SOME METHODS FOR NUMERICAL SOLUTION OF OPTIMAL MODELS IN SPATIAL-PRODUCTION PLANNING

A.E. Bakhtin
Institute of Economics & Industrial Engineering
Siberian Department of the USSR Academy of Sciences
Novosibirsk, USSR

This paper is devoted to problems of numerical solution of some economic-mathematical models used in long-term planning of spatial location of production.

By their structure, models of this kind are of a complicated character and consist as a rule of several closely interconnected submodels. In other words, one has to deal with a system of models. Each model of the system represents certain aspects in a complicated economic system and is meant to clarify certain special problems. The inputs of one model of the system are outputs for other models and affect, therefore, the results obtained with the help of the latter. For this reason, it becomes necessary to have such techniques for the analysis of systems of models and for the synthesis of obtained partial solutions which would allow one to obtain consistent solutions in the optimization system of models under consideration.

The problem of composition is difficult also because the models entering a system are in themselves mathematically difficult objects of investigation:

- they have great dimensionality and a complicated block structure;
- they contain integer conditions or non-linear dependences etc.

Below some examples of models used in long-term spatial-production planning are given and the ideas of the composition of complex solutions are briefly exposed.

Now we shall consider the simplest "production-transportation" model consisting of linear production and transportation submodels.

We introduce the following notation:

i the number of a productive unit located on the territory under consideration, $i = 1, \dots, m$;

k the number of a product (resource) produced (consumed), $k = 1, \dots, l$;

j the number of the way of production, $j = 1, \dots, n_i$;

$A^i = (a^i_{kj})_{\substack{k=1,\dots,l \\ j=1,\dots,n_i}}$ the matrix of the ways of production of the i-th unit;

$a^i = (a^i_k)_{k=1,\dots,l}$ the vector of constraints for the i-th unit;

$x^i = (x^i_j)_{j=1,\dots,n_i}$ the unknown vector of intensities in the use of production ways by the i-th unit;

$y^i = (y^i_k)_{k=1,\dots,l}$ the unknown vector, the k-th component of which equals the import-export balance of the k-th product for the i-th unit.

The sought vectors x^i, y^i must satisfy balance relations:

$$\begin{gathered} A^i x^i + y^i = a^i, \\ x^i \geqslant 0. \quad (i = 1, \dots, m) \end{gathered} \tag{1}$$

We shall designate by P_k $(k = 1, \dots, l)$ a subset of the set $\{(i, j) : i, j = 1, \dots, m\}$, defining the possible exchange (cooperation) by the k-th item between the units, and by x^k_{ij}, $(i, j) \in P_k$, an unknown variable equal to the delivery size of the k-th product by the i-th unit to the j-th unit. The unknown x^k_{ij} must satisfy conditions:

$$\begin{gathered} \sum_{j : (i,j) \in P_k} x^k_{ji} - \sum_{j : (i,j) \in P_k} x^k_{ij} = y^i_k, \\ x^k_{ij} \geqslant 0, \quad i = 1, \dots, m; \quad k = 1, \dots, l. \end{gathered} \tag{2}$$

The production-transportation problem is finding such a composition of $(\boldsymbol{X}, \boldsymbol{Y}, \mathcal{X})$ $(\boldsymbol{X} = (x^1, \ldots, x^m);$

$$\boldsymbol{Y} = (y^1, \ldots, y^m); \ \mathcal{X} = \{x^\kappa_{ij}\}, \ \kappa = 1, \ldots, l, (i, j) \in \boldsymbol{P}_\kappa),$$

which satisfies the conditions of (1)-(2) and minimizes production--transportation costs

$$\sum_{i=1}^{m} (c^i, x^i) + \sum_{\kappa=1}^{l} \sum_{(i,j) \in \boldsymbol{P}_\kappa} c^\kappa_{ij} x^\kappa_{ij} \qquad (3)$$

For the search of the optimal composition $(\hat{X}, \hat{Y}, \hat{\mathcal{X}})$ in problem (1)-(3) a finite iterative method /1, 2/ has been used for which a program has been made in the codes of BESM-6 computer and which is used in practical calculations. With regard to such problems sufficient quantity of statistical data has been accumulated.

This method presupposes a mutual exchange of information between the models of the production-transportation system.

In finding production plans x^i and the sizes of y^i the information about reasonable cooperation of units by different kinds of products is used, and in finding the plan of transportations $\mathcal{X}$ the data from production models about y^i are used.

The successive approximation to the optimal composition $(\hat{X}, \hat{Y}, \hat{\mathcal{X}})$ is made either by adding new methods of manufacture and deleting the bad ones or cancelling non-reasonable deliveries; or by introducing new, more effective deliveries to substitute for low-effective deliveries or reducing inefficient production.

In finding the next direction in improvement use is made of productive and transportation evaluations of products in different points (the solutions of dual systems).

This method has been extended to production-transportation problems /3/ in which balance conditions (2) have been replaced by

$$\sum_{j:(i,j) \in \boldsymbol{P}_\kappa} a^\kappa_{ji} x^\kappa_{ji} - \sum_{j:(i,j) \in \boldsymbol{P}_\kappa} x^\kappa_{ij} = y^i_\kappa \qquad (2')$$

$$x^\kappa_{ij} \geqslant 0, \qquad i = 1, \ldots, m, \quad \kappa = 1, \ldots, l.$$

The above models are linear.

In the practice of long-term planning one can also find such production-transportation models in which the intensities of production ways x_j^i can assume integer values 0 or 1.

Here a situation is discussed where from several ways of a production of unit it is possible to choose only a single method, i.e. to conditions (1)-(3) we add the conditions

$$\sum_{j=1}^{n_i} x_j^i \leq 1, \qquad i=1,\dots,m, \qquad (4)$$
$$x_j^i = 0 \vee 1, \qquad j=1,\dots,n_i.$$

These models are typical of problems about location and specialization of enterprises in an industry or in a group of related industries.

These optimal problems belong to the field of partial integer programming.

The account taken of specific conditions of this problem made it possible in this case also to build up a decomposition solution method acceptable for practical calculations.

The underlying basic ideas were: the implicit enumeration method of E.Balas /4/ and the method of cutting planes /5, 6/.

For certain special cases of models (1)-(4) computational schemes and programs for computers have been made which are employed at this Institute and other institutions in our country. The description and substantiation of the method for the solution of problem (1)-(4) is provided in /7, 8/.

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