A MULTI-AREA APPROACH TO THE ECONOMIC OPTIMIZATION OF ELECTRIC POWER SYSTEM

Radmila Rakić Radivoj Petrović Milan Rakić

Mihailo Pupin Institute,
Belgrade, Yugoslavia

A B S T R A C T

In this paper, the problem of short-term economic dispatch of active power in a combined hydro-thermal electric power system is considered. The problem studied is a 24 hours optimization, with operational cost as associated criterion. It is assumed that consumer´s demand, defined as a number of time functions, as well as technical constraints concerning power production units, and transmission line capabilities are satisfied within the period of optimization. The deterministic time discrete mathematical model of the above stated problem consists of a set of nonlinear algebraic equations and a set of nonequalities. The power system is decomposed into a number of interconnected power areas. Consequently, a number of local less dimensional area optimization subproblems are defined. Each of them is a typical nonlinear programming problem. Coordination among subproblem solutions is performed by a higher level decision making effort. It is done by specially derrived coordination algorithm.

It is shown that the proposed multi-area approach is computationally effective and fast. This property has a particular importance since in practice the system under control is influenced by frequent structural disturbances (failures of production units and/or transmission lines) causing power generation rescheduling. Using the presented algorithm a short term dynamic optimization problem of 220 kV network of Serbia has been solved. The results of this solution are also discussed in the paper.

INTRODUCTION

Considerable activity in the field of electric power systems control in recent years has been devoted to the development and application of various optimization methods to achieve optimum system economy. In studies of economic operation of a combined hydro-thermal electric power system, attention has to be paid to the number of imporant factors such as: operating efficiency of available sources, customer requirements and supply security, water inflows, transmission losses, etc.

The need to solve the problem has rapidly grown with increased use of computers in control of electric power systems, which allow significant savings in system´s operational costs by use of optimization theory in scheduling systems operation. There have been numerous papers discussing the problem of hydro-thermal system optimization and the application of various optimization methods/1,2,3/.

The problem,considered here, appears to be quite common. It is a problem of short-term economic dispatch of active power in a combined hydro-thermal electric power system. The optimization problem is formulated as minimization of the system´s operational cost within a 24 hour period. The main system´s characteristics are:

(a) Consumer demand is known and given by a number of time functions representing power demands in all passive nodes of a network.

(b) The set of thermal power plants belonging to the system is given. Economic efficiency of each power plant is described by generation cost curve.

(c) The hydro power generation schedules are arranged as piece-wise constant during discrete time intervals. The total amounts of water available for hydro power generation during 24 hours are given in advance for each hydro power plant.

(d) Transmission losses are taken into account in the usual simplified quadratic form.

(e) A number of constraints are imposed on both control and state variables: (i) output of each power plant is limited between technical minimum, and technical maximum, (ii) the transmission lines power capacities are limited, (iii) flows through each turbine are limited.

A common way to overcome the computational difficulties, that arise in optimization due to a high problem dimensionality, is to decompose the problem on subproblems and to coordinate the process of obtaining their solutions. Decomposition procedure presented in this paper is not a standard one. It is characterized by adoption of decomposition technique already developed[5], and some kind of its modification. The modification consists of linearization of the integral criterion functional with respect to hydro power generation, and successive hydro power scheduling by use of minimization of a linearized functional. Solution of the problem of optimal power generation scheduling is obtained iteratively. Optimal scheduling of thermal power generation and hydro generation scheduling are the two subroutines that are successively applied until the optimal solution of hydro and thermal power generation schedule has been reached.

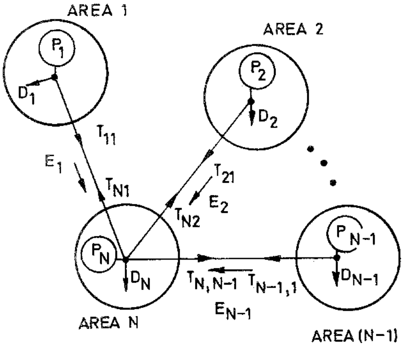

Fig. 1. Schematic representation of an electric power system composed of N interconnected areas

FORMAL PROBLEM STATEMENT

It is assumed that the system under consideration consists of a number of active and passive nodes connected by high voltage lines. It is also assumed that the system is decomposed in N areas, Fig. 1., with sufficiently independent power production and consumption. Each area $i=1,\dots,N$, has numbers h_i and s_i of hydro and steam power plants, respectively. The hydro and steam power generation H_{ij}^t, S_{ij}^t, $\forall i$ and $j=1,\dots,h_i$ (or s_i) of each production plant are discrete functions of time t, $t=1,\dots,24$. They form column vectors of hydro and thermal power production H_i^t, S_i^t, $\forall i,t$.

