ON THE MULTIVARIABLE CONTROL OF NUCLEAR REACTORS USING THE STATE FEEDBACK APPROACH

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ABSTRACT

Recently an effort was made to design nuclear reactor systems via state-variable feedback techniques. On the other hand, a great amount of research has been concentrated on the multivariable state feedback control methodology, for its own, and a variety of useful results have been derived. The purpose of the paper is to apply a general multivariable state feedback control technique to nuclear reactor systems (e.g. multiregion reactors, coupled core reactors, etc.). Two fundamental design tasks are considered, namely, noninteraction and realization of desired transfer functions. This technique requires the system under control to be given in its phase canonical form and provides explicit expressions for the feedback control law matrices required. Two nuclear reactor examples are considered and fully worked out.

1. INTRODUCTION

The application of the state variable feedback control technique to nuclear reactor systems seems to be a promising approach with many advantages over the classical and the optimal control techniques providing a kind of link between them. Some studies concerning the application of this technique to nuclear reactors are involved in [1]-[7]. In general, the objective of this technique is to realize exactly given dynamics by feeding back some or all of the state variables through appropriate gains. The desired system dynamics is usually described by a given transfer function which is completely specified by its zeroes, poles and d.c. gains.

The work described in [1]-[3] is constrained to single-input single-output reactor systems and derives the solution (i.e. the feedback gains) by a direct comparison of the transfer function of the closed-loop system with that of the desired model, and an equation of the equal-power terms. The full theory of this method may be found in [4]. These results were extended in [5] to reactor systems having m inputs and m outputs through Gilbert's technique of canonical decoupling

[9]. A similar technique is described in [6]. A further extension of this method to the case where additional compensation is required to meet the desired criteria was made in [7].

From the point of view of pure control theory this problem has received independent attention and a substantial amount of results are already available [8]-[19]. Particular attention was given to two sub-problems of the general state variable feedback control design problem, namely the decoupling problem [8]-[10], and the eigenvalue control problem [11].

The purpose of the present paper is to investigate the applicability to multi-input multi-output nuclear reactors of a recent state variable feedback control technique [11], [15], [21] which is based on the assumption that the system under control is given in, or transformed into, its phase canonical form. The main problem is that of non-interacting system design, and the method provides simultaneous input-output and state variable decoupling. The pole and d.c. gain assignment can be accomplished simultaneously with the decoupling, but to control the zeros additional compensators are used as in [7]. Decoupling (noninteraction) in an actual coupled-core reactor is obtained by negating the neutron coupling between the cores. For comparison purposes the nuclear reactor examples studied in [5]-[7] are considered and completely worked out by the present technique.

2. STATE EQUATIONS OF MULTIPLE-CORE REACTORS

A coupled-core reactor is a critical reactor composed by two or more independently subcritical cores. The coupling effect is the result of mutual exchange of leakage neutrons between cores. In such reactors in order to apply successful control to the power levels of the cores independently, the effects of the neutron coupling must be balanced, i.e. a decoupled or noninteracting system is to be designed.

Consider a multiple-core reactor in which each core is coupled only with the neighbouring cores. Assume for simplicity one delayed neutron group for each core, and small neutron travel time between cores. Then, including the negative temperature feedback, the state equations (linearized) for a 3-core system are*

*Details of derivation together with an introduction of nuclear reactors in state space are given in [2].

$$\frac{dx_1}{dt} = \frac{D_{11}+\beta_1}{\tau_1} x_1 + \frac{D_{12}}{\tau_2} x_2 + \lambda_1 x_4 - \frac{\alpha_1 n_1^o}{\tau_1} x_7 + \frac{n_1^o}{\tau_1} x_{10}$$

$$\frac{dx_2}{dt} = \frac{D_{21}}{\tau_1} x_1 - \frac{2D_{22}+\beta_2}{\tau_2} x_2 + \frac{D_{23}}{\tau_3} x_3 + \lambda_2 x_5 - \frac{\alpha_2 n_1^o}{\tau_2} x_8 + \frac{n_2^o}{\tau_2} x_{11}$$

