Numerical Estimation of the Impact of Interferences on the Localization Problem in Sensor Networks

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Matthieu Bouget, Pierre Leone, and Jose Rolim

Computer Science Department
University of Geneva
1211 Geneva 4
Switzerland

Abstract. In this paper we numerically analyze the impact of interferences in wireless networks. More particularly, the impact on the probability of success of a localization algorithm. This problem is particularly relevant in the context of sensor networks. Actually, our numerical experiments provide information about this algorithm, even when we do not consider interferences. Indeed, in this case, sharp threshold is observed on the probability of success of the algorithm. Moreover, our numerical computations show that the main harmful interferences are the ones occurring between sensors which get localized at the same time and send in turn their own location. This is demonstrated by varying the time span of the random waiting time before the emissions. We then observe that the longer the waiting time the closer the curves are to the ones obtained without considering interferences. Hence, this proves to be an efficient way of reducing the impact of interferences on the localization algorithm. Moreover, our numerical experiments demonstrate that among the sectors of disk with same area, the one with the smaller radius of emission and larger angle of emission is the more appropriate to the localization algorithm.

1 Introduction

Wireless networks are undoubtedly appealing for many applications. For instance, to provide easy access points to Internet from laptop computer, establishing networks in difficult environment conditions (wireless ad-hoc networks) typically to help the coordination of the action of rescuers operating on a disaster site and are the only alternative in the context of mobile networks. A particularity of wireless networks is that all the transmissions share the same channel of communication. Different protocols are available such as Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA) or Code Division Multiple Access (CDMA), for multiplexing the channel of transmission. All these protocols have the same common limitation, as the number of stations increases the impact of the interferences occurring between simultaneous emission becomes more stringent and can even prevent the efficient establishment of the network. Some applications allow the limitation of the number of stations for instance typical application of Bluetooth based wireless networks. Bluetooth based ad-hoc networks [21] are possible extension of the Bluetooth technology and allow the inter-connexion of clusters of Bluetooth nodes by using two radio per nodes.

In this paper, we numerically analyze the impact of the interferences on the networks performances. Although the numerical tools involved in our analysis can be tailored to deal with the aforementioned protocols we proceed to the analysis of a simpler protocol of transmission. The considered protocol is relevant in the field of sensor networks [1] and we now particularize our discussion to these class of networks. Moreover, the impact of the interferences depends on the statistical occupation of the channels of transmission. Hence, we particularize our analysis to a probabilistic localization algorithm (discussed below) and actually deal with the impact of interferences on the performances of this algorithm.

Two important characteristics of sensor networks are the large number of nodes involved in the composition of the networks and that sensors are usually battery powered, hence limiting the energy consumption is a key issue to make longer the life-time of the networks [27, 28]. Economical factors require that sensor should be as cheap as possible and then, based on limited hardware resources. Also, the protocols involved in the establishment and use of the networks have to be as simple as possible to limit the energy consumption due to the exchange of synchronization messages. These requirements make relevant to consider *random access* channel introduced in [4]. Besides its simplicity random access is relevant in some situations to optimize the transmission delay [7] and hence, can be a better alternative of the different protocols discussed above (TDMA, FDMA). Actually, in this paper we consider a slightly more complicated random access protocols since the stations wait for a random time interval before sending a data. The reason is that in the particular situation we consider, nearby stations are prone to receive a data at the same time and the random waiting time reduces the interferences between such stations [2]. Moreover, to reduce these local interactions the transmissions are directional.

Another important characteristic of sensor networks is that they are *data-centric*, meaning that sensors are less important than the data they convey. Typically, sensors are used to proceed to some measurements and convey the measured values towards one (or more) particular stations which is able to collect and process the data's. However, for the data to be meaningful one should attach to it the location where the measurement was made. This implies that sensors are to be localized. This can be done by adding hardware resources, for instance GPS electronic devices, to sensors. But this would conflict with the requirements of minimizing the energy consumption as well as lowering the price of the entire system. The probabilistic localization algorithm studied in this paper assumes that a few sensors (anchors) are equipped with electronic devices to ensure their localization. The others sensors compute their own position by triangulation (3-lateration) given the position of their localized neighbors. This procedure requires the estimate of the distance between sensors. This can be achieved for instance, by Time of Arrival (ToA) or Received Signal Strength Indicator (RSSI) techniques. The success of the localization algorithm depends on the structure of the communication graph and we focus on the probability of success of the algorithm with respect to the networks parameters.

