# Infinite Qualitative Simulations by Means of Constraint Programming

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Abstract. We introduce a constraint-based framework for studying infinite qualitative simulations concerned with contingencies such as time, space, shape, size, abstracted into a finite set of qualitative relations. To define the simulations we combine constraints that formalize the background knowledge concerned with qualitative reasoning with appropriate inter-state constraints that are formulated using linear temporal logic. We implemented this approach in a constraint programming system (ECL<sup>*i*</sup>PS<sup>*e*</sup>) by drawing on the ideas from bounded model checking. The implementation became realistic only after several rounds of optimizations and experimentation with various heuristics.

The resulting system allows us to test and modify the problem specifications in a straightforward way and to combine various knowledge aspects. To demonstrate the expressiveness and simplicity of this approach we discuss in detail two examples: a navigation problem and a simulation of juggling.

### 1 Introduction

#### 1.1 Background

**Qualitative reasoning** was introduced in AI to abstract from numeric quantities, such as the precise time of an event, or the location or trajectory of an object in space, and to reason instead on the level of appropriate abstractions. Two different forms of qualitative reasoning were studied. The first one is concerned with reasoning about continuous change in physical systems, monitoring streams of observations and simulating behaviours, to name a few applications. The main techniques used are qualitative differential equations, constraint propagation and discrete state graphs. For a thorough introduction see [15].

The second form of qualitative reasoning focuses on the study of contingencies such as time, space, shape, size, directions, through an abstraction of the quantitative information into a finite set of qualitative relations. One then relies on complete knowledge about the interrelationship between these qualitative relations. This approach is exemplified by temporal reasoning due to [1], spatial reasoning introduced in [10] and [19], reasoning about cardinal directions (such as North, Northwest); see, e.g., [17], etc. In this paper we study the second form of qualitative reasoning. Our aim is to show how infinite qualitative simulations can be naturally formalized by means of temporal logic and constraint satisfaction problems. Our approach allows us to use generic constraint programming systems rather than specialized qualitative reasoning systems. By a **qualitative simulation** we mean a reasoning about possible evolutions in time of models capturing qualitative information. One assumes that time is discrete and that only changes adhering to some desired format occur at each stage. Qualitative simulation in the first framework is discussed in [16], while qualitative spatial simulation is considered in [9].

#### 1.2 Approach

In the traditional constraint-based approach to qualitative reasoning the qualitative relations (for example overlap) are represented as constraints over variables with infinite domains (for example closed subsets of  $\mathcal{R}^2$ ) and path-consistency is used as the constraint propagation; see, e.g., [11].

In our approach we represent qualitative relations *as variables*. This allows us to trade path-consistency for hyper-arc consistency which is directly available in most constraint programming systems, and to combine in a simple way various theories constituting the *background knowledge*. In turn, the *domain specific knowledge* about simulations is formulated using the linear temporal logic. These temporal formulas are subsequently translated into constraints.

Standard techniques of constraint programming combined with techniques from bounded model checking can then be used to generate simulations. To support this claim, we implemented this approach in the constraint programming system  $\mathrm{ECL}^i\mathrm{PS}^e$ . However, this approach became realistic only after fine-tuning of the translation of temporal formulas to constraints and a judicious choice of branching strategy and constraint propagation. To show its usefulness we discuss in detail two case studies. In each of them the solutions were successfully found by our implementation, though for different problems different heuristic had to be used.

The program is easy to use and to interact with. In fact, in some of the case studies we found by analyzing the generated solutions that the specifications were incomplete. In each case, thanks to the fact that the domain specific knowledge is formulated using temporal logic formulas, we could add the missing specifications in a straightforward way.

#### 1.3 Structure of the paper

In Section 2 we discuss examples of qualitative reasoning and in Section 3 explain our formalization of the qualitative reasoning by means of constraints. Next, in Section 4 we deal with qualitative simulations by introducing inter-state constraints which connect different stages of simulation and determine which scenarios are allowed. These constraints are defined using linear temporal logic. Their semantics is defined employing the concept of a cyclic path borrowed from the bounded model checking approach (see [5]) for testing validity of temporal formulas.

In Section 5 we explain how the inter-state constraints are translated to constraints of the underlying background knowledge. Next, in Section 6 we discuss technical issues pertaining to our implementation that generates infinite qualitative simulations. In the subsequent two sections we report on our case studies. Finally, in Section 9 we discuss the related work.