$$\frac{dx_3}{dt} = \frac{D_{32}}{\tau_2} x_2 - \frac{D_{33}+\beta_3}{\tau_3} x_3 + \lambda_3 x_6 - \frac{\alpha_3 n_3^o}{\tau_3} x_9 + \frac{n_3^o}{\tau_3} x_{12}$$

$$\frac{dx_4}{dt} = \frac{\beta_1}{\tau_1} x_1 - \lambda_1 x_4, \quad \frac{dx_7}{dt} = k_1 x_1 - m_1 x_7, \quad \frac{dx_{10}}{dt} = x_{13}$$

$$\frac{dx_5}{dt} = \frac{\beta_2}{\tau_2} x_2 - \lambda_2 x_5, \quad \frac{dx_8}{dt} = k_2 x_2 - m_2 x_8, \quad \frac{dx_{11}}{dt} = x_{14} \tag{1}$$

$$\frac{dx_6}{dt} = \frac{\beta_3}{\tau_3} x_3 - \lambda_3 x_6, \quad \frac{dx_9}{dt} = k_3 x_3 - m_3 x_9, \quad \frac{dx_{12}}{dt} = x_{15}$$

$$\frac{dx_{13}}{dt} = -\Theta_1 x_{13} + \mu_1 u_1, \quad \frac{dx_{14}}{dt} = -\Theta_2 x_{14} + \mu_2 u_2, \quad \frac{dx_{15}}{dt} = -\Theta_3 x_{15} + \mu_3 u_3$$

$$y_1 = x_1, \quad y_2 = x_2, \quad y_3 = x_3$$

Here x_1, x_2, x_3 are power levels in cores 1,2,3 correspondingly, x_4, x_5, x_6 are concentrations of delayed neutrons x_7, x_8, x_9 are control rod rates, and u_1, u_2, u_3 are the control inputs for cores 1,2,3 respectively. The parameters involved have the following interpretation with i=1,2,3.

τ_i	prompt neutron generation time in core i
β_i	delayed neutron fraction in core i
n_i^o	steady state neutron power level in core i
D_{ij}	neutron coupling coefficient from core j to core i
λ_i	decay constant of the delayed neutron emitter in core i
α_i	reactivity-temperature proportionality constant of core i
k_i	temperature-power proportionality constant of core i
m_i	heat removal coefficient of core i
Θ_i	inverse time constant of rod controller in core i

In this model the control rods are assumed to be driven by electric motors and hence each error signal produces a proportional motor speed. Since the reactivity is proportional to control rod position (not to control rod speed) one must integrate the output of the control rod driver to obtain the reactivity. A signal flow graph of this model indicating the coupling among the cores as well as the control channels is given in Fig. 1.