2 Related Work

Due to their importance in sensor networks, localization algorithms are widely analyzed in the literature. General considerations and presentations of various strategies can be found in [26, 28, 10]. In [29] the accuracy of *range free* localizations algorithms are analyzed. The main interest of these protocols as opposed to *range based* protocols is that they minimize the required hardware. Particularly, there is no need to estimate the distances between sensors. In [8] numerical evaluation of various protocols is done in the context of optical sensor networks. In [13, 14] the authors consider a particular technique for estimating the distances between sensors and proceed to real experiments based on Motes[1] sensor systems. From a complexity point of view, in [3, 6] NP-hardness results are provided for the localization and connex problems. These results support the application of approximation algorithms and numerical investigations. Concerning the analysis of the impact of interferences, we mention [9, 16] and references therein. These papers are based on a model of interferences called the capture model which assumes that a communication can be established given that the ratio of the signal to noise is large enough. Asymptotic results are provided as the number of sensors goes to infinity. In this paper, we consider a different model called the collision model, at once two

[1] http://www.xbow.com

sensors emits towards a third same sensor there is collision and the data is lost. Moreover, we keep fixed the number of sensors and look for the impact of the networks parameters. The numerical methods we use for the numerical experiments are particular stochastic estimation methods [24, 22]. The general frameworks as well as some applications of the methods are discussed in [19]. The numerical experiments presented in this paper are different than the ones suggested in [19] and actually both papers are complementary. As far as we know, no previous similar works are present in the literature. However, stochastic estimation methods seems to appear sporadically for dynamic control of communications in wireless networks, see for instance[17, 20, 25].

3 Sensor Networks Characteristics and the Localization Protocol

Our model of sensor and sensor networks is based on directional emission of radio frequency signals. The idealized directional emission pattern we consider is a sector of disk of radius r and angle α. Notice that this is general and also incorporate the model of isotropic radiation ($\alpha = 360$ degree).

Sensors composing the networks establish wireless communication through directional antenna. The parameters of the communications are the range (radius) of communication r which is the maximal distance a sensor can send a data, the angle of emission α and the direction of the emission β. The former parameter β is chosen randomly and independently by each sensor with uniform distribution on the circle and the others two parameters r, α are the same for all the sensors. We denote by p the area of the region covered by the emission, see Figure1.

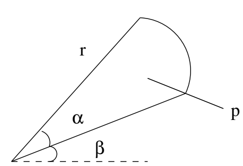

Fig. 1. Sensor with radius of emission r, angle of emission α and orientation of the emission β

The model of interferences is the collision model where a collision occurs at once two stations emit at the same time in the same region.

This is illustrated in Figure 2, x and y are emitters and the hatched region is where collisions occur and no data from x nor y can be received in this region.

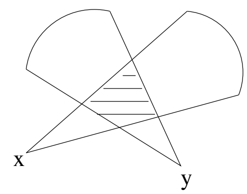

Fig. 2. Collision region (hatched), x and y and the emitters.

The sensors are assumed to be randomly and independently scattered in the unit square region $[0,1] \times [0,1]$ with uniform distribution. Given the relative positions of sensors directional communication can be established between a sensor x toward a sensor y provided the former belongs to area covered by the emission of x, see Figure 1. This defines the directional graph of communication. Two example of such graphs are represented on Figure 3. Up to our knowledge results concerning the connectivity of such graphs are only of asymptotic character, see [23] and references therein. The choice of the uniform distribution is arbitrary and the computation suggested in this paper remain unchanged with a different distribution. However, the computation is based on the results on the simulation of the communication graph and, of course, simulation is to be conducted appropriately.