# 2 Qualitative Reasoning: Setup and Examples

As already said, in qualitative reasoning, one abstracts from the numeric quantities and reasons instead on the level of their abstractions. These abstractions are provided in the form of a finite *set of qualitative relations*, which should be contrasted with the infinite set of possibilities available at the numeric level. After determining the 'background knowledge' about these qualitative relations we can derive conclusions on an abstract level that would be difficult to achieve on the numeric level. The following three examples illustrate the matters.

*Example 1 (Region Connection Calculus).* The qualitative spatial reasoning with topology introduced in [19] and [10] is concerned with the following set of qualitative relations:

 $RCC8 := \{ disjoint, meet, overlap, equal, covers, contains, covered-by, inside \}.$ 

The objects under consideration are here spatial regions, and each region pair is in precisely one RCC8 relation; see Fig. 1.

The background knowledge in this case is the set of possible relation triples pertaining to triples of regions. For example, the relation triple  $\langle meet, meet, meet \rangle$  is possible since there exist three regions pairwise touching each other. In contrast, the triple  $\langle inside, inside, disjoint \rangle$  is impossible since for any three regions A, B, C, if A is inside B and B is inside C, then A cannot be disjoint

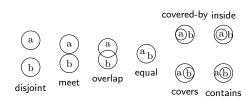


Fig. 1. The eight RCC8 relations

with C. The set of possible triples is called the *composition table*; it is presented in the above two papers. In total, the table lists 193 relation triples.

*Example 2 (Cardinal Directions).* Qualitative reasoning dealing with relative directional information about point objects can be formalized using the set of cardinal directions

$$\mathsf{Dir} := \{\mathsf{N}, \mathsf{NE}, \mathsf{E}, \mathsf{SE}, \mathsf{S}, \mathsf{SW}, \mathsf{W}, \mathsf{NW}, \mathsf{EQ}\},\$$

that consists of the wind rose directions together with the identity relation denoted by EQ; see [12]. The composition table for this form of qualitative reasoning is provided in [17].

*Example 3 (Relative Size).* Qualitative reasoning about relative size of objects is captured by the relations in the set

Size :=  $\{<, =, >\}$ .

The corresponding composition table is given in [13].

Other examples of qualitative reasoning deal with shape, directional information about regions or cyclic ordering of orientations. In some of them the qualitative relations are non-binary and the background knowledge is more complex than the composition table. To simplify the exposition we assume in the following binary qualitative relations.

# 3 Formalization of Qualitative Reasoning

In what follows we follow the standard terminology of constraint programming. So by a **constraint** on a sequence  $x_1, \ldots, x_m$  of variables with respective domains  $dom(x_1), \ldots, dom(x_m)$  we mean a subset of  $dom(x_1) \times \cdots \times dom(x_m)$ . A **constraint satisfaction problem** (CSP) consists of a finite sequence of variables X with respective domains and a finite set of constraints, each on a subsequence of X. A **solution** to a CSP is an assignment of values to its variables from their domains that satisfies all constraints.

We study here CSPs with finite domains and solve them using a top-down search interleaved with constraint propagation. In our implementation we use a heuristics-controlled *domain partitioning* as the branching strategy and *hyper-arc consistency* of [18] as the constraint propagation.

We formalize the qualitative reasoning within the CSP framework as follows. We assume a finite set of objects  $\mathcal{O}$ , a finite set of binary qualitative relations  $\mathcal{Q}$  and a ternary relation CT representing the composition table. Each qualitative relation between objects is modelled as a constraint variable the domain of which is a subset of  $\mathcal{Q}$ . We stipulate such a *relation variable* for each ordered pair of objects and organize these variables in an array *Rel* which we call a *qualitative array*.

For each triple a, b, c of elements of  $\mathcal{O}$  we have then a ternary constraint comp on the corresponding variables:

$$\begin{aligned} \mathsf{comp}(Rel[a,b],\ Rel[b,c],\ Rel[a,c]) &:= \\ CT \cap (dom(Rel[a,b]) \times dom(Rel[b,c]) \times dom(Rel[a,c])). \end{aligned}$$

To assume integrity of this approach we also adopt for each ordered pair a, b of elements of  $\mathcal{O}$ , the binary constraint  $\operatorname{conv}(\operatorname{Rel}[a, b], \operatorname{Rel}[b, a])$  that represents the converse relation table, and postulate that  $\operatorname{Rel}[a, a] = \operatorname{equal}$  for all  $a \in \mathcal{O}$ .

We call these constraints *integrity constraints*.