Each area is characterized by a given demand D_i^t, $\forall i,t$, so that the total system demand at time t equals $D^t = \sum_{i=1}^{N} D_i^t$. Each area exchanges power with other areas. Power exchange of an area i, E_i, $\forall i$, is an algebraic sum of powers through transmission lines connecting i-th areas with others

$$E_i^t = \sum_{r=1}^{r_i} T_{ir}, \quad \forall i, \tag{1}$$

T_{ir} being a power of r-th power line connecting i-th area with other areas. With line losses neglected, the area power exchange satisfy the following relation

$$\sum_{i=1}^{N} E_i^t = 0. \tag{2}$$

Due to the technical characteristics of the interconnection transmission lines, the powers of the interconnection transmission lines are subject to the constraints of the type

$$|T_{ir}^t| \leq |T_{ir}\max|, \quad \forall i,t, \text{ and } r=1,\dots,r_i. \tag{3}$$

It is supposed that hydro power plant is a constant head one. Power production of the j-th power station belonging to the i-th area, at time t, is described by a linear relation

$$H_{ij}^t = k_{ij} q_{ij}^t, \quad \forall i,t \text{ and } j=1,\dots,h_i, \tag{4}$$

where q_{ij}^t is the flow of water through the turbines of the j-th hydro power station in i-th area at time t and k_{ij} constant coeficients $\forall i$ and $j=1,\dots,h_i$. The flow of water is subject to the following constraints

$$0 \leq q_{ij}^t \leq q_{ij\,\max}, \quad \forall i,t, \text{ and } j=1,\dots,h_i, \tag{5}$$

which are a consequence of the power plant characteristics and

$$\sum_{t=1}^{24} q_{ij}^t \leq kQ_{ij}, \quad \forall i, \text{ and } j=1,\ldots,h_i, \tag{6}$$

which are imposed by weekly operation planing. In (5) and (6) $q_{ij\ max}$ is maximum allowed flow of water through the turbines of the j-th plant in the i-th area and Q_{ij} is a volume of water available for hydro power production of the same plant during the whole interval of 24 hours.

Relations (4) can be represented in a simplified matrix notation by introducing a column vector of area water flows through turbines $(q_i^t)' = (q_{i1}^t,\ldots,q_{ih_i}^t)$, $\forall i,t$, ($(\cdot)'$ denoting a transpose operation) and a diagonal area matrix of constant coeficients $k_i = \text{diag}\ (k_{i1},\ldots,k_{ih_i})$. The column vector of area hydro power production is according to (4)

$$H_i^t = k_i q_i^t, \quad \forall i,t. \tag{7}$$

It is convenient to introduce a column vector of area thermal power generation $(S_i^t)' = (S_{i1}^t, S_{i2}^t,\ldots,S_{is_i}^t)$, $\forall i,t$. Thermal power production is normally subject to the constraints of the type

$$S_{i\ min} \leq S_i^t \leq S_{i\ max}, \quad \forall i,t, \tag{8}$$

where $S_{i\ min}$ and $S_{i\ max}$ are corresponding column vectors representing technical minimum and maximum outputs of thermal power stations belonging to i-th area. It is also convenient to write the state of the energy production and interchange for each area and time as a triplet $P_i^t = (S_i^t, H_i^t, T_i^t)$, where T_i^t represents a vector of power flows in the transmission lines connecting i-th area with other areas.

According to H. Happ/5,6/, the transmission losses in the whole system can be written, in terms of area hydro and thermal generation powers and transmission line powers, in the usual quadratic form

$$L^t = \sum_{i=1}^{N} L_i^t = \sum_{i=1}^{N} (P_i^t)' B_i P_i^t \tag{9}$$

where B_i are matrices of known B-coefficients for each area $i=1,\ldots,N$.

Balance equations for the first N-1 areas are the following

$$D_i^t = \sum_{j=1}^{h_i} H_{ij}^t + \sum_{j=1}^{s_i} S_{ij}^t - L_i^t - E_i^t, \quad i=1,\ldots,N-1, \ \forall t, \tag{10}$$

and for the N-th area, due to the relation (1)

$$D_N^t = \sum_{j=1}^{h_N} H_{Nj}^t + \sum_{j=1}^{s_N} S_{Nj}^t - L_N^t + \sum_{i=1}^{N-1} E_i^t, \quad \forall t. \tag{11}$$

The associated performance criterion functional, representing system operational costs written in discrete form, is

$$F = \sum_{t=1}^{24} \sum_{i=1}^{N} \sum_{j=1}^{s_i} F_{ij}(S_{ij}^t). \tag{12}$$

In order to simplify notation, it is convenient to introduce column vector functions for area production costs, defined by the cost functions of thermal power plants belonging to the area

$$(F_i(S_i^t))' = (F_{i1}(S_{i1}^t), \ldots, F_{is_i}(S_{is_i}^t)), \quad \forall i,t. \tag{13}$$

The dynamic optimization problem is stated as follows. Find S_{ij}^t, and H_{ij}^t, $\forall i,j,t$, that minimize functional F defined by (12), taking into account the conditions (10) and (11) which have to be satisfied for any $t=1,\ldots,24$, and the constraints (3), (5), (6) and (8), which are imposed on values of state and control variables.

A METHOD TO SOLVE A PROBLEM

Stated problem belongs to the class of nonlinear dynamic optimization problems. Problems of this class are intensively treated in literature[2,3]. In a number of papers[5,6,10] decomposition technique has been applied in determination of optimal control of strictly thermal power systems. However, this technique is useful in treating combined hydro-thermal power systems[9].