The localization protocol we wish to numerically analyze the performances works as follows. We consider a given sensor network composed of n sensors. We choose randomly a fixed number l among them and assume that they are localized sensors. Practically, this corresponds to scatter randomly n sensors among which l are equipped with electronic devices to ensure the localization. Each localized sensor choose randomly and independently a waiting time w in $0, \ldots, \log(n)$ and wait for w clock. Then, it sends to all its neighbors its location (coordinate in the plane) and we assume that the receivers are able to estimate the distance from them to the sender. Once a sensor receives the coordinates of three localized neighbors it computes its own location by triangulation (3 lateration) and becomes localized. Hence, it chooses a waiting time and send its coordinate in turn. The algorithm is **successful** if more than 90% of the

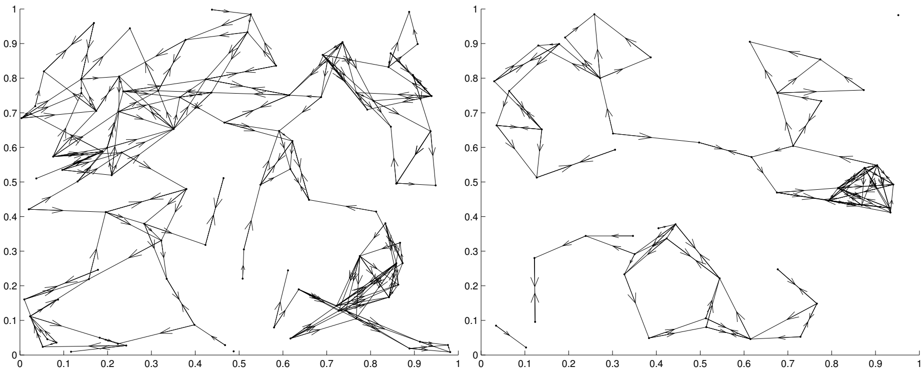

Fig. 3. n=100 r=0.2 a=1.5; n=50 r=0.2 R=3.14

sensors manage to compute their location. Notice that 90% is arbitrarily chosen. However, it is important not to consider the situation where all the sensors have to be localized for the success of the localizations algorithm. Indeed, if the number of sensors is reasonable $(50, 100)$ some of them can even be isolated. The 90% is chosen in order to avoid taking into account such sensors in our numerical computations. The process is probabilistic since it depends on the random location of sensors as well as on the random set of initially localized sensors. Hence, it does make sense to look for the probability of success of the algorithm. Our interest here is to numerically investigate the probability of success of the algorithm with respect to the network parameters.

Although the algorithm makes use of a clock we do not assume that the network is synchronized. Actually, we assume that the time-span of an emission is contained in a clock interval. So, only emissions occurring in a given clock interval can lead to collision. It is important to notice that the main impact of the waiting time is to reduce the collisions occurring between sensors which are close from each others. Indeed, these sensors are prone to receive location data's at the the same at hence, to become localized at the same time. The choice of the waiting time belonging to $[0, \log(n)]$ is motivated in [2].

4 Numerical Methods

The numerical methods we use to compute the chance of success with respect of the networks parameters of the localization algorithm are particularization of stochastic estimation procedures. These methods were first introduced in [24]. Since their introduction these methods have been widely applied to deal with stochastic estimation problems, see [22].

To apply these methods, we first need to define the probabilistic framework. In our problem what is probabilistic is the communication graph (which depends on the sensors location as well as on the orientation of the emission) and on the initially localized sensors. We denote by Adj the set of adjacency matrix which corresponds to possible communication graphs. This set is a probabilistic space and there exists a probability measure on it which does not need to be explicited. We denote by Ω the set of subset of sensors which are initially localized. This set is also a probabilistic space embodied with a probability measure corresponding to choosing l of them among the set of sensors. In a given set of computation, all the network parameters are kept constant except the angle on transmission α. Then, the localization algorithm can be seen as a map