# 4 Specifying Simulations using Temporal Logic

In our framework we assume a *conceptual neighbourhood* between the qualitative relations. This is a binary relation neighbour between the elements of the relation set Q describing which *atomic* changes in the qualitative relations are admissible. So only 'smooth' transitions are allowed. For example, in the case of the Region Connection Calculus from Example 1, the relation between two regions can change from disjoint to overlap only indirectly via meet. The neighbourhood relation for RCC8 has 22 elements such as (disjoint, meet), (meet, meet), (meet, overlap) and their converses and is shown in Fig. 2.

We assume here that objects can change size during the simulation. If we wish to disallow this possibility, then the pairs (equal, covered-by), (equal, covers), (equal, inside), (equal, contains) and their converses should be excluded from the conceptual neighbourhood relation.

In what follows we represent each stage t of a simulation by a CSP  $\mathcal{P}_t$  uniquely determined by a qualitative array  $Q_t$  and its integrity constraints. Here t is a variable ranging over the set of natural numbers that represents discrete time.

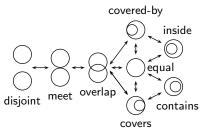


Fig. 2. The RCC8 neighbourhood relation

Instead of  $Q_t[a, b]$  we also write Q[a, b, t], as in fact we deal with a *ternary* array.

The stages are linked by *inter-state constraints* that determine which scenarios are allowed. The inter-state constraints always include constraints stipulating that the atomic changes respect the conceptual neighbourhood relation. Other inter-state constraints are problem dependent.

A qualitative simulation corresponds then to a CSP consisting of *stages* all of which satisfy the integrity constraints and the problem dependent constraints, and such that the inter-state constraints are satisfied. To describe the inter-state constraints we use *atomic formulas* of the form

$$Q[a,b] \in \mathcal{R}, \ Q[a,b] \notin \mathcal{R}, \ Q[a,b] = q, \ Q[a,b] \neq q,$$

where  $\mathcal{R} \subseteq \mathcal{Q}$  and  $q \in \mathcal{Q}$ . As the latter three forms reduce to the first one, we deal with the first form only.

We employ a propositional linear temporal logic with four *temporal operators*,  $\diamond$  (eventually),  $\bigcirc$  (next time),  $\Box$  (from now on) and U (until), and with the usual connectives. We use bounded quantification as abbreviations, e.g.,  $\phi(o_1) \lor \ldots \lor \phi(o_k)$  abbreviates to  $\exists A \in \{o_1, \ldots, o_k\}$ .  $\phi(A)$ .

Given a finite set of temporal formulas formalizing the inter-state constraints we wish then to exhibit a simulation in the form of an infinite sequence of 'atomic' transitions which satisfies these formulas and respects the integrity constraints. In the Section 5 we explain how each temporal formula is translated into a sequence of constraints.

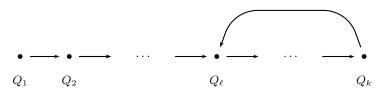


Fig. 3. A  $(k - \ell)$ -loop

**Paths and loops** We now proceed by explaining the meaning of a temporal formula  $\phi$  with respect to an arbitrary infinite sequence of qualitative arrays,

$$\pi := Q_1, Q_2, \ldots,$$

that we call a *path*. Our goal is to implement this semantics, so we proceed in two stages:

- First we provide a definition with respect to an arbitrary path.
- Then we limit our attention to specific types of paths, which are unfoldings of a loop.

In effect, we use here the approach employed in *bounded model checking*; see [5]. Additionally, to implement this approach in a simple way, we use a recursive definition of meaning of the temporal operators instead of the inductive one.

We write  $\models_{\pi} \phi$  to express that  $\phi$  holds along the path  $\pi$ . We say then that  $\pi$  satisfies  $\phi$ . Given  $\pi := Q_1, Q_2, \ldots$  we denote by  $\pi_i$  the subpath  $Q_i, Q_{i+1}, \ldots$ . Hence  $\pi_1 = \pi$ . The semantics is defined in the standard way, with the exception that the atomic formulas refer to qualitative arrays. The semantics of connectives is defined independently of the temporal aspect of the formula. For other formulas we proceed by recursion as follows:

$\models_{\pi_i} Q[a,b] \in \mathcal{R}$	if	$Q[a,b,i] \in \mathcal{R};$
$\models_{\pi_i} \bigcirc \phi$	if	$\models_{\pi_{i+1}} \phi;$
$\models_{\pi} \Box \phi$	if	$\models_{\pi} \phi \text{ and } \models_{\pi} \bigcirc \Box \phi;$
$\models_{\pi} \diamondsuit \phi$	if	$\models_{\pi} \phi \text{ or } \models_{\pi} \bigcirc \Diamond \phi;$
$\models_{\pi} \chi \cup \phi$	if	$\models_{\pi} \phi \text{ or } \models_{\pi} \chi \land \bigcirc (\chi \cup \phi).$

Next, we limit our attention to paths that are loops. Following [5] we call a path  $\pi := Q_1, Q_2, \ldots$  a  $(k - \ell)$ -loop if

$$\pi = u \cdot v^*$$
 with  $u := Q_1, \dots, Q_{\ell-1}$  and  $v := Q_\ell, \dots, Q_k;$ 

see Fig. 3. By a general result, see [5], for every temporal formula  $\phi$  if a path exists that satisfies it, then a loop path exists that satisfies  $\phi$ . This is exploited by our algorithm. Given a finite set of temporal formulas  $\Phi$  it tries to find a path  $\pi := Q_1, Q_2, \ldots$  consisting of qualitative arrays that satisfies all formulas in  $\Phi$ , by repeatedly trying to construct an infinite  $(k - \ell)$ -loop. Each such  $(k - \ell)$ -loop can be finitely represented using k qualitative arrays. The algorithm is discussed in Section 6.

# 5 Temporal Formulas as Constraints

A temporal formula restricts the sequence of qualitative arrays at consecutive stages (time instances). We now show how to translate these formulas to constraints in a generic target constraint language. The translation is based on unravelling the temporal operators into primitive Boolean constraints and primitive constraints accessing the qualitative arrays. Furthermore, we discuss a variation of this translation that retains more structure of the formula, using non-Boolean array constraints.

We assume that the target constraint language has primitive Boolean constraints and reified versions of simple comparison and arithmetic constraints. (Recall that a reified constraint generalizes its base constraint by associating with it a Boolean variable reflecting its truth.)

Paths with and without loops. Both finite and infinite paths can be accommodated within one constraint model. To this end, we view a finite sequence of qualitative arrays together with their integrity constraints as a single CSP. The sequence  $Q_1, \ldots, Q_k$  can represent both

an infinite path  $\pi = (Q_1, \ldots, Q_{\ell-1}) \cdot (Q_\ell, \ldots, Q_k)^*$ , for some  $\ell \ge 1$  and  $k \ge \ell$ , or a finite path  $\pi = Q_1, \ldots, Q_k$ .

To distinguish between these cases, we interpret  $\ell$  as a constraint variable. We define  $\ell = k+1$  to mean that there is no loop, so we have  $dom(\ell) = \{1, \ldots, k+1\}$ . A new placeholder array  $Q_{k+1}$  is appended to the sequence of qualitative arrays, without integrity constraints except the neighbourhood constraints connecting it to  $Q_k$ . Finally, possible looping is realized by conditional equality constraints

$$(\ell = j) \rightarrow (Q_j = Q_{k+1})$$

for all  $j \in \{1, ..., k\}$ . Here  $Q_p = Q_q$  is an equality between qualitative arrays, i.e., the conjunction of equalities between the corresponding array elements.

Translation into constraints. We denote by  $cons(\phi, i) \equiv b$  the sequence of constraints representing the fact that formula  $\phi$  has the truth value b on the path  $\pi_i$ . The translation of a formula  $\phi$  on  $Q_1, \ldots, Q_k$  is initiated with  $cons(\phi, 1) \equiv 1$ .

We define the constraint translation inductively as follows.

#### Atomic formulas:

 $cons(true, i) \equiv b$  translates to b = 1; $cons(Q[a_1, a_2] \in R, i) \equiv b$  translates to  $Q[a_1, a_2, i] = q, (q \in R) \equiv b.$ 

#### **Connectives:**

 $cons(\neg \phi, i) \equiv b$  translates to  $(\neg b') \equiv b, cons(\phi, i) \equiv b';$ 

other connectives are translated analogously.

