

# Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

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Samuel H. Fuller

Analysis of Drum and  
Disk Storage Units

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Springer-Verlag  
Berlin · Heidelberg · New York 1975

**Editorial Board:** P. Brinch Hansen · D. Gries  
C. Moler · G. Seegmüller · N. Wirth

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**Library of Congress Cataloging in Publication Data**

Fuller, Samuel, 1946-  
Analysis of drum and disk storage units.  
  
(Lecture notes in computer science ; 31)  
Bibliography: p.  
Includes index.  
1. Magnetic memory (Calculating-machines)--  
Mathematical models. 2. Computer storage devices--  
Mathematical models. I. Title. II. Series.  
TK7895.M3F84                      001.6'442                      75-25523

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AMS Subject Classifications (1970): 60K25, 60K30, 68-00,  
CR Subject Classifications (1974): 2.44, 4.40, 4.6, 5.30, 6.35

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ISBN 3-540-07186-5 Springer-Verlag Berlin · Heidelberg · New York  
ISBN 0-387-07186-5 Springer-Verlag New York · Heidelberg · Berlin

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Printed in Germany

Offsetdruck: Julius Beltz, Hemsbach/Bergstr.

## PREFACE

Many computer systems are operating significantly below the potential of their central processors because of inadequate support from the auxiliary storage units. Most storage units, e.g. drums and disks, store the information on rotating surfaces, and the delays associated with retrieving information from these devices are substantial; access times on the order of ten to one hundred milliseconds are not uncommon, while many central processors are capable of executing well over a million operations per second. Unless unusual care is taken in the organization and scheduling of these rotating storage units, they will become the dominant bottleneck in the computer system.

A number of problems concerning the scheduling, organization, and configuration of auxiliary storage units are analyzed in this monograph. Stochastic, combinatorial, or simulation techniques are applied, depending on the assumptions and complexity of the particular problem. For the relatively simple scheduling disciplines of first-in-first-out (FIFO) and shortest-latency-time-first (SLTF), stochastic models are used. The starting addresses of I/O requests to a file (nonpaging) drum are modeled as random variables that are uniformly distributed about the circumference of the drum; the lengths of I/O requests are modeled as random variables that are exponentially distributed. This model of I/O requests is based upon measurements from an operational computer system. The arrival times of I/O requests are first modeled as a Poisson process and then generalized to the case of a computer system with a finite degree of multiprogramming. Well-known results in queueing theory are sufficient for some models, but in other cases original approaches are required. In particular, a new model of the SLTF file drum is developed, is compared with

previous models of the SLTF file drum as well as a simulation model, and is found to be a more accurate model than previously available. Furthermore, it is easily integrated into queueing network models used to analyze complete computer systems. Several simple, cyclic queueing networks that include this new model of the SLTF file drum are analyzed.

Another practical problem that is discussed is an I/O channel serving several, asynchronous paging drums. The analysis leads to the Laplace-Stieltjes transform of the waiting time and a significant observation is that the expected waiting time for an I/O request can be divided into two terms: one independent of the load of I/O requests to the drum and another that monotonically increases with increasing load. Moreover, the load-varying term of the waiting time is nearly proportional to  $(2 - 1/k)$  where  $k$  is the number of paging drums connected to the I/O channel.

In addition to the FIFO and SLTF scheduling disciplines, a new scheduling discipline is presented to minimize the total amount of rotational latency (and processing time) for an arbitrary set of  $N$  I/O requests and the algorithm that is developed to implement this minimal-total-processing-time (MTPT) scheduling discipline has a computational complexity on the order of  $N \log N$ . The MTPT scheduling algorithm was implemented, and for more than three or four records, the most time-consuming step is the initial sorting of the records, a step also present in SLTF scheduling algorithms. It is also shown that the least upper bound of the difference between the SLTF and MTPT schedules is one drum revolution and the expected difference is the average record length.

Finally, this monograph includes an empirical study of the MTPT and SLTF scheduling disciplines. It is discovered that the MTPT discipline offers substantial advantages over the SLTF discipline for the intra-cylinder scheduling in moving-head disks. For fixed-head drums, there are many situations

in which the MTPT discipline is superior to the SLTF discipline, but it is necessary to use a more sophisticated MTPT scheduling algorithm than the one shown here to have a computational complexity on the order of  $N \log N$ .

The material in this monograph is organized into relatively independent chapters. This has several implications: some definitions and concepts must be introduced more than once to minimize the dependence among the chapters, but more importantly, this redundancy in preliminary material allows individuals to immediately turn to the chapters they are interested in and begin reading, without the annoyance of being required to scan through previous chapters to find definitions and notational conventions. References to figures, tables, and equations follow a two-level structure that refers to the section within a chapter and the item within a section; for instance, Fig. 4.7 and Eq. (10.2) refer to the seventh figure of section four and the second equation of section ten respectively. Such references refer only to items within the current chapter and hence ambiguities between identically labeled items in different chapters are avoided.

This monograph is based on a Ph.D dissertation submitted to the Dept. of Electrical Engineering at Stanford University in 1972. I am deeply indebted to Edward J. McCluskey, Forest Baskett, and Harold S. Stone (members of my reading committee) for the assistance and encouragement they provided during the initial years of my career. Harold Stone also deserves thanks for encouraging me to investigate what has now become the topic of this monograph. The research reported in Chapter 2 was done in collaboration with Forest Baskett. This research has also benefited from discussions with Norman R. Nielsen, Thomas H. Brettt, Robert Fabry, Thomas Price, and Neil Wilhelm.

I gratefully acknowledge the financial support provided by fellowships from the National Science Foundation and the Fannie and John Hertz Foundation.

I am indebted to the Computation Center of the Stanford Linear Accelerator Center, and in particular its director, Charles R. Dickens, for the support and insight gained through the use and measurement of its computer facility. This work has been partially supported by the Joint Services Electronics Program under Contract N-00014-67-A-0112-0044. Also, I want to thank my wife Carol and my children, Amy, Deborah, and Matthew for their patience and understanding during the many hours this work has kept me from them.

Pittsburgh, Pennsylvania  
April 1975

Samuel H. Fuller

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