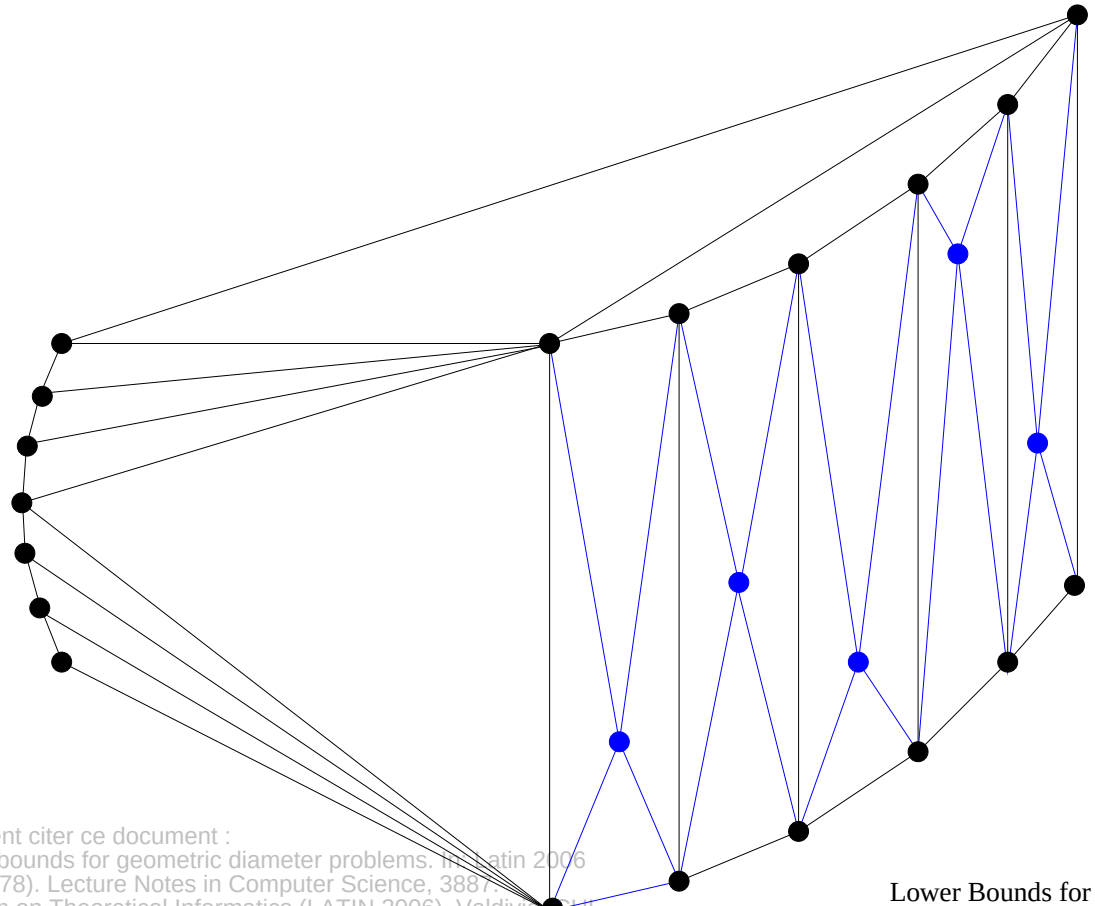

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