OPTIMUM ALLOCATION OF INVESTMENTS IN A TWO - REGION ECONOMY

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1. INTRODUCTION

The problem considered in this paper is that of resource allocation between two regions in an economy. Neo-Classic Macro Economic Growth models, which by definition |1| are formulated at an highly aggregated level, are used to analyse optimal policies which, under a suitable set of constraints, allocate investments to two regions with different economic characteristics. A criterion of social welfare is defined and the resulting optimal control problem, which in same cases admits singular arcs, is resolved This paper provides a framework within which such important resource allocation problems may be better understood.

The problem considered is of interest, for example, in a planified economy, where one possible objective of the planning authority could be that of reducing economic differentials between the two regions by a suitable choice of investments, based on the total available capital.

Previous work, using macro economic models for the economic analysis of the allocation problem between two regions, have been presented by Rahman |2|, Intriligator |3|, and Takayama |4|.

In this paper, a general two-region macro economic model is developed and assumptions on the production functions are introduced. Under suitable hypothesis, the proposed model can be modeled by a bilinear continuous dynamic system.

A criterion of social welfare is introduced, which takes into account the need of a balanced growth of the two regions and a maximization of both the final capital stock and of the social consumptions.

The Maximum Principle of Pontryagin |5| is then applied to determine the optimal allocation policy on respect to a general functional which takes into account the criterion of social welfare.

The solutions thus obtained are analyzed.

2. THE GENERAL TWO REGION GROWTH MODEL

In a two region economy, national income, Y, is the sum of regional incomes, $Y_i (i = 1,2)$:

$$Y = Y_1 + Y_2 \tag{1}$$

Each of the regional incomes is assumed |2,3,4| to be determined as a product of the regional capital stock, k_i, and the constant regional output-capital ratio, α_i:

$$Y_i = \alpha_i k_i \tag{2}$$

National investment, I, the sum of regional increases in capital stocks, $\dot{k}_i$, equals national savings, S, the sum of the products of incomes and constant regional savings ratio, s_i:

$$I \equiv \dot{k}_1 + \dot{k}_2 = S \equiv s_1 Y_1 + s_2 Y_2 \tag{3}$$

Using the production function (2):

$$\dot{k}_1 + \dot{k}_2 = g_1 k_1 + g_2 k_2 \tag{4}$$

where $g_i = s_i \alpha_i$ is the constant regional growth rate.

Two allocation parameters $u_i (i = 1,2)$ are defined as the proportions of investment allocated to region i, leaving $(1 - u_1 - u_2)$ as the proportion allocated to a third sector which refer to social consumptions.

Assuming that there is neither investments external to the system, or delay between investment and capital stock, or shift of capital from one region to another, when placed in either region, the equations of the system become:

$$\begin{aligned} \dot{k}_1 &= u_1 (g_1 k_1 + g_2 k_2) - \mu_1 k_1 \\ \dot{k}_2 &= u_2 (g_1 k_1 + g_2 k_2) - \mu_2 k_2 \\ \dot{k}_3 &= (1 - u_1 - u_2)(g_1 k_1 + g_2 k_2) - \nu k_3 \end{aligned} \tag{5}$$

with:

$$\begin{aligned} &0 \leq u_i \leq 1 \qquad i = 1,2 \\ &0 \leq u_1 + u_2 \leq 1 \end{aligned} \tag{6}$$

In the following, for simplicity, it is assumed that ν is negligible. The case of $\nu \neq 0$ is not substantially different.

The problem facing the economic planner is then to choose an optimal time path for the allocation parameters, $u_1(t)$ and $u_2(t)$, which achieves some objectives of the economy subject to the above constraints and certain initial conditions on capital stocks and social consumptions:

$$k_i(o) = k_{io} \tag{7}$$

The system to be studied is represented in a schematic way in fig.1.