$$L : Adj \times \Omega \times [0, 2\pi] \to \{0, 1\} \tag{1}$$

where $L(a, \omega, \alpha)$ is 0 or 1 depending on the success (more than 90% of localized sensors) or failure of the localization algorithm. The fixed network parameters are omitted to simplify the notation. Given a particular value $p \in [0, 1]$, our problem is to find the particular value of α such that

$$\mathrm{Prob}(L(\cdot, \cdot, \alpha) = 1) = p. \tag{2}$$

In the formula above, the probability is taken with respect to the $Adj \times \Omega$ space and α is a definite numerical value. This is why we use the 'dot' notation $L(\cdot, \cdot, \alpha)$. Although the underlying probability space has to be taken in mind, we also use the simplified notation $L(\alpha)$. The computations introduced below are based on simulations of the localization algorithm. At each step of the computations a communication graph is generated, a fixed number of initially localized sensors are determined and the process is simulated. This leads to an observed success or failure of the localization algorithm. This observation is denoted by $L(\alpha)$.

To solve this problem we use stochastic estimation methods in the region where $\frac{d}{d\alpha}\mathrm{Prob}(L(\alpha))$ is large since the convergence speed is proportional to this term (see later). In the regions where the above mentioned derivative is small, hence slowing the convergence rate of the stochastic estimation method, we use the typical estimator which consists in averaging the results of a large number of observations. In both cases, we check that the results belong to a confidence interval of 3 degrees with probability 95% [15]. To construct the confidence interval we basically use 20 estimates and the mean value as the result when the confidence interval is much smaller than 3 degrees. Basically, the number of iterations of the stochastic estimation methods to compute one estimate is about 30'000 ($\alpha_{30'000}$ in Theorem **??**). In the region where the method becomes not efficient enough we use the mean of about 100'000 observations to reach the fixed confidence interval.

To proceed to the computation with the stochastic estimation method, we first fix some parameters, namely n the number of sensors and r the radius of emission. Then, we choose a value p of the probability of success and look for the corresponding value of the angle of emission α. To avoid the choice of a value of p too big and hence not corresponding to any α, we compute $\tilde{\alpha}$ such that $m(\tilde{\alpha})$ is maximal with a Kiefer-Wolfovitz algorithm [18] and estimate its value by averaging about $100'000$ observations (the procedure is done about 20 times to construct the confidence interval). The stochastic estimation algorithm is based on the hypothesis that there exist a value α^* of the parameter such that

$$m(\alpha^*) = \operatorname{Prob}\big(L(\alpha^*) = 1\big) = p, \tag{3}$$

and

$$m'(\alpha^*) > 0. \tag{4}$$

Then, it can be proven [24, 31] that the sequence $(\alpha_n)_{n\geq 0}$ recursively defined by

$$\alpha_{n+1} = \alpha_n + \frac{1}{n}\big(p - L(\alpha_n)\big) \tag{5}$$

converges to α^*, i.e. $(\alpha_n \to \alpha^*)$. Moreover,

$$\sqrt{n}(\alpha_n - \alpha^*) \to N\left(0, \frac{\sigma^2}{(2m'(\alpha^*) - 1)}\right)$$

At each step of the computation a random communication graph has to be generated with the corresponding value of the parameter α_n. The localization algorithm is then simulated and the result is denoted by $L(\alpha_n)$ which is 0 or 1 accordingly to the success or failure of the algorithm. The fact that the function $m(\alpha)$ is differentiable is discussed in [19]. It is due to the fact that the state of outcomes is finite. Actuallay, the same argument is applicable to the situation where collisions are taken into account. By a suitable change of sign in the formula (5), one can cope with the situation where $m'(\alpha) < 0$.

5 Numerical Results

Besides the dependence on the network parameters, we are interested in observing the impact on the interferences between simultaneous emissions. Hence, we provide two set of experiments with the same parameters, one assumes that the communications are not altered by interferences and the second one takes into account interferences with the collisions model.