The method presented in this paper represents imbeding of a previously developed method of a multi-area approach to a hydro-thermal power system optimization with hydro power generation schedule given in advance[10]. It should be noted that a multi-area approach in its original versions[5,6] was not used for dynamic optimization of hydro-thermal power systems.

Method developed to handle complete process of optimal power generation scheduling, control and rescheduling, consists of three subroutines:

(1) Subroutine for Initial Hydro Power Generation Scheduling.

(2) Subroutine for Optimal Thermal Power Generation Scheduling by use of Multi-Area Approach.

(3) Subroutine for Hydro Power Generation Rescheduling.

These subroutines are incorporated in algorithm for a multi-area dynamic optimization of a combined hydro-thermal power system as shown in Fig.2.

Each subroutine is briefly described in the following part of the paper.

Initial Scheduling of Hydro Power Generation (ISHPG)

Initial schedules for hydro power generation are usually determined by area dispatchers, and their determination is a matter of skill and experience. If initial schedules are not given, they can be determined by use of the following simple algorithm.

An initial hydro power production schedule of the j-th hydro power station, located in the i-th area, $\forall i$ and $j=1,\ldots,h_i$, is

$$H_{ij}^{t} = \begin{cases} \xi_{ij}(D_i^t - W_i), \text{ if } 0 \leq \xi_{ij}(D_i^t - W_i) \leq H_{ij\,max}, \\ H_{ij\,max}, \quad \text{if} \quad \xi_{ij}(D_i^t - W_i) \geq H_{ij\,max}, \\ 0 \quad \text{if} \quad \xi_{ij}(D_i^t - W_i) < 0, \end{cases} \quad \forall i, t \text{ and } j=1,\ldots,24, \tag{14}$$

where:

$$\xi_{ij} = \frac{k_{ij}Q_{ij}}{\sum_{j=1}^{h_i} k_{ij}Q_{ij}}, \quad \forall i \text{ and } j=1,\ldots,h_i, \text{ are constants,}$$

W_i is adjusted by the algorithm so that, taking into account relations between powers of hydro power stations, H_{ij}^t, and flow of water through turbines, q_{ij}^t (relation (4)), hydro power generation H_{ij}^t, obtained by (14), satisfy constraints (5), and (6) imposed on water flow q_{ij}^t.

The subroutine constructed on the basis of (14) provides initial schedules of hydro power generation such that every thermal power plant operates on a constant level, during the interval of optimization, unless the constraints (5) and (6) are violated.

The flow chart of the subroutine for initial area hydro power production scheduling, constructed according to equations (14), (4), (5) and (6) is illustrated on the Fig. 3.

Initial hydro power production schedules obtained by use of the described subroutine are not necessarily close to the optimal schedules of hydro power generation. Since the schedules made by area dispatchers are often rather good, the subroutine should be applied only if

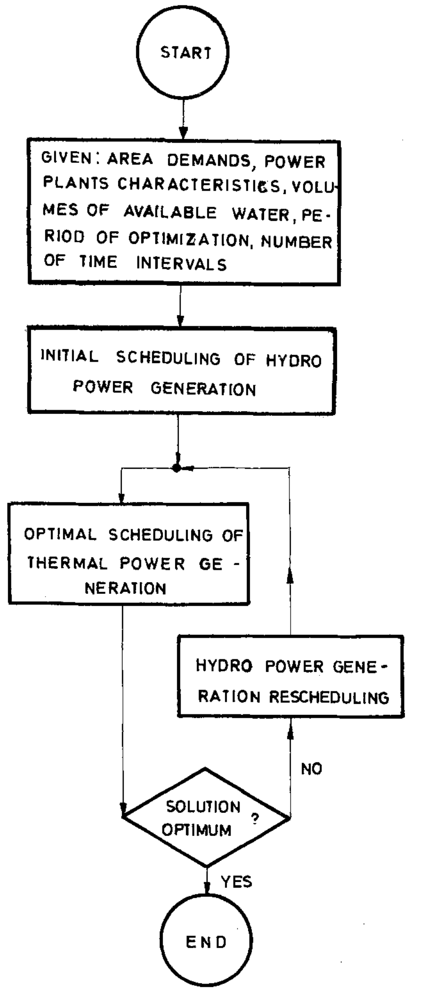

Fig. 2. Flow chart of algorithm for multi-area dynamic optimization of a combined hydro-thermal power system

initial schedules do not exist.

Optimal Schedule for Thermal Power Generation (OSTPG)

Given consumers demand and hydro power stations generation for every hour, the problem of dynamic optimization of hydro-thermal power system can be solved as a sequence of static otpimization problems. Instead of one dynamic optimization problem, 24 static thermal power system otpimization problems, one for each hour, have to be solved.

It is assumed, that the problem of static optimization of a hydro-thermal power system is a complex one. That is the reason why a multi-area approach and Diakoptics method /4,7/ in problem formulation and decomposition method in the determination of optimal powers of thermal power plants /6,9,10/ are applied.