A constant rate of growth n is assumed for the whole populations in both regions, $N_i (i = 1,2)$:

$$\dot{N}_1 = n\, N_i \qquad i = 1,2 \tag{8}$$

$N_{io} (i = 1,2)$ are the populations at time $t = 0$

3. THE CHOICE OF A CRITERION OF SOCIAL WELFARE

For economic planners the choice of a criterion of social welfare which can guide the establishment of economic policies is a complex problem.

It is possible to evaluate in different ways the performance of an economic system. Some Authors want to maximize a functional of the global consumption flow per capita or of the global income or of a linear combination of the two, over the planning horizon considered. As a matter of fact, global indices may cause large unbalances between the two regions. In order to consider this fact, in this paper a general functional is considered which takes into account the following three factors, all along the fixed planning horizon T:

1) a "balanced growth", taken into account by minimizing, all along the fixed planning horizon T, the quadratic deviation between the incomes of the two regions, weighted according to their population:

$$\int_o^T \left(\frac{N_1 \alpha_1 k_1}{N_1 + N_2} - \frac{N_2 \alpha_2 k_2}{N_1 + N_2}\right)^2 dt \tag{9}$$

Taking into account (8) and with $\gamma = \frac{N_1(t)}{N_2(t)} \frac{\alpha_1}{\alpha_2} = \frac{N_{10}}{N_{20}} \frac{\alpha_1}{\alpha_2} > o$, (9) can be written:

$$\left(\frac{N_{10}\alpha_1}{N_{10}+N_{20}}\right)^2 \int_0^T \left(k_1 - \frac{k_2}{\gamma}\right)^2 dt \qquad (10)$$

2) a global income at time T, to be maximized:

$$\alpha_1 k_1(T) + \alpha_2 k_2(T) \qquad (11)$$

3) the social consumption, to be maximized at the final time T

$$k_3(T) \qquad (12)$$

eventually discounted along the planning horizon:

$$\frac{1}{N_o} \int_0^T (1 - u_1 - u_2)(g_1 k_1 + g_2 k_2)\, e^{-(\nu+\rho)t}\, dt \qquad (13)$$

where ρ is a suitable discounting factor.

For simplicity, only the first case is considered. The second case does not lead to solutions substantially different once a current value Hamiltonian |6| is considered.

The hole functional considered is:

$$\max_{\substack{u_i \\ i=1,2}} \left\{ I = -\omega \int_0^T \left(k_1 - \frac{k_2}{\gamma}\right)^2 dt + (1-\beta)\,|\alpha_1 k_1(T) + \alpha_2 k_2(T)| + \beta k_3(T) \right. \qquad (14)$$

where ω and β are suitable weights, which satisfy:

$$0 \le \omega \le \infty$$
$$0 \le \beta \le 1 \qquad (15)$$

4. THE OPTIMAL CONTROL PROBLEM

Once some values for ω and β are chosen, the maximization of I (14) subject to the dynamic state equations (5) along with the constrants on the controls (6) defines an optimal control problem. Since the controls u_1 and u_2 appear linearly in the state equations and the performance index, the problem could admit singular extremal arcs. Con sider the Hamiltonian:

$$H = -\ \omega(k_1 - \frac{k_2}{\gamma})^2 + |(\lambda_1 - \lambda_3)u_1 + (\lambda_2 - \lambda_3)u_2|\ |g_1k_1 + g_2k_2| +$$
$$+ \lambda_3(g_1k_1 + g_2k_2) - \mu_1\lambda_1k_1 - \mu_2\lambda_2k_2 \tag{16}$$

The three adjoint variables are defined by:

$$\lambda_1 = -\ \frac{\partial H}{\partial k_1} = -\ g_1|(\lambda_1 + \beta)\ u_1 + (\lambda_2 + \beta)\ u_2| + \lambda_1\mu_1 + g_1\beta + 2\omega(k_1 - \frac{k_2}{\gamma})$$

$$\lambda_2 = -\ \frac{\partial H}{\partial k_2} = -\ g_2|(\lambda_1 + \beta)\ u_1 + (\lambda_2 + \beta)\ u_2| + \lambda_2\mu_2 + g_2\beta - \frac{2}{\gamma}\ \omega(k_1 - \frac{k_2}{\gamma})$$