If interferences are not taken into account the set of random communication graphs on which the localization algorithm succeeds is monotone. Broadly speaking, this means that by adding new edges to a (successful) communication graph leads again to a communication graph on which the localization algorithm succeeds. In our setting of experiments, edges are added by increasing the angle of emission α. This is a particular situation for which there are general applicable theoretical results. In particular, sharp threshold is expected as the number of sensors increases [11, 12, 5, 30]. This means that if the number of sensors is large enough we should observe that, as the angle of emission increases, the probability of success suddenly changes from small values (close to 0) to larger values (close to 1). However, that such a sharp threshold can be observed as the interferences are taken into account is no at all an evidence from a theoretical point of view.

To conduct the numerical experiments, we fix n the number of sensors as well as the radius of emission r and the number of initially localized sensors (anchors). Then, we look for the curve describing the probability of success of the localization protocol with respect to the angle of emission α, i.e. $\text{Prob}(L(\alpha) = 1)$. The numerical experiments are conducted once considering the interferences between simultaneous emissions and once without. This allows measuring the impact of the interferences on the localization algorithm. On a same figure, the experiments are repeated with different values of the radius of emission r, keeping n constant, to measure the impact on increasing the radius on emission. Proceeding this way, we have to keep in mind that increasing the radius of emission increases the energy consumption. Indeed, the energy consumed increases as r^γ, where $2 \leq \gamma \leq 5$ depending on the environment conditions. This point is very important to be considered since minimizing the energy consumption is a key issue in wireless sensor networks.

The numerical experiments introduced above are repeated with different number of sensors n and different number of anchors.

In Figure 4, the number of sensors is $n = 50$ and 5 of them are anchors. The right picture shows the numerical results without collisions.

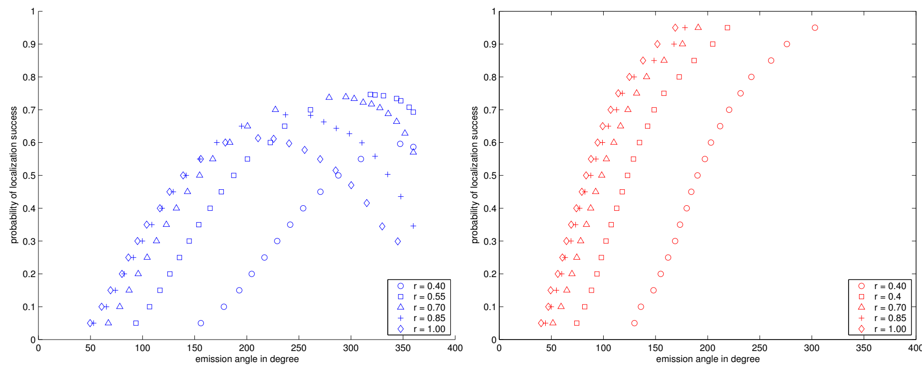

Fig. 4. $n = 50$ sensors, 5 anchors, with collisions (left) and without (right). The curves depicted from right to left are going in increasing the radius of emission. Waiting time in $[0, 4]$

It is observed, as theoretically expected, that as the radius of emission increases the angle of emission necessary to ensure the same probability of success decreases. As colisions are taken into account one observe on Figures 4,5,6 that increasing the angle of emissions increases initially the chance of success up to a maximal value and afterwards the chance of succes decreases. On these figures one can also observe that for large value of the emission angle, the performances are better with small radius of emission. The qualitative behavior does not change as the number of anchors changes.

It is worth to stress the importance of the waiting time, compare Figures 4 and 7 as well as the results plotted in Figures 8,9. Sensors which receive a location data at a same time wait for a random time in $0, \ldots, \log_e(n)$, in order to reduce the interferences between such stations [2]. Our numerical computations confirm that this waiting time is efficient in reducing the impact of the interferences on the localization algorithm. Indeed, for large values of n the number of sensors composing the networks, we observe, see in particular Figure 9, that the performances of the algorithm with and without interferences are very close from each other.

Although this observation is intuitively clear when assuming no interferences, it is not at all evident that this can still be observed when considering the interferences between simultaneous emissions. However, it is a general observation (see Figures 4,8,9,5,6,7) that increasing the angle of emission increases the chance of success of the localization algorithm.