**Formula**  $\bigcirc \phi$ : The next-time operator takes potential loops into account.

$$\begin{aligned} cons(\bigcirc \phi, i) \equiv b & \text{translates to} \\ & \text{if } i < k \text{ then} \\ & cons(\phi, i+1) \equiv b; \\ & \text{if } i = k \text{ then} \\ & \ell = k+1 \to b = 0, \\ & \ell \leqslant k & \to b = \bigwedge_{j \in \{1, \dots, k\}} \left(\ell = j \to cons(\phi, j)\right). \end{aligned}$$

**Formula**  $\Diamond \phi$ : We translate  $\Diamond \phi$  by unravelling its recursive definition  $\phi \lor \bigcirc \Diamond \phi$ . It suffices to do so a finite number  $n_{\text{unravel}}$  of steps beyond the current state, namely the number of steps to reach the loop,  $\max(0, \ell - i)$ , plus the length of the loop,  $k - \ell$ . A subsequent unravelling step is unneeded as it would reach an already visited state. We find

 $n_{\text{unravel}} = k - \min(\ell, i).$ 

This equation is a simplification in that  $\ell$  is assumed constant. For a variable  $\ell$ , we 'pessimistically' replace  $\ell$  here by the least value in its domain,  $min(\ell)$ .

**Formulas**  $\Box \phi$  and  $\phi \cup \psi$ : These formulas are processed analogously to  $\Diamond \phi$ .

The result of translating a formula is a set of primitive reified Boolean constraints and accesses to the qualitative arrays at certain times.

Translation using array constraints. Unravelling the temporal operators leads to a creation of several identical copies of subformulas. In the case of the  $\diamond$  temporal operator where the subformulas in essence are connected disjunctively, we can do better by translating differently. The idea is to push disjunctive information inside the variable domains. We use **array constraints**, which treat array lookups such as  $x = A[y_1, \ldots, y_n]$  as a constraint on the variables  $x, y_1, \ldots, y_n$ and the (possibly variable) elements of the array A. Array constraints generalize the classic element constraint.

Since we introduce new constraint variables when translating  $\Diamond \phi$  using array constraints, one needs to be careful when  $\Diamond \phi$  occurs in the scope of a negation. Constraint variables are implicitly existentially quantified, therefore negation cannot be implemented by a simple inversion of truth values. We address this difficulty by first transforming a formula into a negation normal form, using the standard equivalences of propositional and temporal logic.

The constraint translations using array constraints (where different from above) follow. The crucial difference to the unravelling translation is that here i is a constraint variable.

**Formula**  $\Diamond \phi$ : A fresh variable *j* ranging over state indices is introduced, marking the state at which  $\phi$  is examined. The first possible state is the current position or the loop start, whichever is earlier. Both  $\ell$  and *i* are constraint

variables, therefore their least possible values  $min(\ell)$ , min(i), respectively, are considered.

 $cons(\Diamond \phi, i) \equiv b$  translates to new j with  $dom(j) = \{1, \dots, k\},$  $\min(min(\ell), min(i)) \leq j,$  $cons(\phi, j) \equiv b.$ 

**Formula**  $\bigcirc \phi$ : This case is equivalent to the previous translation of  $\bigcirc \phi$ , but we now need to treat *i* as a variable. So both "if ... then" and  $\rightarrow$  are now implemented by Boolean constraints.

# 6 Implementation

Given a qualitative simulation problem formalized by means of integrity constraints and inter-state constraints formulated as temporal formulas, our program generates a solution if one exists or reports a failure. During its execution a sequence of CSPs is repeatedly constructed, starting with a single CSP that is repeatedly step-wise extended. The number of steps that need to be considered to conclude failure depends on the temporal formulas and is finite [5]. The sequence of CSPs can be viewed as a single finite CSP consisting of finite domain variables and constraints of a standard type and thus is each time solvable by generic constraint programming systems. The top-down search is implemented by means of a regular backtrack search algorithm based on a variable domain splitting and combined with constraint propagation.

The variable domain splitting is controlled by domain-specific heuristics if available. We make use of the specialized reasoning techniques due to [20] for RCC8 and due to [17] for the cardinal directions. In these studies maximal tractable subclasses of the respective calculi are identified and corresponding polynomial decision procedures for non-temporal qualitative problems are discussed. In our terminology, if the domain of each relation variable in a qualitative array belongs to a certain class, then a certain sequence of domain splittings intertwined with constraint propagation finds a solving instantiation of the variables without backtracking if one exists. However, here we deal with a more complex set-up: sequences of qualitative arrays together with arbitrary temporal constraints connecting them. These techniques can then still serve as heuristics. We use them in our implementation to split the variable domains in such a way that one of the subdomains belongs to a maximal tractable subclass of the respective calculus.