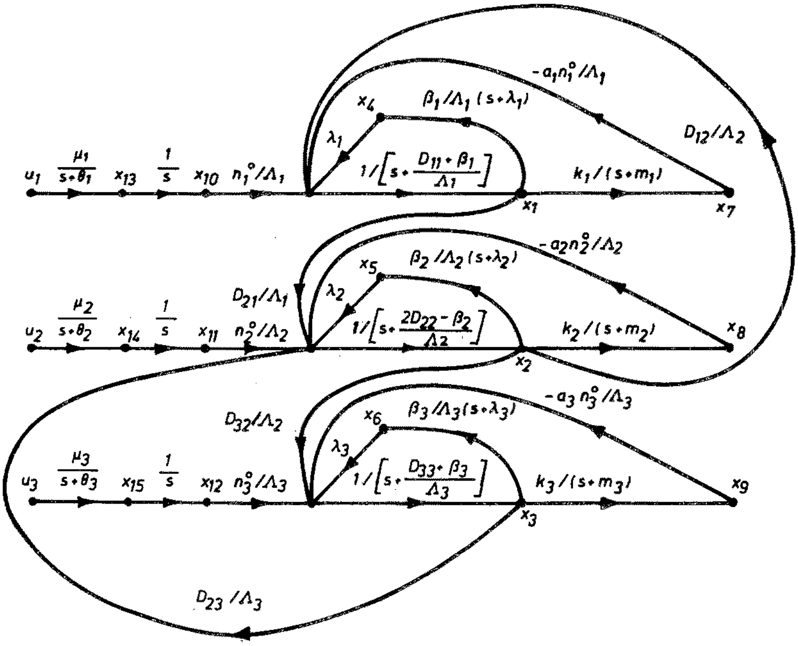

Fig. 1. Signal flow graph of the 3-coupled-core reactor

In matrix form this 3-core reactor system (as any multiple core reactor system) can be written as:

$$\frac{dx(t)}{dt} = Ax(t)+Bu(t), \quad y(t) = Cx(t) \tag{2}$$

where $x(t) = [x_1, x_2, \ldots, x_{15}]^T$ is the state vector, $u(t) = [u_1, u_2, u_3]^T$ is the input vector, $y(t) = [y_1, y_2, y_3]^T$ is the output vector, and the matrices A,B,C have obvious definitions. It is the state-space model (2) which will be utilized in the present paper.

3. THE STATE-VARIABLE FEEDBACK CONTROL PROBLEM

3.1 Statement of the problem

In general u(t) is an m-dimensional and y(t) is a p-dimensional vector. Introducing the linear state feedback control law

$$u(t) = Kx(t) + Nw(t) \tag{3}$$

where w(t) is a new input vector of dimensionality m', and K,N are matrix-valued gains of appropriate dimensions, yields the closed-loop system

$$dx_c/dt = (A+BK)x_c + BNw, \quad y = Cx_c \tag{4}$$

Here it is assumed that p=m'=m. The problem under consideration is to calculate the numerical values of the gain matrices K and N which cause the system to be input-output and state variable decoupled, and to possess required dynamic performance.

Mathematically, input-output decoupling (or noninteraction) implies that the input-output transfer matrix is diagonal, whereas state variable decoupling implies that in state space the system is composed by m noninteracting subsystems each one having one input-output pair. The transfer matrix of the closed-loop system (4) is equal to $H_c(s) = C(sI-A-BK)^{-1}BN$. It is well known that $H_c(s)$ is invariant under a nonsingular similarity transformation $x_c' = Qx$. In fact, the transformed closed-loop system is $dx_c'/dt = Q(A+BK)Q^{-1}x_c'+QBNw$, $y= CQ^{-1}x_c'$ and has the transfer matrix $H_c'(s)=CQ^{-1}\left[(sI-Q(A+BK)Q^{-1}\right]^{-1}QBN = H_c(s)$. It is assumed here that the system (2) is transformed in its input-Luenberger canonical form prior to the application of the state feedback control law. In this canonical form the matrices A,B have the form $A = \left[A_{ij}\right]$, $B = B_i$, where the blocks A_{ij} and B_i are

$$A_{ii} = \left[\begin{array}{c|c} 0 & I \\ \hline \alpha_{ii}^{1} \cdots & \alpha_{ii}^{\sigma_i} \end{array}\right] = \left[\begin{array}{c} A_{ii}^{*} \\ \hline A_{ii}^{**} \end{array}\right] \qquad B_i = \left[\begin{array}{c} 0 \\ \hline 0 \ldots 1 \ldots 0 \end{array}\right] = \left[\begin{array}{c} B_i^{*} \\ \hline B_i^{**} \end{array}\right]$$

$\sigma_i - 1$, σ_i, m, ith position (5)

$$A_{ij} = \left[\begin{array}{c} 0 \\ \hline \alpha_{ij}^{1} \cdots \alpha_{ij}^{\sigma_i} \end{array}\right] = \left[\begin{array}{c} A_{ij}^{*} \\ \hline A_{ij}^{**} \end{array}\right]$$

and the matrix C is not required to have any special form.