$$\lambda_3 = -\ \frac{\partial H}{\partial k_3} = 0 \tag{17}$$

with transversality conditions:

$$\lambda_1(T) = (\beta - 1)\alpha_1$$

$$\lambda_2(T) = (\beta - 1)\alpha_2 \tag{18}$$

$$\lambda_3(T) = -\ \beta$$

From the last equ. of (17), one gets:

$$\lambda_3(t) = \text{const} = -\ \beta \tag{19}$$

the maximization of H with respect to (u_1,u_2) gives rise to the following possibilities:

$$\begin{array}{llll} \text{A:} & \lambda_1 < -\ \beta, & \lambda_1 < \lambda_2 \Rightarrow & u_1^* = 1,\ u_2^* = 0 \\ \text{B:} & \lambda_2 < -\ \beta, & \lambda_2 < \lambda_1 \Rightarrow & u_1^* = 0,\ u_2^* = 1 \\ \text{C:} & \lambda_1 > -\ \beta, & \lambda_2 > -\ \beta \Rightarrow & u_1^* = 0,\ u_2^* = 0 \end{array} \tag{20}$$

$$\text{D:}\quad \lambda_1 = \lambda_2 < -\ \beta \quad \Rightarrow \quad u_1^* + u_2^* = 1 \tag{21}$$

$$E: \quad \lambda_2 = -\beta, \quad \lambda_1 > -\beta \quad \Rightarrow \quad u_1^* = 0, \; u_2^* \in |0,1| \tag{22}$$

$$F: \quad \lambda_1 = -\beta, \quad \lambda_2 > -\beta \quad \Rightarrow \quad u_2^* = 0, \; u_1^* \in |0,1| \tag{23}$$

$$G: \quad \lambda_1 = \lambda_2 = -\beta \quad \Rightarrow \quad u_i^* \in |0,1| \; i = 1,2 \tag{24}$$

In cases D,E,F,G, singular arcs are defined. To study the optimal strategies from these conditions, one can perform a phase plane analysis of $\underline{\lambda}$. The preceding cases correspond to different regions in this plane. Following the optimal trajectories of $\underline{\lambda}$, when one goes from one region to another, there is a switching in the optimal solution.

5. THE OPTIMAL SOLUTION WHEN W = 0

5.1 Optimal Control

The analysis of the behaviour of $\underline{\lambda}$, in the cases considered in the preceding paragraph is done under the following hypothesis:

1. $\omega = 0$
2. region two has a larger growth rate ($g_2 > g_1$)
3. growth rates are larger than the corresponding depreciations, in both regions ($g_i > \mu_i$, $i = 1,2$).

$\underline{A}$: $\lambda < -\beta \quad \lambda_1 < \lambda_2$

$$\begin{aligned} \dot{\lambda}_1 &= -g_1\lambda_1 + \mu_1\lambda_1 \\ \dot{\lambda}_2 &= -g_2\lambda_1 + \mu_2\lambda_2 \end{aligned} \tag{25}$$

The eigenvalues are:

$$\begin{aligned} \xi_1 &= -(g_1 - \mu_1) < 0 \\ \xi_2 &= \mu_2 \qquad\quad > 0 \end{aligned} \tag{26}$$

which correspond to a saddle point with asymptotes:

$$\begin{aligned} \lambda_1 &= 0 \\ \lambda_2 &= \lambda_1 \frac{g_2}{g_1 - \mu_1 + \mu_2} \end{aligned} \tag{27}$$