We implemented the algorithm and both translations of temporal formulas to constraints in the ECL<sup>i</sup>PS<sup>e</sup> constraint programming system [22]. The resulting program is about 2000 lines of code. We used as constraint propagation hyperarc consistency algorithms directly available in  $ECL^iPS^e$  in its fd and propia libraries and for array constraints through the implementation discussed in [6]. In the translations of the temporal formulas, following the insight from bounded model checking, redundancy in the resulting generation of constraints is reduced by sharing subformulas.

### 7 Case Study 1: Navigation

Consider a ship and three buoys forming a triangle. The problem is to generate a cyclic route of the ship around the buoys. We reason qualitatively with the cardinal directions of Example 2.

- First, we postulate that all objects occupy different positions:

 $\Box \, \forall a, b \in \mathcal{O}. \ a \neq b \ \rightarrow \ Q[a, b] \neq \mathsf{EQ}.$ 

- Without loss of generality we assume that the buoy positions are given by

 $\Box Q[buoy_a, buoy_c] = \mathsf{NW}, \ \Box Q[buoy_a, buoy_b] = \mathsf{SW}, \ \Box Q[buoy_b, buoy_c] = \mathsf{NW}$ 

and assume that the initial position of the ship is south of buoy c:

 $Q[ship, buoy_c] = S.$ 

 To ensure that the ship follows the required path around the buoys we stipulate:

$$\begin{split} \Box \left( Q[\text{ship}, \text{buoy}_c] = \mathsf{S} &\to & \Diamond (Q[\text{ship}, \text{buoy}_a] = \mathsf{W} \land \\ & \Diamond (Q[\text{ship}, \text{buoy}_b] = \mathsf{N} \land \\ & \Diamond (Q[\text{ship}, \text{buoy}_c] = \mathsf{E} \land \\ & \Diamond (Q[\text{ship}, \text{buoy}_c] = \mathsf{S} )))) \right). \end{split}$$

In this way we enforce an infinite circling of the ship around the buoys.

When fed with the above constraints our program generated the infinite path formed by the cycle through thirteen positions depicted in Fig. 4. The positions required to be visited are marked by bold circles. Each of them can be reached from the previous one through an atomic change in one or more qualitative relations between the ship and the buoys. One hour running time was not enough to succeed with the generic first-fail heuristic, but it took only 20s to find the cycle using the Dir-specific heuristic. The array constraint translation reduced this slightly to 15 s.

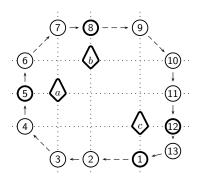


Fig. 4. Navigation path

The cycle found is a shortest cycle satisfying the specifications. Note that other, longer cycles exist as well. For example, when starting in position 1 the ship can first move to an 'intermediate' position between positions 1 and 2, characterized by:

 $Q[ship, buoy_c] = SW, \ Q[ship, buoy_a] = SE, \ Q[ship, buoy_b] = SE.$ 

We also examined a variant of this problem in which two ships are required to circle around the buoys while remaining in the N or NW relation w.r.t. each other. In this case the shortest cycle consisted of fifteen positions.

# 8 Case Study 2: Simulating of Juggling

Next, we consider a qualitative formalization of juggling. We view it as a process having an initialization phase followed by a proper juggling phase which is repeated. As such it fits well our qualitative simulation framework.

We consider two kinds of objects: the hands and the balls. For the sake of simplicity, we only distinguish the qualitative relations 'together', between a ball and a hand that holds it or between two touching balls, and 'apart'. This allows us to view the juggling domain as an instance of an existing topological framework: we identify 'together' and 'apart' with the relations meet and disjoint of the RCC8 calculus.

In our concrete study, we assume a single juggler (with two hands) and three balls. We aim to recreate the three-ball-cascade, see [14, p. 8]. So we have five objects:

 $\mathcal{O} := Hands \cup Balls,$ Hands := { left-hand, right-hand }, Balls := { ball\_i | i \in \{1, 2, 3\} }.

The constraints are as follows.

- We only represent the relations of being 'together' or 'apart':

 $\Box \forall x, y \in \mathcal{O}. \ (x \neq y \rightarrow Q[x, y] \in \{\mathsf{meet}, \mathsf{disjoint}\}).$ 

- The hands are always apart:

 $\Box Q[left-hand, right-hand] = disjoint.$ 

- A ball is never in both hands at the same time:

 $\Box \forall b \in Balls. \neg (Q[left-hand, b] = meet \land Q[right-hand, b] = meet).$ 

 From some state onwards, at any time instance at most one ball is in any hand:

- Two balls touch if and only if they are in the same hand:

 $\Box \, (\forall b_1, b_2 \in Balls. \ b_1 \neq b_2 \ \rightarrow$ 

 $(Q[b_1,b_2] = \mathsf{meet} \ \leftrightarrow \ \exists h \in \mathit{Hands}. \ (Q[h,b_1] = \mathsf{meet} \ \land \ Q[h,b_2] = \mathsf{meet}))).$ 

 A ball thrown from one hand remains in the air until it lands in the other hand:

$$\label{eq:constraint} \begin{array}{l} \Box \ (\forall b \in Balls. \ \forall h_1, h_2 \in Hands. \ h_1 \neq h_2 \land Q[h_1, b] = \mathsf{meet} \rightarrow \\ Q[h_1, b] = \mathsf{meet} \ \ \mathsf{U} \ \ (Q[h_1, b] = \mathsf{disjoint} \ \land \ \ Q[h_2, b] = \mathsf{disjoint} \ \land \\ (Q[h_1, b] = \mathsf{disjoint} \ \ \mathsf{U} \ \ Q[h_2, b] = \mathsf{meet}))). \end{array}$$

- A ball in the air will land before any other ball that is currently in a hand,

$$\begin{split} & \Box \left( \forall h_1, h_2 \in \mathit{Hands.} \; \forall b_1, b_2 \in \mathit{Balls.} \; Q[h_1, b_1] = \mathsf{disjoint} \land Q[h_2, b_2] = \mathsf{meet} \rightarrow \\ & Q[h_2, b_2] = \mathsf{meet} \; \mathsf{U} \left( (\forall h \in \mathit{Hands.} \; Q[h, b_2] = \mathsf{disjoint}) \\ & \mathsf{U} \left( \exists h \in \mathit{Hands.} \; Q[h, b_1] = \mathsf{meet}) \right) ). \end{split}$$

- No two balls are thrown at the same time:

$$\label{eq:constraint} \begin{split} & \Box \left( \forall b_1, b_2 \in Balls. \ b_1 \neq b_2 \rightarrow \forall h_1, h_2 \in Hands. \\ & \neg (Q[b_1, h_1] = \mathsf{meet} \land \bigcirc Q[b_1, h_1] = \mathsf{disjoint} \land \\ & Q[b_2, h_2] = \mathsf{meet} \land \bigcirc Q[b_2, h_2] = \mathsf{disjoint})). \end{split}$$

- A hand can interact with only one ball at a time:

$$\label{eq:constraint} \begin{array}{l} \Box \; \forall h \in Hands. \; \forall b_1 \in Balls. \\ (Q[h, b_1] = \mathsf{meet} \; \land \; \bigcirc Q[h, b_1] = \mathsf{disjoint} \; \lor \\ Q[h, b_1] = \mathsf{disjoint} \; \land \; \bigcirc Q[h, b_1] = \mathsf{meet}) \; \rightarrow \\ \forall b_2 \in Balls. \; b_1 \neq b_2 \; \rightarrow \\ (Q[h, b_2] = \mathsf{meet} \; \rightarrow \; \bigcirc Q[h, b_2] = \mathsf{meet}) \; \land \\ (Q[h, b_2] = \mathsf{disjoint} \; \rightarrow \; \bigcirc Q[h, b_2] = \mathsf{disjoint}). \end{array}$$

Initially balls 1 and 2 are in the left hand, while ball 3 is in the right hand:

 $Q[left-hand, ball_1] = meet, Q[left-hand, ball_2] = meet, Q[right-hand, ball_3] = meet.$ 

Note that the constraints enforce that the juggling continues forever. Our program finds an infinite simulation in the form of a path [1..2][3..8]<sup>\*</sup>; see Fig. 5. The running time was roughly 100s using the generic first-fail heuristic; the RCC8-specific heuristic, resulting in 43 min, was not useful.

We stress the fact that the complete specification of this problem is not straightforward. In fact, the interaction with our program revealed that the initial specification was incomplete. This led us to the introduction of the last constraint.

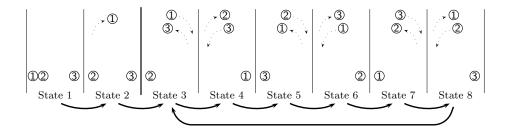


Fig. 5. Simulation of Juggling

#### Aspect Integration: Adding Cardinal Directions

The compositional nature of the 'relations as variables' approach makes it easy to integrate several spatial aspects (e.g., topology *and* size, direction, shape etc.) in one model. For the non-temporal case, we argued in [7] that the background knowledge on linking different aspects can be viewed as just another integrity constraint. Here we show that also qualitative simulation and aspect integration combine easily, by extending the juggling example with the cardinal directions.