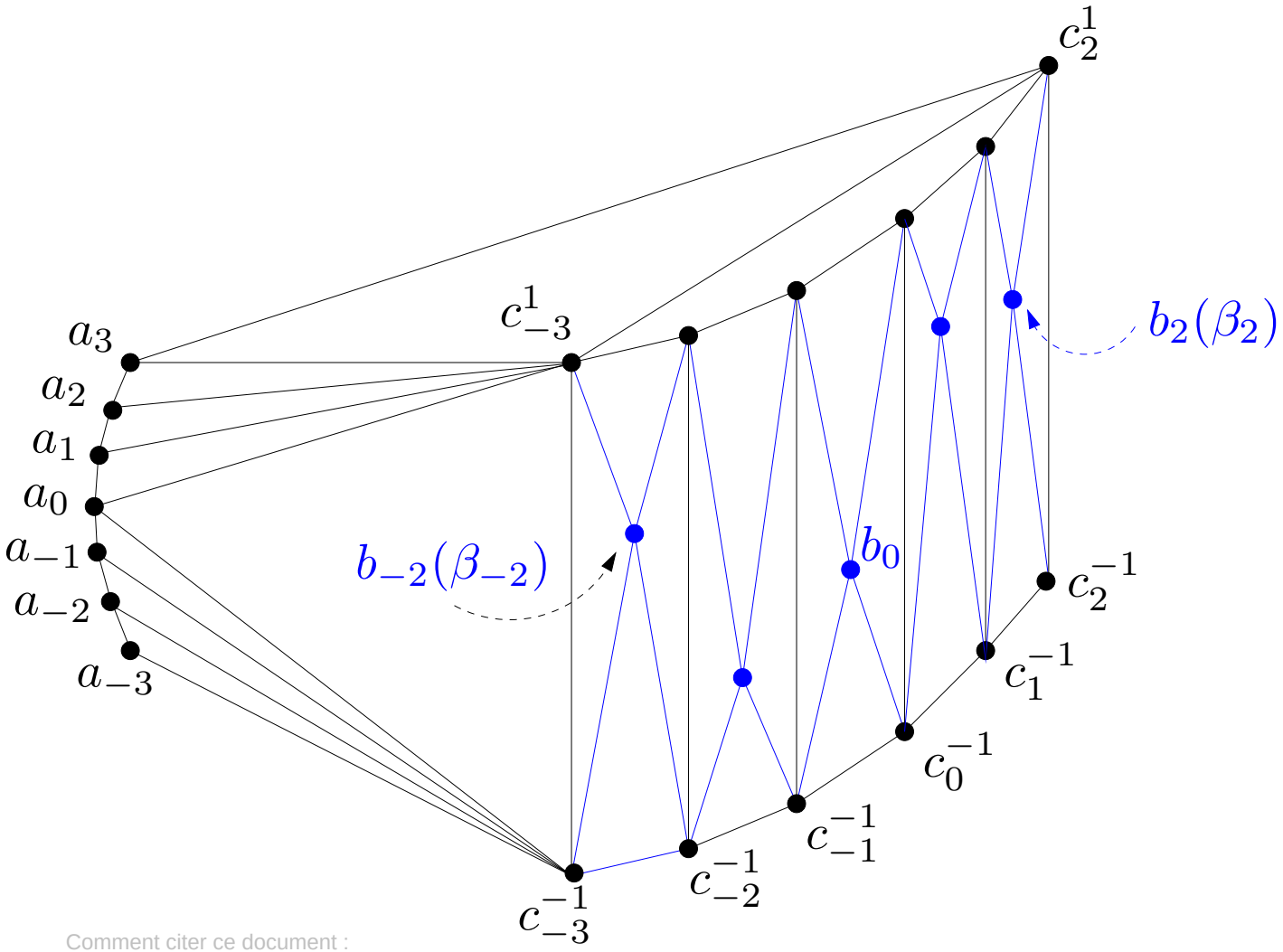
Notation

- Example where $n = 3$.