$\underline{B}$: $\lambda_2 < -\beta, \quad \lambda_2 < \lambda_1$

$$\dot{\lambda}_1 = -g_1\lambda_2 + \mu_1\lambda_1 \qquad (28)$$

$$\dot{\lambda}_2 = -(g_2 - \mu_2)\lambda_2$$

The eigenvalues are:

$$\xi_1 = \mu_1 \qquad > 0$$

$$\xi_2 = -(g_2 - \mu_2) \quad < 0 \qquad (29)$$

which correspond to a saddle point with asymptotes:

$$\lambda_1 = 0 \qquad (30)$$

$$\lambda_2 = \lambda_1 \frac{g_2 - \mu_1 + \mu_2}{g_1}$$

$\underline{C}$: $\lambda_1 > -\beta, \quad \lambda_2 > -\beta$

$$\dot{\lambda}_1 = \mu_1\lambda_1 + g_1\beta \qquad (31)$$

$$\dot{\lambda}_2 = \mu_2\lambda_2 + g_2\beta$$

with an equilibrium point:

$$\lambda_1 = -\frac{g_1\beta}{\mu_1} \quad \text{and} \quad \lambda_2 = -\frac{g_2\beta}{\mu_2} \qquad (32)$$

after a translation of the origin to point (32), equ. (31) become:

$$\dot{\bar{\lambda}}_1 = \mu_1\bar{\lambda}_1 \qquad (33)$$

$$\dot{\bar{\lambda}}_2 = \mu_2\bar{\lambda}_2$$

with eigenvalues:

$$\xi_1 = \mu_1 \quad > 0 \qquad (34)$$

$$\xi_2 = \mu_2 \quad > 0$$

both positive, so that one has an instable node.

Since the rates of capital depreciation could be very similar in both sectors, il could be $\mu_1 = \mu_2 = \mu$, which correspond to an unstable star. In fig. (2), this case is shown when $g_2 > g_1$.

The remaining cases correspond to possible singular solutions:

$$\underline{D}: \lambda_1 = \lambda_2 < -\beta$$

$$\begin{aligned} \dot{\lambda}_1 &= -g_1\lambda_1 + \mu_1\lambda_1 \\ \dot{\lambda}_2 &= -g_2\lambda_2 + \mu_2\lambda_2 \end{aligned} \tag{35}$$

which is admissible only if:

$$-g_1 + \mu_1 = -g_2 + \mu_2 \quad \Rightarrow \quad g_1 - \mu_1 = g_2 - \mu_2 \tag{36}$$

$$\begin{aligned} &\underline{E}: \lambda_2 = -\beta, \quad \lambda_1 > -\beta \\ &\dot{\lambda}_1 = \mu_1\lambda_1 + g_1\beta \\ &\dot{\lambda}_2 = \mu_2\lambda_2 + g_2\beta = 0 = -\beta\mu_2 + g_2\beta \end{aligned} \tag{37}$$

admissible only if $g_2 = \mu_2$, a rather unrealistic case.

$$\underline{F}: \lambda_1 = -\beta, \quad \lambda_2 > -\beta$$

similar to the previous case and admissible only if $g_1 = \mu_1$, which is unrealistic.

$$\underline{G}: \lambda_1 = \lambda_2 = -\beta$$

admissible only if $g_1 = \mu_1$ and $g_2 = \mu_2$, which once again is unrealistic.

As a conclusion, when $\omega = 0$, singular solutions are not present, in general.

5.2. State Phase Plane Analysis.

To analyse the optimal solution on the state phase plane, it is necessary to study the behaviour of the equations (5) in the various cases considered in the preceding paragraph.

Attention is restricted only to the admissible cases A,B, and C. Only the first two equations are considered, since k_3 does not influen ce them.

A:

$$\begin{aligned} \dot{k}_1 &= (g_1 - \mu_1)k_1 + g_2k_2 \\ \dot{k}_2 &= \qquad\qquad -\mu_2k_2 \end{aligned} \tag{38}$$

The eigenvalues are:

$$\begin{aligned} \xi_1 &= g_1 - \mu_1 > 0 \\ \xi_2 &= - \mu_2 \quad < 0 \end{aligned} \tag{39}$$

which correspond to a saddle point with asymptotes:

$$\begin{aligned} k_2 &= 0 \\ k_2 &= - \frac{g_1 - \mu_1 + \mu_2}{g_2} k_1 \end{aligned} \tag{40}$$

B:

$$\begin{aligned} \dot{k}_1 &= - \mu_1k_1 \\ k_2 &= g_1k_1 + (g_2 - \mu_2)k_2 \end{aligned} \tag{41}$$