By using a new similarity transformation $x_c=M\hat{x}$ to the closed-loop system (4) yields the system

$$d\hat{x} = \hat{A}\hat{x} + \hat{B}w, \quad y=\hat{C}\hat{x} \tag{6}$$

where

$$\hat{A} = M^{-1}(A+BK)M, \quad \hat{B} = M^{-1}BN, \quad \hat{C} = CM \tag{7}$$

Clearly, a Luenberger canonical system with matrices $\hat{A} = \mathrm{diag}\left[\hat{A}_1,\ldots,\hat{A}_m\right]$, $\hat{B}^T = \left[\hat{B}_1^T,\ldots,\hat{B}_m^T\right]$, $\hat{C} = \left[\hat{C}_1,\ldots,\hat{C}_m\right]$, where σ_i are defined in [16], and

$$\hat{A}_i = \left[\begin{array}{c|c} 0 & I \\ \hline \multicolumn{2}{c}{\hat{\alpha}_{i1}\cdots\hat{\alpha}_{i\sigma_i}} \end{array}\right]\Big\}\sigma_i-1\,, \quad \hat{B}_i = \begin{bmatrix} 0 \\ \hline 0\ldots\underset{\substack{\uparrow \\ \text{ith position}}}{1}\ldots 0 \end{bmatrix}\,, \quad \hat{C}_j = \begin{bmatrix} 0 \\ \hline \hat{c}_{j1}\cdots\hat{c}_{j\sigma_i} \\ \hline 0 \end{bmatrix} \tag{8}$$

is input-output decoupled and consists of m decoupled (noninteracting) single input single-output subsystems.

Hence, the combined input-output and state variable decoupling problem under consideration here is reduced to that of selecting K,N, and M so as to satisfy the conditions in (7), with $\hat{A},\hat{B},\hat{C}$ having the form (8). The control of the system poles is accomplished by suitably choosing the parameters $\hat{\alpha}_{ij}$, $j=1,2,\ldots,\sigma_i$, $i=1,2,\ldots,m$, whereas the d.c. gains are controlled by suitably selecting $\hat{c}_{jk}$. Of course it must be noted here that not all of $\hat{c}_{jk}$ are free to be selected arbitrarily, since they are constrained by the zeros of the system under control, i.e. by the structure of system (2).

3.2 Solution of the problem

The pure input-output decoupling problem has been studied by Falb and Wolovich [8], and the canonical decoupling problem has been considered by Gilbert [9]. They show that the necessary and sufficient condition for a matrix pair $\{K,N\}$ to exist such that the state feedback control (3) yields an input-output decoupled closed-loop system is the nonsingularity of the matrix

$$D = \begin{bmatrix} c_1A^{d_1}B \\ \vdots \\ c_mA^{d_m}B \end{bmatrix} \tag{9}$$

where the indexes $d_i (i=1,2,\ldots,m)$ are defined as

$$d_i = \begin{cases} \min\{\, j: c_iA^jB \neq 0, j = 0,1,\ldots,n-1\} \\ n-1, \text{ if } c_iA^jB = 0 \text{ for all } j \end{cases} \tag{10}$$

The present method is based on the fact that the decoupleability of system (2) by the control law (3), as well as the indexes d_i are invariant under a non-singular similarity transformation M. We shall consider two cases: (i) the system (2) has no inherent coupling in the sense of Gilbert 9 i.e. $|D|\neq 0$, and (ii) the system has weak inherent coupling, i.e. $|D|\neq 0$ but $|H(s)|\neq 0$.