To reinforce this observation we run simulations of the localization algorithm without waiting time and observe that in the same condition the localization algorithm fails nearly all the time to locate a significant set of sensors. Moreover, the bound $\log_e(n) \approx 4$ for the waiting time is obtained with an asymptotic analysis and hence, is valid as the number of sensors is large enough. Actually, we observe as the number of sensors is smaller, Figure 4, that the impact of the interferences on the performance of the localization algorithm is much more important. We ran a new set of simulations with $n = 50$ sensors and a waiting time bounded by 10. The results are plotted in Figure 7 and comparing the results obtained without interferences (right of Figure 4) it is clear that the impact of a longer waiting time on improving the performances of the algorithm is important.

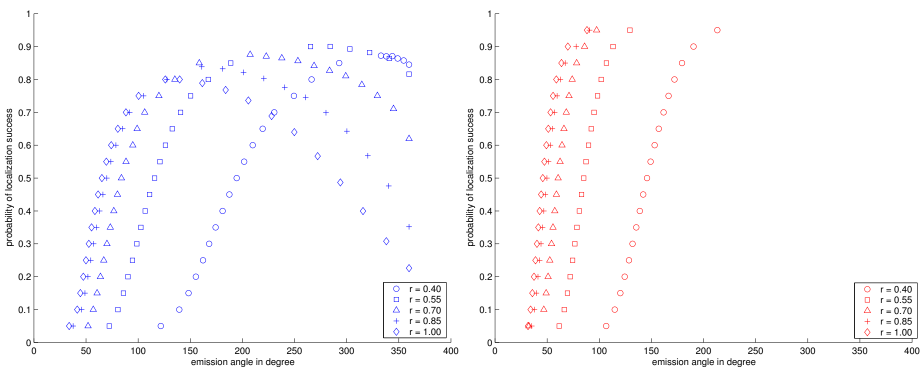

Fig. 5. $n = 50$ sensors, 10 anchors, with collisions (left) and without (right). The curves depicted from right to left are going in increasing the radius of emission. Waiting time in $[0, 4]$

In Figures 4, 5, 6, the probability of success of the algorithm is plotted with $n = 50$ sensors with respectively 5, 10 and 15 anchors. Only minor improvements can be observed by comparing the numerical results.

With the numerical experiments of Figure 4, 8, 9 it is possible to estimate the impact of the shape of the emission pattern on the success of the localization algorithm. For this purpose, we consider a value of $p = 0.45$ choosen arbitrarily. For each couple (r_i, α_i) leading to a probability of success of $p = 0.45$ we compute $\alpha_i \times r_i^2$ which is proportional to the area of the emission pattern. The results are plotted in Figures 10, 11, 12 corresponding respectively to $n = 50, 100, 1000$ sensors, i.e. couples (r_i, α_i) are measured on the Figures 4, 8, 9, respectively. The numerical results show that the probability of success depends on the product $\alpha \times r^2$. Actually, the probability depends on the shape of the emission pattern

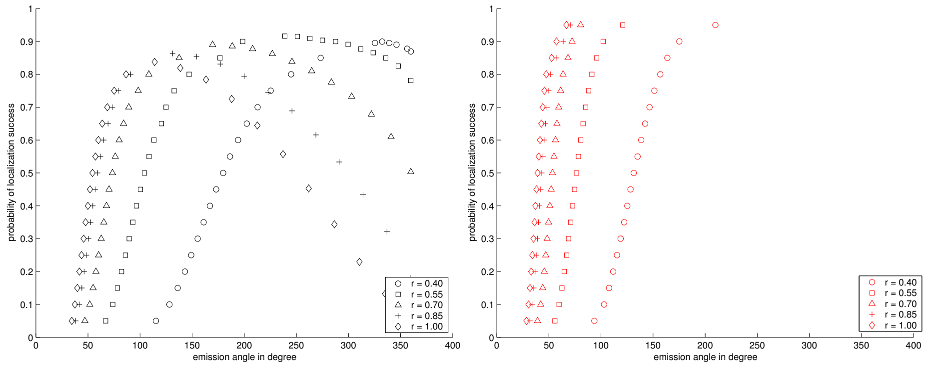

Fig. 6. $n = 50$ sensors, 15 anchors, with collisions (left) and without (right). The curves depicted from right to left are going in increasing the radius of emission. Waiting time in $[0, 4]$