The eigenvalues are:

$$\begin{aligned} \xi_1 &= - \mu_1 < 0 \\ \xi_2 &= (g_2 - \mu_2) > 0 \end{aligned} \tag{42}$$

which correspond to a saddle point with asymptotes:

$$\begin{aligned} k_1 &= 0 \\ k_2 &= - \frac{g_1}{g_2 - \mu_2 + \mu_1} k_1 \end{aligned} \tag{43}$$

C:

$$\begin{aligned} k_1 &= - \mu_1k_1 \\ k_2 &= \qquad - \mu_2k_2 \end{aligned} \tag{44}$$

The eigenvalues are:

$$\xi_1 = -\mu_1$$
$$\xi_2 = -\mu_2 \tag{45}$$

If $\mu_1 = \mu_2$, one has a stable star centered in the origin.

It is now possible to perform an analysis of the results. The case of a region 2 with a larger rate of growth ($g_2 > g_1$) is considered. The behaviour of the optimal solution can be studied by following the trajectories of λ in their phase plane. Taking into account the economical interpretation of $\lambda(t)$ as shadow prices, one should consider only negative values for both λ_1 and λ_2. As a matter of fact, the final values of $\lambda(t)$ (18):

$$\lambda_1(T) = -(1-\beta)\alpha_1 < 0$$

$$\lambda_2(T) = -(1-\beta)\alpha_2 < 0$$

are negative. A study of the optimal trajectories also show rather clearly that it is possible to arrive to them only from negative values of $\lambda(t)$. The actual trajectory depends on $\lambda(t)$ and on the lenght of the planning horizon T.

In fig.2, the phase plane of $\underline{\lambda}$ is divided in regions named according to the cases considered in paragraph 4. In principle, three cases are possible:

1. $\lambda(t)$ is situated in A, that is the output-capital ratio is larger in region 1 and $-(1-\beta)\alpha, < -\beta$.

Two subcases are possible according to the lenght of the planning period T:

a. $u_1 = 1 \quad \forall \quad t \in |0,T|$

b. $u_2 = 1 \quad \forall \quad t \in |0,t^*), \; u_1 = 1 \quad \forall \quad t \in (t^*,T|$

t^*, obtained by solving the adjoint equs., is given by:

$$t^* = T - \frac{1}{g_1} \ln \left[\frac{\frac{\alpha_2}{\alpha_1} - \frac{g_2}{g_1}}{1 - \frac{g_2}{g_1}} \right] \tag{46}$$

Of course, b. is possible only if $t^* > 0$.

Possible trajectories for the two cases considered are shown in fig.3, for the state variables k_1 and k_2. The optimal policy consists in investing in the region with a larger output-capital ratio. If planning horizon is long enough it could be convenient to invest before in region 2 and then in region 1.

2. $\lambda(T)$ is situated in B, that is the output-capital ratio is larger in region 2 $(\frac{\alpha_2}{\alpha_1} > 1)$, and $-(1-\beta)\alpha_2 < -\beta$. In such a case, the optimal trajectory is always $u_2 = 1 \ \forall \ t \in |0,T|$.

The optimal policy consists in investing always in the region with a regional growth rate larger $(g_2 > g_1)$ and a regional output-capital ratio also larger $(\alpha_2 > \alpha_1)$.

A possible optimal trajectory is shown in fig.4

2. $\lambda(T)$ is situated in B, that is: $\frac{\alpha_2}{\alpha_1} > 1$, $-(1-\beta)\alpha_2 < -\beta$. In such a case the optimal trajectory is always $u_2 = 1$ for $t \in |0,1|$.

3. $\lambda(T)$ is situated in C, that is: $\alpha_2 < \frac{\beta}{1-\beta}$; $\alpha_1 < \frac{\beta}{1-\beta}$.