As the subject of this paper is modelling and solving, not the actual interaspect background knowledge, we only explain the integration of the three relations meet, disjoint, equal with the cardinal directions Dir. We simply add

$$\mathsf{link}(Q[a,b], Q_{\mathsf{Dir}}[a,b]) := (Q[a,b] = \mathsf{equal}) \leftrightarrow (Q_{\mathsf{Dir}}[a,b] = \mathsf{EQ})$$

as the aspect linking constraint. It refers to the two respective qualitative arrays and is stated for all spatial objects a, b. We add the following domain-specific requirements to our specification of juggling:

$$Q_{\mathsf{Dir}}[left-hand, right-hand] = \mathsf{W};$$

$$\Box \forall b \in Balls. \forall h \in Hands. Q[b, h] = \mathsf{meet} \rightarrow Q_{\mathsf{Dir}}[b, h] = \mathsf{N};$$

$$\Box \forall b \in Balls. Q[b, left-hand] = \mathsf{disjoint} \land Q[b, right-hand] = \mathsf{disjoint} \rightarrow Q_{\mathsf{Dir}}[b, left-hand] \neq \mathsf{N} \land Q_{\mathsf{Dir}}[b, right-hand] \neq \mathsf{N}.$$

We state thus that a ball in a hand is 'above' that hand, and that a ball is not thrown straight upwards.

This simple augmentation of the juggling domain with directions yields the same first simulation as in the single-aspect case, but now with the RCC8 and Dir components. The ball/hand relation just alternates between N and NW (or NE).

We emphasize that it was straightforward to extend our implementation to achieve the integration of two aspects. The constraint propagation for the link constraints is achieved by the same generic hyper-arc consistency algorithm used for the single-aspect integrity constraints. This is in contrast to the 'relations as constraints' approach which requires new aspect integration *algorithms*; see, e.g., the bipath-consistency algorithm of [13].

# 9 Conclusions

**Related Work** The most common approach to qualitative simulation is the one discussed in [15, Chapter 5]. For a recent overview see [16]. It is based on a qualitative differential equation model (QDE) in which one abstracts from the usual differential equations by reasoning about a finite set of symbolic values (called *landmark values*). The resulting algorithm, called *QSIM*, constructs the tree of possible evolutions by repeatedly constructing the successor states. During this process CSPs are generated and solved. This approach is best suited to simulate the evolution of physical systems.

Our approach is inspired by the qualitative spatial simulation studied in [9], the main features of which are captured by the composition table and the neighbourhood relation discussed in Example 1. The distinction between the integrity and inter-state constraints is introduced there; however, the latter only link consecutive states in the simulation. As a result, our case studies are beyond their reach. Our experience with our program moreover suggests that the algorithm of [9] may not be a realistic basis for an efficient qualitative reasoning system.

To our knowledge the '(qualitative) relations as variables' approach to modelling qualitative reasoning was first used in [21], to deal with the qualitative temporal reasoning due to [1]. In [20] this approach is used in an argument to establish the quality of a generator of random scenarios, whilst the main part of this paper uses the customary 'relations as constraints' approach. In [2, pages 30-33] we applied the 'relations as variables' approach to model a qualitative spatial reasoning problem. In [7] we used it to deal in a simple way with aspect integration and in [3] to study qualitative planning problems.

In [8] various semantics for a programming language that combines temporal logic operators with constraint logic programming are studied. Finally, in the TLPLAN system of [4] temporal logic is used to support the construction of control rules that guide plan search. The planning system is based on an incremental forward-search, so the temporal formulas are unfolded one step at a time, in contrast to the translation into constraints in our constraint-based system.

**Summary** We introduced a constraint-based framework for describing infinite qualitative simulations. Simulations are formalized by means of inter-state constraints that are defined using linear temporal logic. This results in a high degree of expressiveness. These constraints are translated into a generic target constraint language. The qualitative relations are represented as domains of constraint variables. This makes the considered CSPs finite, allows one to use hyper-arc consistency as constraint propagation, and to integrate various knowledge aspects in a straightforward way by simply adding linking constraints.

We implemented this approach in a generic constraint programming system,  $\mathrm{ECL}^i\mathrm{PS}^e$ , using techniques from bounded model checking and by experimenting with various heuristics. The resulting system is conceptually simple and easy to use and allows for a straightforward modification of the problem specifications. We substantiated these claims by means of two detailed case studies.

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