After the schedules of hydro power plants are determined, the determinati-

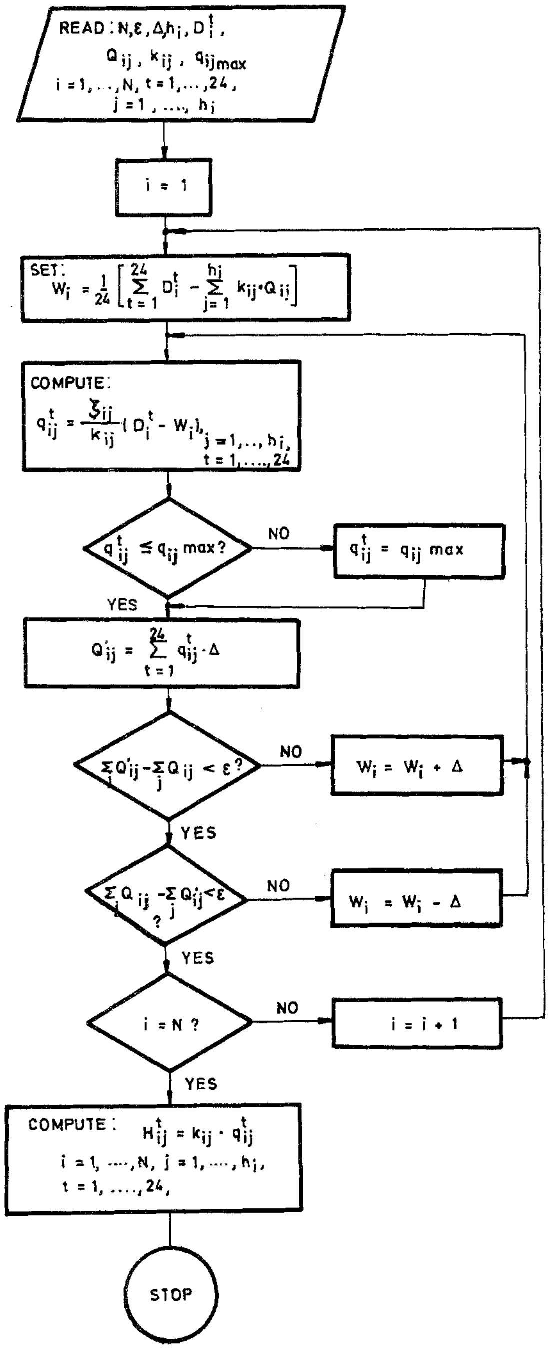

Fig. 3. Initial hydro power generation scheduling

on of the corresponding thermal power schedules becomes a nonlinear programming problem. If a Lagrangian functional is formed from criterion function (12), relations (10) and (11), and constraints (3) and (8), optimal values of variables representing powers of thermal power stations and powers of transmission lines have to satisfy the following sets of equations representating necessary conditions for optimality:

(a) The set of area vector equations

$$\frac{dF_i(S_i^t)}{dS_i^t} - \lambda_i^t\left(1 - \frac{\partial L_i^t}{\partial S_i^t}\right) + \mu_i^t(S_{imax} + S_{imin} - 2S_i^t) = 0, \quad \forall i,t, \tag{15}$$

(b) The set of inter-area equations

$$\lambda_i^t + \sum_{m=1}^{N-1} \lambda_m^t \frac{\partial L_m^t}{\partial E_i^t} = \lambda_N^t - \lambda_N^t \frac{\partial L_N^t}{\partial E_i^t} + 2\sum_{r=1}^{r_i} \nu_{ir}^t T_{ir}^t \frac{\partial T_{ir}^t}{\partial E_i^t}, \quad \forall t \text{ and } i=1,\dots,N-1, \tag{16}$$

(c) The set of equations (10) and (11) representing area balance between production and consumption of electric power.

(d) The set of constraints obtained from Kuhn-Tucker conditions

$$\begin{aligned}
(S_{ijmax} - S_{ij}^t)(S_{ij}^t - S_{ijmin}) &\geq 0, \\
\mu_{ij}(S_{ijmax} - S_{ij}^t)(S_{ij}^t - S_{ijmin}) &= 0, \\
(T_{irmax}^2 - (T_{ir}^t)^2) &\geq 0, \\
\nu_{ir}^t(T_{irmax}^2 - (T_{ir}^t)^2) &= 0,
\end{aligned} \tag{17}$$

$$\forall i,t \text{ and } j=1,\dots,s_i, \quad r=1,\dots,r_i.$$

In equations (15) to (17) λ_i^t, μ_{ij}^t, ν_{ir}^t, $\forall i,t$ and $j=1,\dots,s_i$, $r=1,\dots,r_i$, are Lagrange multipliers, and μ_i^t and ν_i^t are row vector multipliers, $\mu_i^t = (\mu_{i1}^t,\dots,\mu_{is_i}^t)$, $\nu_i^t = (\nu_{i1}^t,\dots,\nu_{ir_i}^t)$.