This consists in giving a larger weight in the optimization of the social consumptions on respect to the increase in the level of capital in both regions, If the planning horizon T is long enough, the following cases are possible:

a. $\lambda(T) \in C - c^*$ that is $\alpha_2 < \frac{\beta}{1-\beta}$ and $\alpha_1 < \frac{g_1+\mu}{g_2+\mu}\alpha_2 + \frac{g_1-g_2}{g_2+\mu} \quad \frac{\beta}{1-\beta}$

$$u_2(t) = 1 \quad \forall \quad t \in |0,t^*)$$

$$u_3(t) = 1 \quad \forall \quad t \in (t^*,T|$$

where:

$$t^* = T - \frac{1}{\mu}\ln\left(\frac{\frac{g_2}{\mu} + (1-\frac{1}{\beta})\alpha_2}{\frac{g_2}{\mu} - 1}\right) \qquad (47)$$

and $t^* < T$ for $\alpha_2 < \frac{\beta}{1-\beta}$

$u_2 = 1$ is possible only if $t^* > 0$

b. $\lambda(T) \in C^*$, that is $\alpha_1 < \frac{\beta}{1-\beta}$ and $\alpha_2 < \frac{g_2+\mu}{g_1+\mu}\alpha_1 - \frac{g_2-g_1}{g_1+\mu} \quad \frac{\beta}{1-\beta}$

$$u_3(t) = 1 \quad \forall \quad t \in |0,t^*)$$

$$u_1(t) = 1 \quad \forall \quad t \in (t^*,t^{**})$$

$$u_3(t) = 1 \quad \forall \quad t \in (t^{**},T|$$

where:

$$t^{**} = T - \frac{1}{\mu} \ln\left(\frac{\frac{g_1}{\mu} + (1-\frac{1}{\beta})\alpha_1}{\frac{g_1}{\mu} - 1}\right) \tag{48}$$

$$t^* = t^{**} - \frac{1}{g_1}\ln\left(\frac{\frac{\alpha_2}{\alpha_1} - \frac{g_2}{g_1}}{1 - \frac{g_2}{g_1}}\right) = T - \frac{1}{\mu}\ln\left(\frac{\frac{g_1}{\mu} + (1-\frac{1}{\beta})\alpha_1}{\frac{g_1}{\mu} - 1}\right) - \frac{1}{g_1}\ln\left(\frac{\frac{\alpha_2}{\alpha_1} - \frac{g_2}{g_1}}{1 - \frac{g_2}{g_1}}\right) \tag{49}$$

Clearly, if $t^{**} < 0$ only the policy $u_3(t) = 1$ and if $t^* < 0$ $u_1(t) = 1 \quad \forall \quad t \in |0,t^{**})$ and $u_3(t) = 1 \quad \forall \quad (t^{**},T|$.

Possible trajectories for the two cases considered are shown in fig.5. In both of them, one begins always investing in the two regions to increase the global income and then in social consumptions $(u_3 = 1)$.

If, in the problem considered, one sets $\beta = 0$, one is considering the maximization of the global income at T. Now, only the cases I and II are possible. The problem reduces to that examined in |2,3,4|. All the results derived there are now particular cases.

CONCLUSIONS

In this paper an economy has been modeled with a two-sector Macro Economic model where one sector, disaggregated in two regions, produces goods which are destined to be either invested or consumed, while the other sector, considered globally for the entire nation, produces goods which can only be used as social consumptions. The policy variable is the allocation of investments between sectors and regions.

A criterion of social welfare has been defined and the resulting optimal control problem is risolved.

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FIGURE CAPTIONS

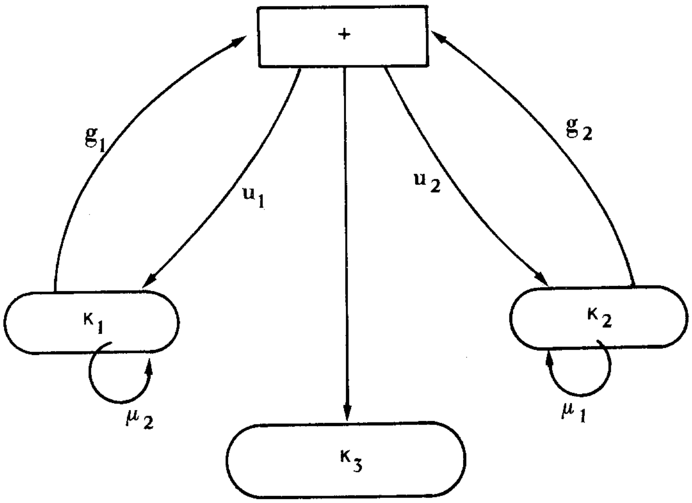
Fig.1 The economical system considered.