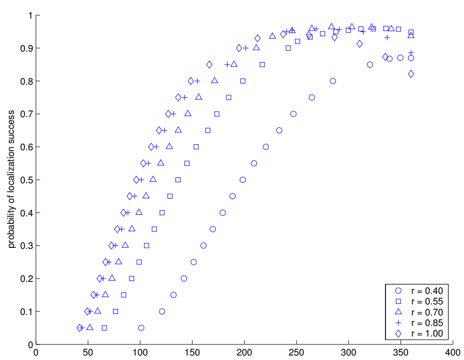

Fig. 7. $n = 50$ sensors, 5 anchors with collisions. The curves depicted from right to left are going in increasing the radius of emission. Waiting time in $[0, 10]$

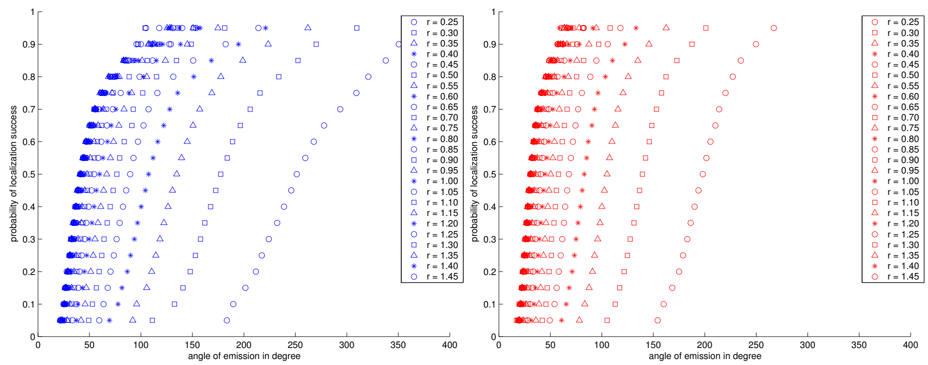

Fig. 8. $n = 100$ sensors, 5 anchors, with collisions (left) and without (right). The curves depicted from right to left are going in increasing the radius of emission. Waiting time in $[0, 5]$

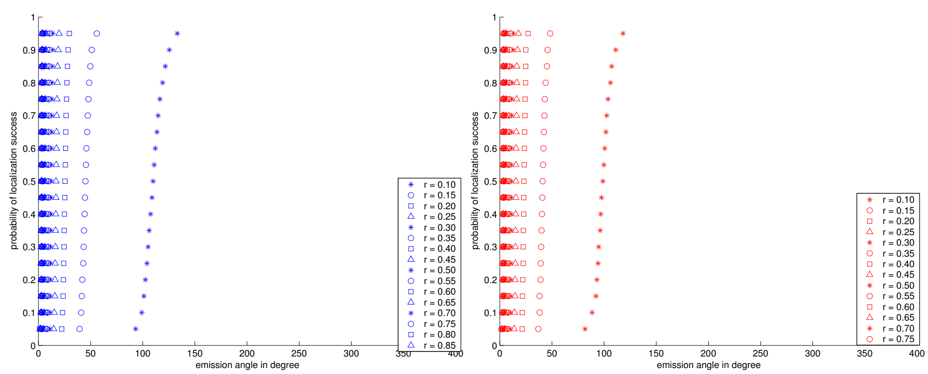

Fig. 9. $n = 1000$ sensors, 5 anchors, with collisions (left) and without (right). The curves depicted from right to left are going in increasing the radius of emission. Waiting time in $[0, 7]$