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Notation

- $\bar{a} := (a_{-n}, a_{-n+1} \ldots, a_n)$.
- $A := \{a_{-n}, a_{-n+1}, \ldots, a_n\}$.
- $\bar{\beta} := (\beta_{-n+1}, \ldots, \beta_{n-1})$.
- $B(\bar{\beta}) := \{b_{-n+1}(\beta_{-n+1}), \ldots, b_{n-1}(\beta_{n-1})\}$.
- $\bar{b}(\bar{\beta}) := (b_{-n+1}(\beta_{-n+1}), \ldots, b_{n-1}(\beta_{n-1}))$.
- $\bar{c} := (c^{-1}_{-n}, c^{-1}_{-n+1}, \ldots, c^{-1}_{n-1}, c^{1}_{-n}, c^{1}_{-n+1}, \ldots, c^{1}_{n-1})$.
- $C := \{c^{-1}_{-n}, c^{-1}_{-n+1}, \ldots, c^{-1}_{n-1}, c^{1}_{-n}, c^{1}_{-n+1}, \ldots, c^{1}_{n-1}\}$.
- $P(\bar{\beta}) := \mathrm{CH}(A \cup B(\bar{\beta}) \cup C)$.

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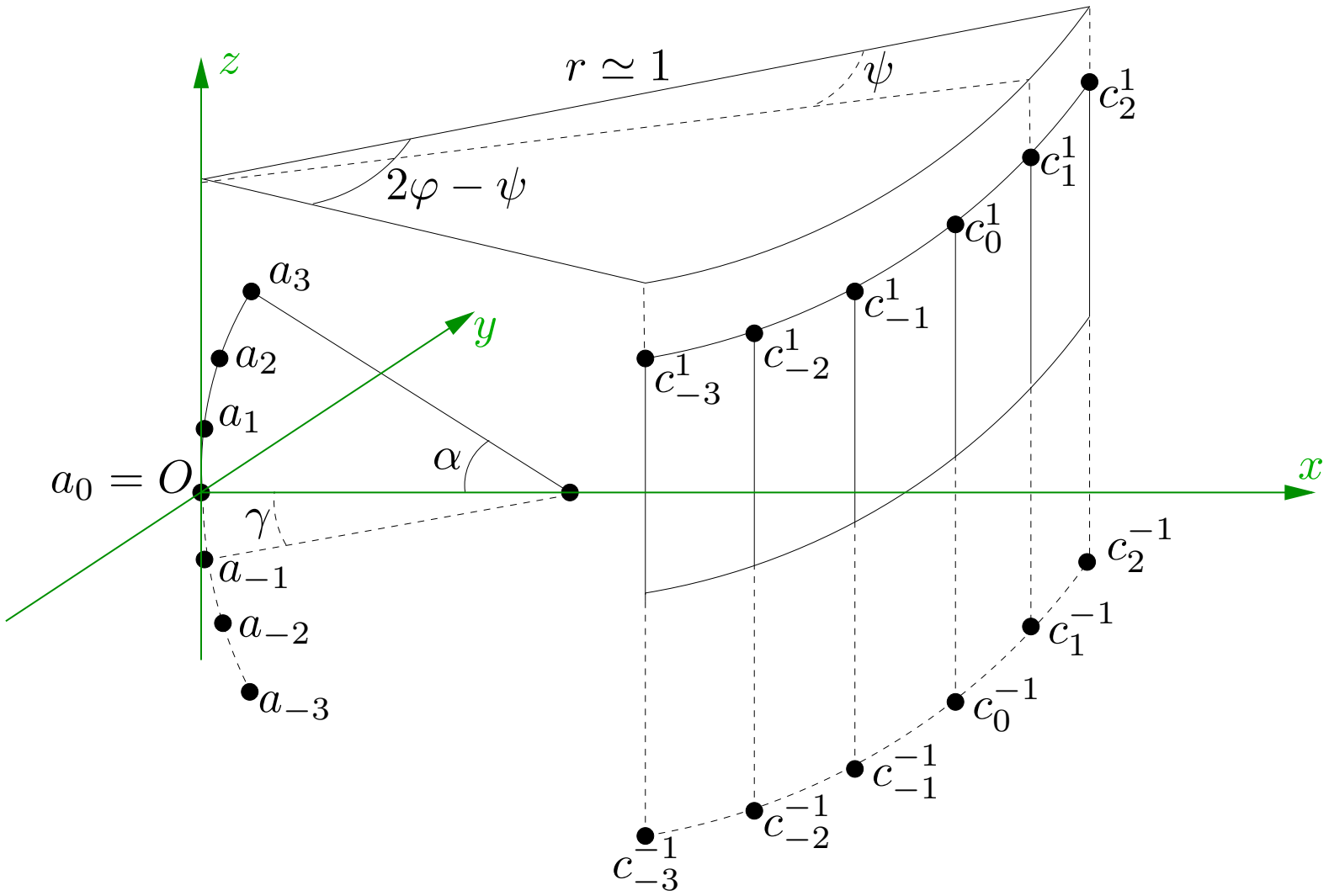
Point sets A and C