The optimal schedules of thermal power generation for given hydro power schedules are determined iteratively. One of areas (say area N) is chosen to be a reference area, and initial values of the corresponding reference multipliers λ_N^t, $\forall t$, are sellected.

Given λ_N^t, initial values for thermal power generation are arbitrarily chosen $\forall t$ and values of multipliers for other areas are obtained from the system of N-1 equations (16). It can be represented in explicit form by use of multipliers ratio vector[6,11]

$$\begin{bmatrix} \lambda_1^t \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{N-1}^t \end{bmatrix} = \begin{bmatrix} \text{multipli-} \\ \text{ers ratio} \\ \text{vector} \end{bmatrix} \cdot \lambda_N^t + R(T_1^t,\dots,T_{N-1}^t), \tag{18}$$

where R is a column vector which is zero vector when all T_i^t satisfy conditions (17).

Given λ_i^t, consumer demand D_i^t, and initial hydro generation schedule of every hydro power plant H_{ij}^t, $\forall i,t$ and $j=1,\dots,h_i$, optimal values of S_i^t are obtained by solving Equations (15) and (17). If area balance Equations (10) and (11) are not satisfied, new values of λ_N^t, $\forall t$ are determined using a single area iterative algorithm[1].

New value of λ_N^t at iteration k is calculated according to

$$(\lambda_N^t)^k = (\lambda_N^t)^{k-1} + (P_T^d - P_T^{k-1})\frac{(\lambda_N^t)^{k-1} - (\lambda_N^t)^{k-2}}{P_T^{k-1} - P_T^{k-2}}, \tag{19}$$

where superscript k indicates the iteration being started,

$k-1$ = iteration just completed

$k-2$ = preceeding iteration

$P_T^k = \sum_{i=1}^{N} \sum_{j=1}^{s_i} (S_{ij}^t)^k$ = total thermal power generation at iteration k

$P_T^d = \sum_{i=1}^{N} [D_i^t - \sum_{j=1}^{h_i} H_{ij}^t]$ = total desired thermal power generation.

Multipliers ratio matrix is computed for new values of S_i^t, and new values of multipliers of other areas are obtained by use of (18). The optimal schedules of thermal power generation are obtained when constraints (10) and (11) are satisfied $\forall i$ and $\forall t$. The flow chart for thermal power generation scheduling is presented on the Fig. 4.

Hydro Power Generation Rescheduling (HPGR)

In order to obtain optimal hydro and thermal power generation schedule, the nonlinear dynamic optimization problem is linearized in a neighbourhood of initial solution of hydro power generation schedule, obtained by ISHPG subroutine, and optimal solution of thermal power generation schedule, obtained by OSTPG subroutine.

If all but j-th thermal power station in the i-th area are assumed to generate power at constant level, small variations of its power can be compensated by variations of the production of hydro power stations in the same area so that Equations (10) and (11) are always satisfied. If the powers of interconnection lines are constant, for a given demand, sufficiently small variation of thermal power can be compensated by variations of outputs of hydro power stations belonging to the same area according to

$$dS^t_{ij} = \frac{1}{\left(1 - \frac{\partial L^t_i}{\partial S^t_{ij}}\right)} \sum_{r=1}^{h_i} \left(1 - \frac{\partial L^t_i}{\partial H^t_{ir}}\right) dH^t_{ir}, \qquad (20)$$

$\forall i,t$ and $j=1,\ldots,s_i$.