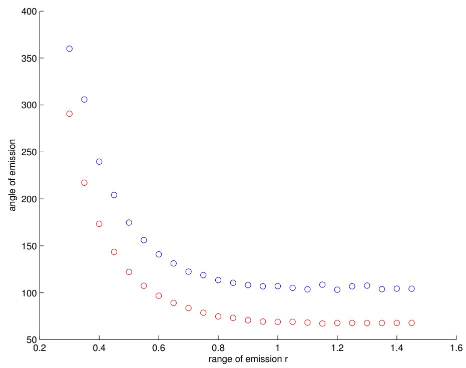

Fig. 10. $n = 50$ sensors, 5 anchors, with collisions (blue) and without (red). Variation of the area of the emission pattern (with $p = 0.45$ constant) as a function of r. Plot of $\alpha(x) \times x^2$, $x = 6, 7, \ldots$ with respect to x

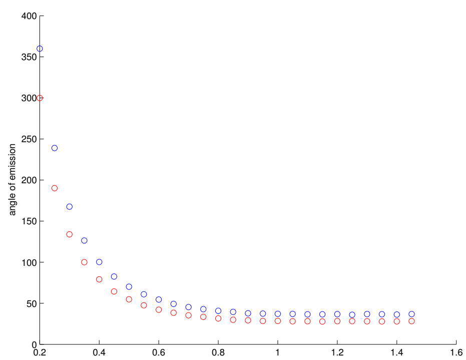

Fig. 11. $n = 100$ sensors, 5 anchors, with collisions (blue) and without (red). Variation of the area of the emission pattern (with $p = 0.45$ constant) as a function of r. Plot of $\alpha(x) \times x^2$, $x = 6, 7, \ldots$ with respect to x

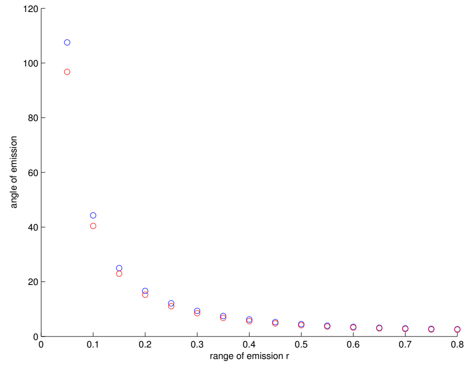

Fig. 12. $n = 1000$ sensors, 5 anchors, with collisions (blue) and without (red). Variation of the area of the emission pattern (with $p = 0.45$ constant) as a function of r. Plot of $\alpha(x) \times x^2$, $x = 6, 7, \ldots$ with respect to x

and not uniquely on the area covered. This leads to the question whether there exists an optimal shape of the emission pattern? We can obtain a first answer by restricting the shape of the emission pattern to be a sector of the disk and assume that the energy consumption is proportional to the area covered by the emission pattern. The numerical results in Figures 10, 11, 12 show that to minimize the energy consumption, keeping the probability of success constant, the radius of emission has to be chosen as small as possible (and increase the angle of emission to keep constant the probability of success). In our setting this implies that the optimal shape of the emission pattern for the localization problem is the circle. More generally, we can postulate that the symmetric radiation pattern is much more appropriate to the localization algorithm.

We should mention that the numerical methods used in this paper prove to work well and lead to some useful numerical results. The computations show clearly the relevance of the waiting time before emitting. This is also relevant to model dynamical behavior of sensor networks.

The threshold behavior of the probability of success of the localization algorithm as interferences are not taken into account can be theoretically explained. Such results are general and based on monotone events. In our setting, an event such as the success of the localization protocol, is monotone if by adding some edges the localization process is still successfull. Once we consider the interferences, the event is no longer monotone since adding some edges can lead to harmful interferences preventing the success of the localization algorithm.

In [19] it is pointed out that the slope of the probability of success is related to the expected number of pivotal sensors, i.e. sensors which cannot be removed without preventing the success of the localization algorithm. Numerical methods are used to estimate this slope. The practical interest of estimating the expected number of pivotal sensors is due to the fact that this leads to an estimate of the network robustness. The less pivotal sensors, the more robust is the network. Again, this result, based on an application of Russo's formula, is theoretically validated only when no interferences are taken into account and development of a theoretical framework as interferences are considered would be of practical interest in sensor networks.

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