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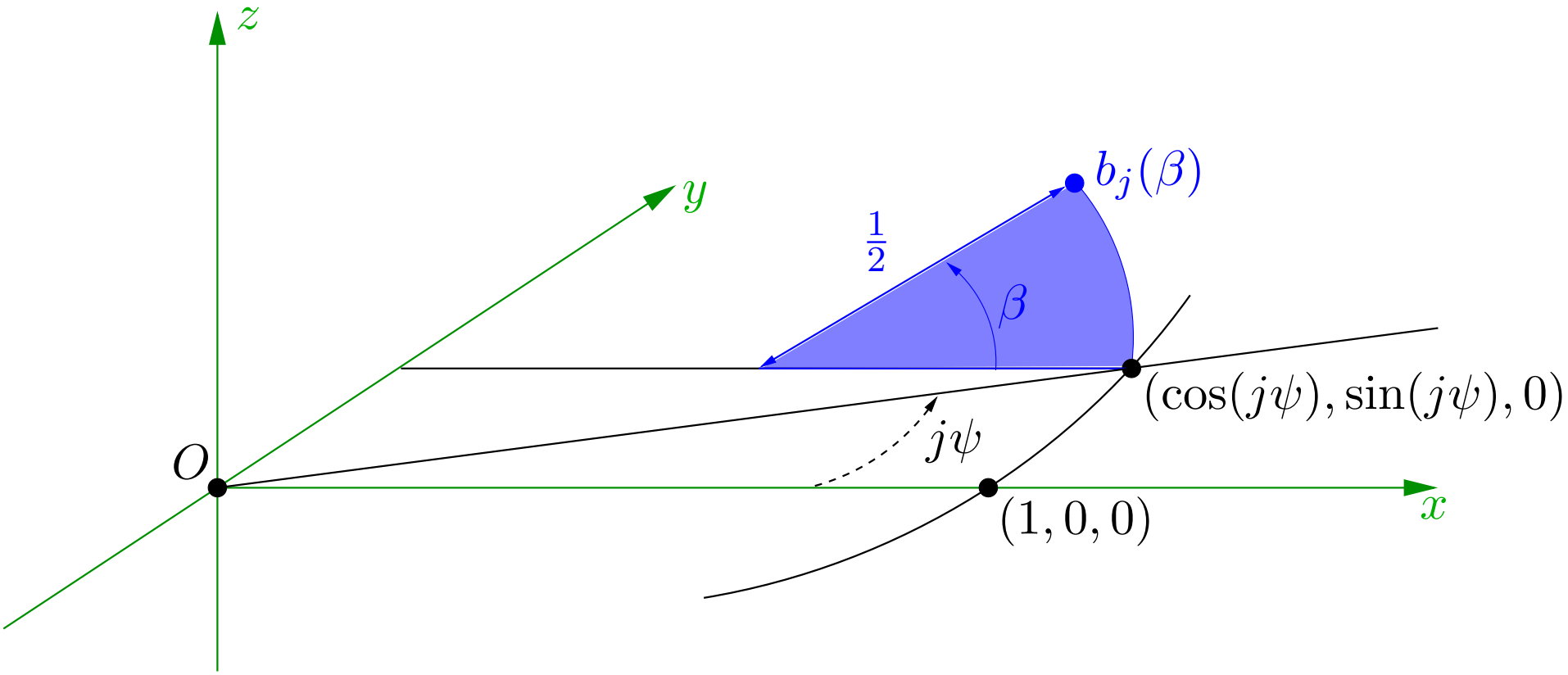
Point $b_j(\beta_j)$