No inherent coupling

Decompose the matrix M in blocks M_{ij}, M^*_{ij}, and M^{**}_{ij}, equidimensional with the corresponding blocks A_{ij}, A^*_{ij}, and A^{**}_{ij} in the Luenberger form of (2), and write $M = \left[M_{ij}\right]$ and $M^T_{ij} = \left[M^{*T}_{ij} \,\vdots\, M^{**T}_{ij}\right]$. Introducing the matrices

$$\underline{A}^* = \left[A^*_{ij}\right], \quad \underline{B}^* = \left[B^*_i\right], \quad \underline{M}^* = \left[M^*_{ij}\right], \quad \underline{A}^{**} = \left[A^{**}_{ij}\right], \quad \underline{B}^{**} = \left[B^{**}_i\right], \quad \underline{M}^{**} = \left[M^{**}_{ij}\right]$$

the first two conditions in (7) can be grouped as

$$\underline{A}^* + \underline{B}^*K = \underline{M}^*\hat{A}M^{-1}, \quad \underline{A}^{**} + \underline{B}^{**}K = \underline{M}^{**}\hat{A}M^{-1}, \quad \underline{B}^*N = \underline{M}^*\hat{B}, \ \underline{B}^{**}N = \underline{M}^{**}\hat{B} \quad (11)$$

Clearly, $\underline{B}^{**} = I$ (unity matrix), and hence the two relations in (11) involving $\underline{B}^{**}$ give

$$K = \underline{M}^{**}\hat{A}M^{-1} - \underline{A}^{**}, \quad N = \underline{M}^{**}\hat{B} \quad (12)$$

Taking into account the fact that $\underline{B}^* = 0$ the other two relations in (11) yield

$$\underline{A}^*M = \underline{M}^*\hat{A}, \quad \underline{M}^*\hat{B} = 0 \quad (13)$$

The third condition (7) together with conditions (13) constitute the set of equations which determine M. If this set of linear algebraic equations has a solution matrix M with $|M| \neq 0$ then (12) provides the desired state feedback matrix pair required.

Weak inherent coupling

In this case prior to applying the feedback law (3) with K,N being given by (12), one uses a $\bar{n}$-dimensional precompensator of the type

$$d\bar{x}/dt = \bar{A}\bar{x} + \bar{B}\bar{u} \quad \text{with} \quad u = F_1\bar{u} + F_2\bar{x} \quad (14)$$

and obtains an overall precompensated system with state vector x and matrices A,B,C, where

$$\tilde{x} = \left[\begin{array}{c} x \\ \hline \bar{x} \end{array}\right], \quad \tilde{A} = \left[\begin{array}{c|c} A & BF_2 \\ \hline 0 & \bar{A} \end{array}\right], \quad \tilde{B} = \left[\begin{array}{c} BF_1 \\ \hline \bar{B} \end{array}\right], \quad \tilde{C} = \left[\begin{array}{c|c} C & 0 \end{array}\right]$$

Of course care must be taken here to transform the matrices A and B in the Luenberger canonical form. Usually, one starts by using a 1-dimensional precompensator (i.e. with $\bar{n}=1$). If there still exists weak inherent coupling one uses a 2-dimensional precompensator, and so on until the resulting system has no inherent coupling.

3.3 Control of zeros

The state variable feedback is adequate if one desires, simultaneously with the decoupling, to control the poles and the d.c. gains only. To control one or more zeros one must use suitable precompensators which implies that the state dimensionality of the overall system is increased. However, when introducing precompensators prior to decoupling special care is required, since even if the uncompensated system has not inherent coupling, the compensated one may have. A first method of overcoming this difficulty was proposed in [7] and is summarized in the following theorem. "Given a system of the type (2) having no inherent coupling, the series compensator $\dot{\bar{x}} = \bar{A}\bar{x}+\bar{B}\bar{u}$, $u=\bar{x}+\bar{E}\bar{u}$, where $\bar{A}$,$\bar{B}$ and $\bar{E}$ are diagonal matrices of dimensions mxm and $\bar{B}$ is nonsingular, does not introduce inherent coupling if (i) $\bar{E}=0$ or (ii) $\bar{E}$ is nonsingular". This theorem implies that to preserve decoupling when adding precompensators every compensator must involve both a pole and a zero or only a pole. The drawback of this method is that there is the possibility of loosing zeros. This is avoided if the system is decoupled prior to the introduction of the precompensators, since the addition of series precompensators to a system that is already decoupled does not influence decoupling.