Fig.2 $\underline{\lambda}$ phase plane analysis for $\omega = 0$.

Fig.3 Phase plane analysis of the state variables (k_1,k_2) when $\frac{\alpha_2}{\alpha_1} < 1$ and $-(1-\beta)\alpha_1 < -\beta$.

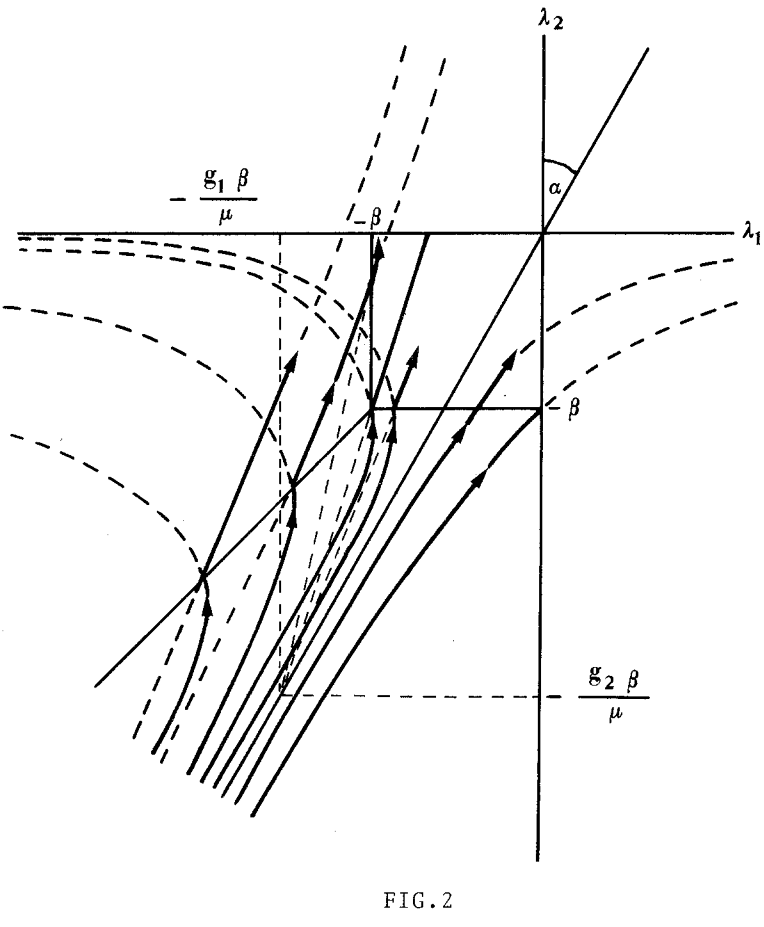
Fig.4 Phase plane analysis of the state variables (k_1,k_2) when $\frac{\alpha_2}{\alpha_1} > 1$ and $-(1-\beta)\alpha_2 < -\beta$.

Fig.5 Phase plane analysis of the state variables (k_1,k_2) when $\alpha_i < \frac{\beta}{1-\beta}$ $(i = 1,2)$.