- The blue region is parallel to Oxz.
- $\beta \in [-\alpha, \alpha]$

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Coordinates of points in A, $B(\bar{\beta})$ and C

- $$a_i := \begin{pmatrix} \frac{1}{2}(1-\cos(i\gamma)) \\ 0 \\ \frac{1}{2}\sin(i\gamma) \end{pmatrix}$$
- $$c_i^s := \begin{pmatrix} r\cos\left(\left(i+\frac{1}{2}\right)\psi\right) \\ r\sin\left(\left(i+\frac{1}{2}\right)\psi\right) \\ \frac{1}{2}s\alpha \end{pmatrix}$$
- $$b_j(\beta) := \begin{pmatrix} \cos(j\psi) - \frac{1}{2}(1-\cos\beta) \\ \sin(j\psi) \\ \frac{1}{2}\sin(\beta) \end{pmatrix}$$

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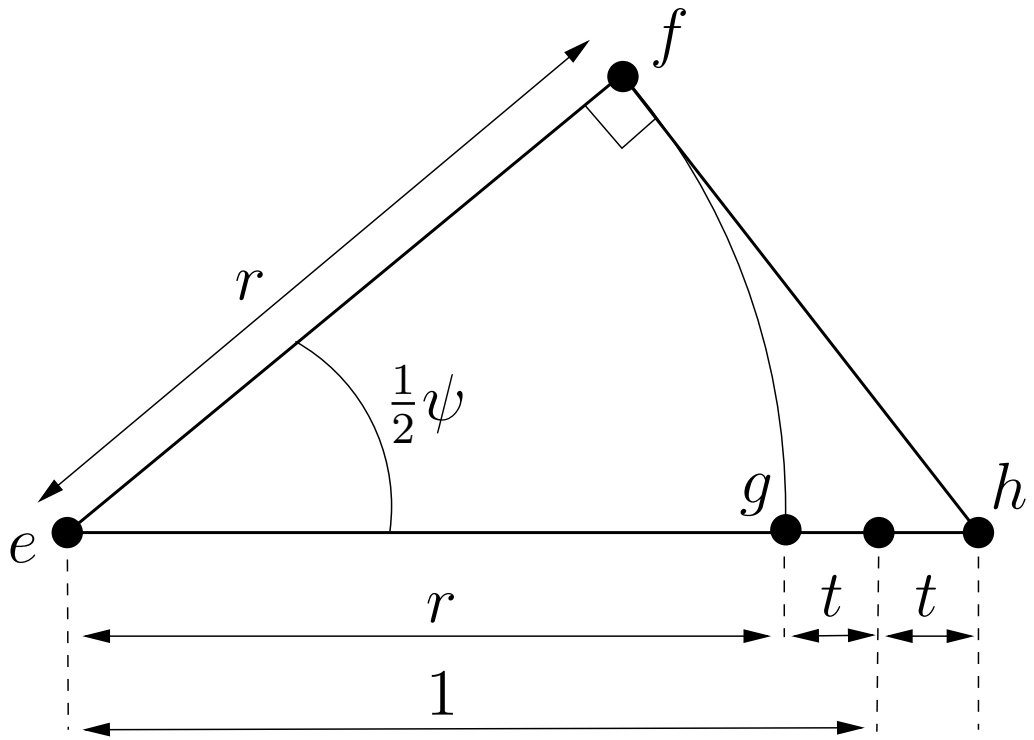