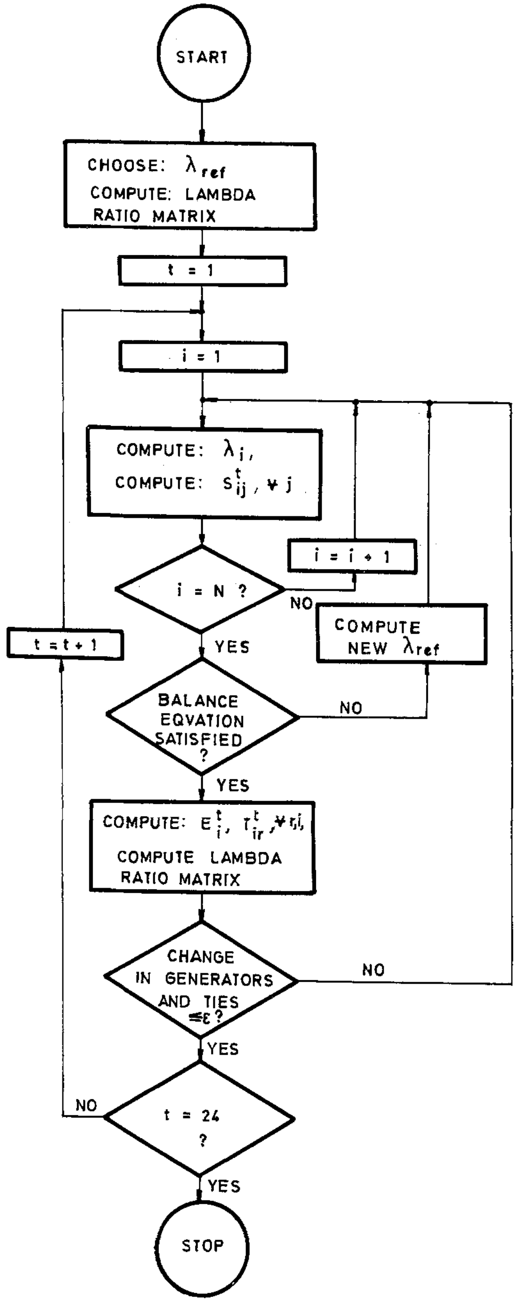

Fig. 4. The flow chart of optimal scheduling for thermal power generation

Since the flow of water through turbines of each hydro power station should satisfy integral constraints (6), flow of water variations and the corresponding hydro power generation variations have to be considered for the whole period of optimization. Since current flow of water schedules satisfy constraints (6), variations of hydro power generation should satisfy

$$\sum_{t=1}^{24} dH_{ij}^{t} = 0, \ \forall i \text{ and } j=1,\ldots,h_i. \tag{21}$$

Taking into account the expression (20), an "incremental worth of water" can be introduced for each hydro power station at every hour. Incremental worth of water of the j-th hydro power station, belonging to i-th area,at the t-th time interval, $\forall i,t$ and $j=1,\ldots,h_i$, calculated at the present levels of thermal and hydro power generation is

$$C_{ij}^{t} = \left(1 - \frac{\partial L_i^t}{\partial H_{ij}^t}\right) \sum_{j=1}^{s_i} \frac{F_i(S_{ij}^t)}{dS_{ij}^t} \Big/ \left(1 - \frac{\partial L_i^t}{\partial S_{ij}^t}\right)\Bigg|_{S_{ij}^t, H_{ij}^t}, \ \forall i,t \tag{22}$$

Given initial hydro power generation schedule and corresponding schedules of thermal power generation, incremental worth of water, C_{ij}^t, is determined according to (22). Given values of incremental worth of water, C_{ij}^t, calculated for a given hydro and thermal power generation schedules, H_{ij}^t and S_{ij}^t, problem of optimal hydro power generation rescheduling can be formulated as minimization of a linear form representing variation of power production costs in some neighbourhood of current values of hydro power generation schedule

$$\min dF = \min_{\{dH_{ij}^t\}} \left\{ \sum_{t=1}^{24} \sum_{i=1}^{N} \sum_{j=1}^{h_i} C_{ij}^t \cdot dH_{ij}^t \right\}, \tag{23}$$

where variations of hydro power generation have to satisfy constraints defined by (21) and the constraints corresponding to those defined by (5)

$$0 \le H_{ij}^t + dH_{ij}^t \le H_{ij\max}, \quad \forall i,t \text{ and } j=1,\ldots,h_i \tag{24}$$

For a small finite variations of dH_{ij}^t in the neighbourhood of current hydro generation schedule

$$dH_{ij}^t \varepsilon [H_{ij}^t - \delta, \quad H_{ij}^t + \delta], \quad \forall\, i,t, \text{ and } j=1,\ldots,h_i, \tag{25}$$

the problem of hydro generation schedule improvement is a linear programming problem vith the criterion function defined by (23) and constraints given by (21), (24) and (25).

The constraints (21), (24) and (25) are such that instead of one, N smaller linear programming subproblems, one for each area, have to be solved. The i-th subproblem, $\forall i$, has the following form

$$\min Z_i = \min\{ \sum_{t=1}^{24} \sum_{j=1}^{h_i} C_{ij}^t \cdot dH_{ij}^t \}, \tag{26}$$

under constraints

$$\sum_{t=1}^{24} dH_{ij}^t = 0$$

$$0 \le H_{ij}^t + dH_{ij}^t \le H_{ijmax}$$

and

$$dH_{ij}^t \varepsilon [H_{ij}^t - \delta, H_{ij}^t + \delta].$$

The problem of hydro power generation rescheduling defined by (21), (23) and (24) is solved if N linear programming subproblems defined by (26) were solved independently since

$$\min dF = \sum_{i=1}^{N} \min Z_i. \tag{27}$$

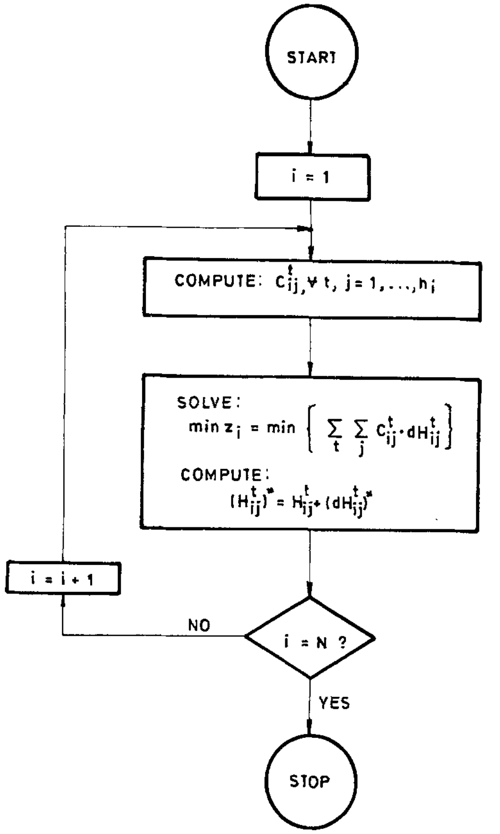

Fig. 5. The flow chart of hydro power generation rescheduling subroutine

After the defined linear programming problem is solved, a new hydro power generation schedule is obtained

$$(H_{ij}^{t})^{*} = H_{ij}^{t} + (dH_{ij}^{t})^{*}, \quad \forall i,t \text{ and } j=1,\ldots,h_i. \tag{28}$$

The flow chart of HPGR subroutine as presented on the Fig. 5.