+
g_1
u_1
u_2
g_2
κ_1
κ_2
μ_2
μ_1
κ_3

FIG.1



λ_2
α
$-\frac{g_1\beta}{\mu}$
$-\beta$
λ_1
$-\beta$
$-\frac{g_2\beta}{\mu}$

FIG.2

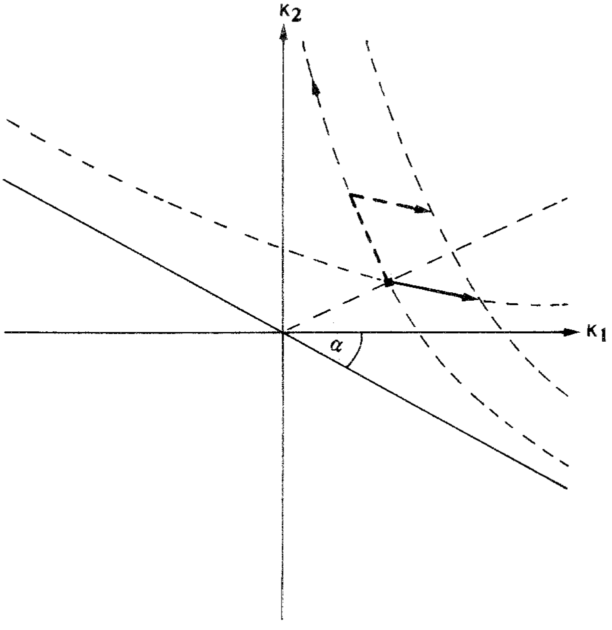


FIG.3

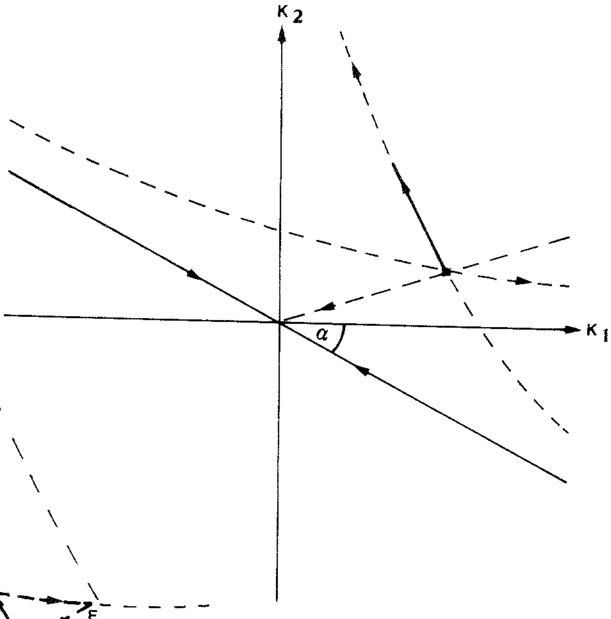


FIG.4

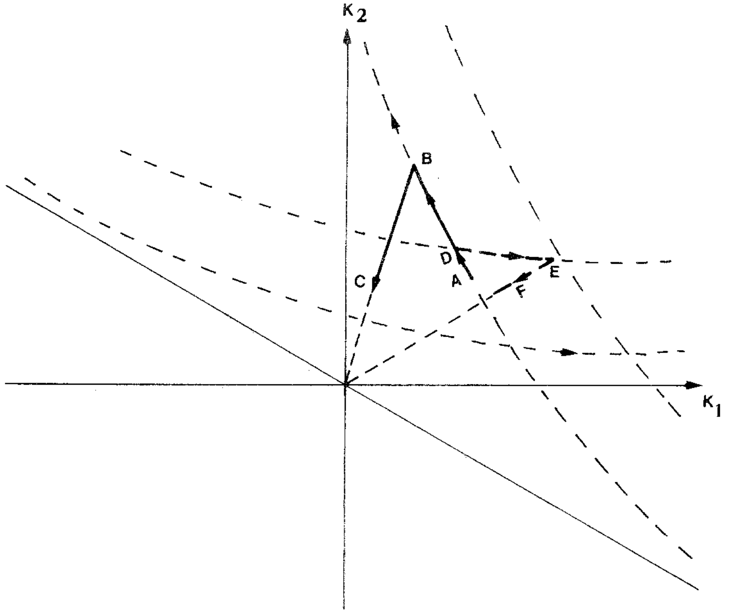


FIG.5