- $\varphi = 1/4n$
- α is small.
- $\psi = \varphi/n$
- $\gamma = \alpha/n$

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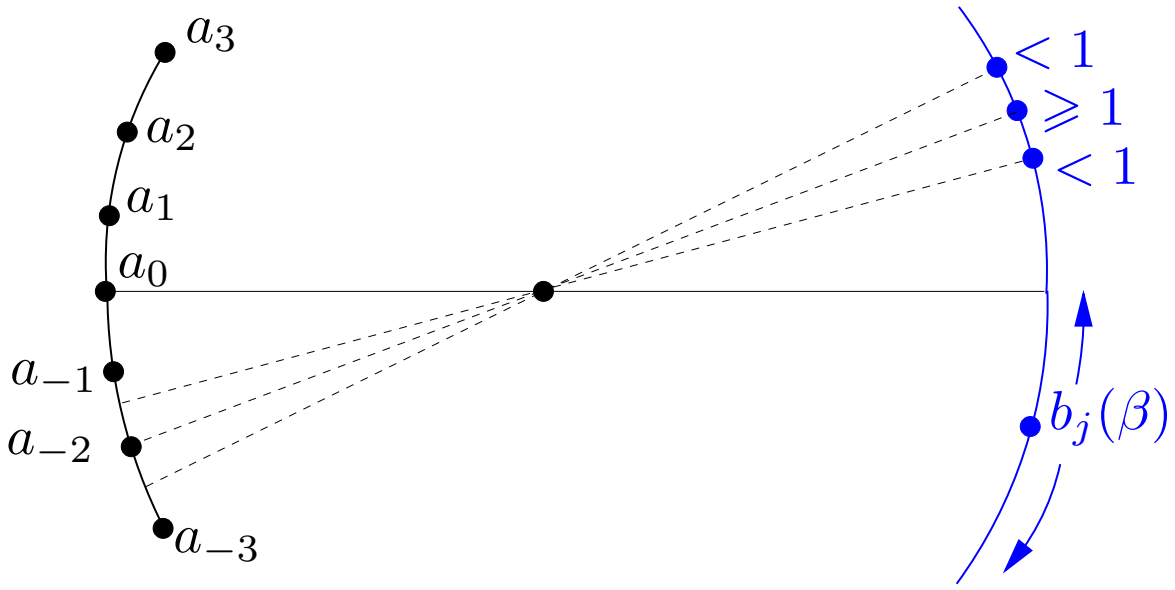
Proof

- Notation: $\mathrm{diam}(E, F) := \max\{\mathrm{d}(e, f) \mid (e, f) \in E \times F\}$.

Lemma 1 *The set*

$$\{b_j(\beta) \mid \beta \in [-\alpha, \alpha] \textit{ and } \mathrm{diam}(A, \{b_j(\beta)\}) < 1\}$$

has at least $2n$ *connected components.*



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Proof

- Proof of Lemma 1: Calculations, until the second-order terms.

Lemma 2 *The combinatorial structure of* $\mathrm{CH}(A \cup B(\bar{\beta}) \cup C)$ *is independent of* $\bar{\beta}$.

- We denote $P(\bar{\beta}) = \mathrm{CH}(A \cup B(\bar{\beta}) \cup C)$.

Lemma 3 $\mathrm{diam}(A \cup B(\bar{\beta}) \cup C) = \mathrm{diam}(A, B(\bar{\beta}))$.

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Proof

- Definitions:

$$\mathcal{S}_n = \{(\bar{a}, \bar{b}(\bar{\beta}), \bar{c}) \mid \bar{\beta} \in [-\alpha, \alpha]^{2n-1}\}$$
$$\mathcal{E}_n = \{(\bar{a}, \bar{b}(\bar{\beta}), \bar{c}) \mid \bar{\beta} \in [-\alpha, \alpha]^{2n-1} \text{ and } \operatorname{diam}(P(\bar{\beta})) < 1\}$$

- Notice that $\mathcal{E}_n \subset \mathcal{S}_n \subset \mathbb{R}^{24n}$.
- **Lemma 4** *The set $\mathcal{S}_n$ can be decided by an ACT with depth $O(n)$.*
- **Lemma 5** *Any ACT that decides $\mathcal{E}_n$ has depth $\Omega(n \log n)$.*
  Proof: By lemmas 1 and 3, $\mathcal{E}_n$ has at least $(2n)^{2n-1}$ connected components. Apply Ben-Or's bound (Theorem 1).