A new hydro generation schedule is used as input to OSTPG subroutine, which is previously described. OSTPG and HPGR subroutines are applied successively until an optimal solution of hydro-thermal power generation schedule is obtained for the whole area.

Example

On the basis of algorithm which is explained in previous part of the paper, a computer program was developed and test example of short term dynamic optimization of 220 kV network for Serbia is solved on a computer IBM 370/135.

A 220 kV Serbian network is schematically represented on the Fig. 6. The 220 kV system includes 2 thermal power stations, 3 hydro power stations and 8 consumers with considerable consumption connected to the network. The whole system is decomposed into 2 areas, as it is shown on the Fig. 6. The corresponding area demands, and B-coeficient matrices, for each area, as well as coefficient matrix of an interconnection subsystem, are calculated.

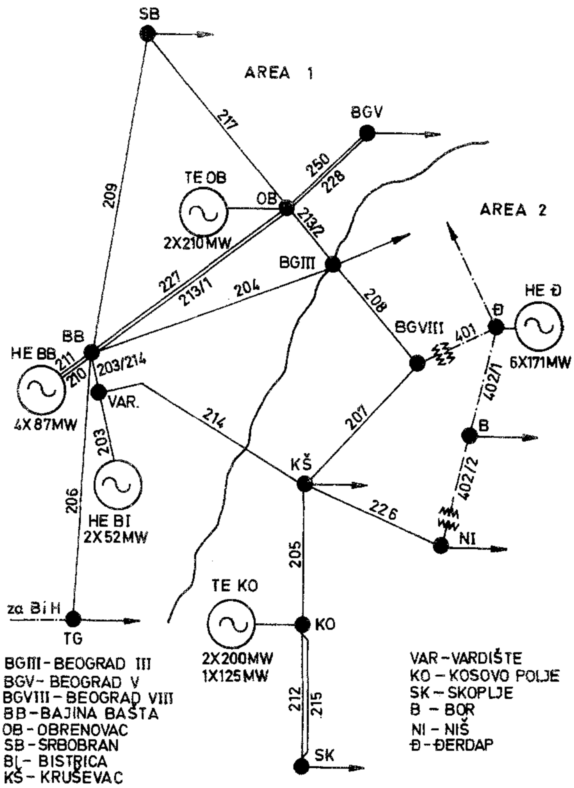

Fig. 6. A schematic representation of a 220 kV Serbian network

For a given area demands D_1^t and D_2^t, Fig. 7. (a), and total volumes of water available for hydro generation for every hydro power plant, accor-