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End of the proof

Theorem 2 *Assume that an algebraic computation tree T_n decides whether the diameter of a 3-polytope is smaller than 1. Then T_n has depth $\Omega(n \log n)$.*

- Proof: Take as input graph of T_n the the graph of $P(\bar{\beta})$. Denote by U_n the ACT of Lemma 4. Plug U_n into the accepting leaves of T_{8n}. It gives an ACT with depth the depth of $T_{8n} + O(n)$ that decides $\mathcal{E}_n$. Apply Lemma 5.

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Related work

- (Chazelle) The convex hull of two 3-polytopes can be computed in linear time.
- (Chazelle et al.) It is not known whether the convex hull of a subset of the vertices of a 3-polytope can be computed in linear time.
- (Chazelle et al.) However, we can compute in linear time the Delaunay triangulation of a subset of the vertices of a Delaunay triangulation.

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Diameter is harder than Hopcroft's problem

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Hopcroft's problem

- P is a set of n points in $\mathbb{R}^2$.
- L is a set of n lines in $\mathbb{R}^2$.
- Problem: decide whether

$$\exists (p, \ell) \in P \times L : p \in \ell.$$

- An $o(n^{4/3} \log n)$ algorithm is known. (Matoušek).
- No $o(n^{4/3})$ algorithm is known.
- We give a linear-time reduction from Hopcroft's problem to the diameter problem in $\mathbb{R}^7$.
- We first give a reduction to the *red-blue diameter* problem in $\mathbb{R}^6$: compute $\mathrm{diam}(E, F)$ when E and F are n-point sets in $\mathbb{R}^6$.

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Proof

- $\theta(x,y,z) := \frac{1}{x^2+y^2+z^2}(x^2, y^2, y^2, \sqrt{2}xy, \sqrt{2}yz, \sqrt{2}zx)$.
- Note that $\|\theta(x,y,z)\| = 1$.
- For $1 \leqslant i \leqslant n$
  - $p_i = (x_i, y_i, 1)$
  - $\ell_i = (u_i, v_i, w_i)$ is the line $\ell_i : u_i x + v_i y + w_i = 0$.
- Let $p'_i := \theta(p_i)$ and $\ell'_j = \theta(\ell_j)$.
- We get

$$\begin{aligned} \|p'_i - \ell'_j\|^2 &= \|p'_i\|^2 + \|\ell'_j\|^2 - 2 < p'_i, \ell'_j > \\ &= 2 - 2\frac{< p_i, \ell_j >^2}{\|p_i\|^2 \|\ell_j\|^2} \end{aligned}$$

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Proof

- Note that $p_i \in \ell_j$ iff $< p_i, \ell_j >= 0$.
- Thus, there exists i, j such that $p_i \in \ell_j$ if and only if $\mathrm{diam}(\theta(P), \theta(L)) = 2$.
- $\theta(P)$ and $\theta(L)$ are n-point sets in $\mathbb{R}^6$.
- Similarly, we can get a reduction from Hopcroft's problem to the diameter problem in $\mathbb{R}^7$, using this linearization:

$$\theta(x, y, z) := \frac{1}{x^2 + y^2 + z^2}(x^2, y^2, y^2, \sqrt{2}xy, \sqrt{2}yz, \sqrt{2}zx)$$

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Related work

- The red-blue diameter in $\mathbb{R}^4$ can be computed in $O(n^{4/3}\text{polylog } n)$ (Matoušek and Scharzkopf). It would be interesting to get a reduction from Hopcroft's problem.
- Erickson gave reduction from Hopfcroft problem to other computational geometry problems.

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