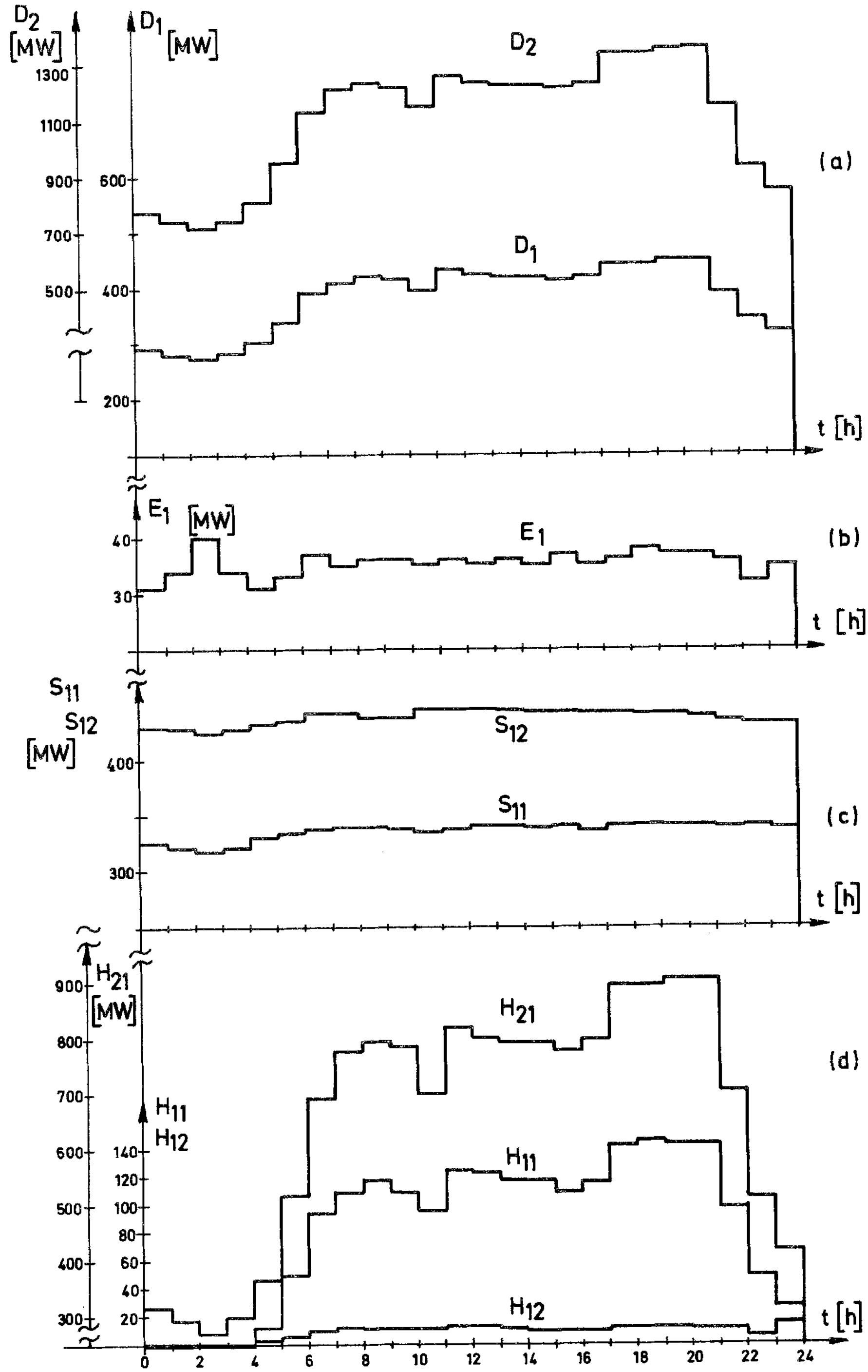

Fig. 7. Diagram of consumer demand and optimal values of power generation

ding to the algorithm presented on the Fig. 2., optimal values of hydro and thermal power generation, S_{ij}^t, Fig. 7 (c), and H_{ij}^t, Fig. 7 (d), for every power plant are determined. Corresponding optimal values of area power exchange for every interconnection line and total power exchange between the two areas presented on the Fig. 7 (b) are determined.

The whole problem is solved in 2 minutes, which is a rather good guarantee that by the algorithm explained larger power network could be easily handled in a reasonable computer time that enables the use of short-term schedules in on-line control of a power system.

List of symbols

t = current index denoting time (hour of a day), $t=1,\dots,24$,
N = number of interconnected areas composing power system,
i = current index denoting area number, $i=1,\dots,N$,
D_i^t = i-th area power demand at time t,
h_i = number of hydro power plants in i-th area,
s_i = number of thermal power plants in i-th area,
r_i = number of transmission lines connecting i-th area with other ones,
j = current index denoting plant number within area; for hydro power plants $j=1,\dots,h_i$, for thermal power plants $j=1,\dots,s_i$,
r = current index denoting number of a transmission line connecting area with other ones, $r=1,\dots,r_i$,
H_{ij}^t = active power output of (i,j) hydro power plant at time t,
H_i^t = h_i-dimensional vector of active power outputs of hydro power plants in the i-th area at time t,
q_{ij}^t = flow of water through turbines of (i,j) hydro power plant at time t, q_{ijmax} being maximum allowed flow of water,
Q_{ij} = a volume of water available for hydro power production of (i,j) hydro power plant,
S_{ij}^t = active power output of (i,j) thermal power plant at time t; S_{ijmin} and S_{ijmax} being technical minimal and maximal output powers, respectively,
S_i^t = s_i-dimensional vector of active power outputs of thermal power plants in the i-th area at time t,
T_{ir}^t = active power of (i,r) transmission line at time t, T_{irmax} being maximal allowed active power,
T_i^t = r_i-dimensional vector of powers in transmission lines connecting i-th area with all others at time t,
P_i^t = $(s_i+h_i+r_i)$-dimensional vector of power production and interchange

in the i-th area at time t, $P_i^t = (S_i^t, H_i^t, T_i^t)$,

E_i^t = active power exchange of i-th area at time t,

L_i^t = transmission losses in the i-th area at time t,

L^t = transmission losses in the whole system at time t,

B_i = $(h_i+s_i+r_i) \times (h_i+s_i+r_i)$ - dimensional matrix of B-coefficients of area i,

$F_{ij}(S_{ij}^t)$ = operational cost of (i,j) thermal power plant at time t, if active oputput power is S_{ij}^t,

$F_i(S_i^t)$ = s_i-dimensional column vector function of area production costs,

λ_i^t, μ_{ij}^t, ν_{ir}^t = Lagrange multipliers,

μ_i^t = s_i-dimensional row vector multiplier,

ν_i^t = r_i-dimensional row vector multiplier,

k = current index denoting the number of iteration,

C_{ij}^t = worth of water of (i,j) hydro power plant at time t.

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