The second method of introducing precompensators is based on exactly this observation and was also proposed in [7] in the form of the following theorem. "Suppose that the kth decoupled subsystem of a system decoupled by state feedback, or by other means, has z_k zeros and n_k poles, whereas the kth precompensator has z_k^c zeros and n_k^c poles. Then the $n_k+n_k^c$ poles of each augmented subsystem can be controlled by state feedback, but the $z_k+z_k^c$ zeros are not affected by state feedback". In fact, by reordering the state variables, the kth precompensated subsystem has the transfer function $F_k(s)\ F_k^c(s)$, where $F_k(s)$ is the transfer function of the kth subsystem of the original decoupled uncompensated system, and $F_k^c(s)$ is the transfer function of the kth cascade compensator. Now, from single-input single-output state variable control theory it is known that all $n_k+n_k^c$ poles are controlled, but the zeros are fixed to be the zeros of $F_k(s)F_k^c(s)$. Thus, combining the results of section 3.2 with the second method outlined here, the following control design procedure is proposed.

<u>Step 1</u>: Transform the system under control into its Luenberger input canonical form.

<u>Step 2</u>: Specify the Luenberger canonical decoupled model by using the desired poles and d.c. gains.

Step 3: Compute the required state feedback pair $\{K,N\}$ and the similarity transformation M.

Step 4: Introduce cascade precompensators to the decoupled single output subsystems in order to control the zeros as desired.

4. APPLICATION EXAMPLES

Example 1

As a first example consider a special case of the coupled-core reactor model (1), namely one with identically-coupled identical cores, in which the delayed neutrons and the control rod dynamics are neglected. By a convenient relabelling of the state variables the state equations of this system take the form

$$\dot{x}_1 = -\frac{2D}{\tau}x_1 - \frac{\alpha n_o}{\tau}x_2 + \frac{D}{\tau}x_3 + \frac{D}{\tau}x_5 + \frac{n_o}{\tau}u_1, \quad \dot{x}_2 = kx_1 - mx_2$$

$$\dot{x}_3 = \frac{D}{\tau}x_1 - \frac{2D}{\tau}x_3 - \frac{\alpha n_o}{\tau}x_4 + \frac{D}{\tau}x_5 + \frac{n_o}{\tau}u_2, \quad \dot{x}_4 = kx_3 - mx_4 \tag{15}$$

$$\dot{x}_5 = \frac{D}{\tau}x_1 + \frac{D}{\tau}x_3 - \frac{2D}{\tau}x_5 - \frac{\alpha n_o}{\tau}x_6 + \frac{n_o}{\tau}u_3, \quad \dot{x}_6 = kx_5 - mx_6$$

where D, τ, m, k, and n_o are the common neutron coupling coefficient, effective generation time, heat removal coefficient, power-temperature proportionality constant, and steady state power level, respectively. Here x_1, x_3, x_5 are the neutron power levels, and x_2, x_4, x_6 the temperatures in cores 1,2,3 respectively. The parameter values are $D=0.1$, $\tau=0.1$ sec, $k=10^{-5}$, $n_o=10^5$W, $\alpha=10^{-3}$/deg and $m=10^{-2}\text{sec}^{-1}$.
Hence, the state equations in (15) take the form

$$\dot{x}_1 = 2x_1 - 10^3x_2 + x_3 + x_5 + 10^6u_1, \quad \dot{x}_2 = 10^{-5}x_1 - 10^{-2}x_2$$

$$\dot{x}_3 = x_1 - 2x_3 - 10^3x_4 + x_5 + 10^6u_2, \quad \dot{x}_4 = 10^{-5}x_3 - 10^{-2}x_4$$

$$\dot{x}_5 = x_1 + x_3 - 2x_5 - 10^3x_6 + 10^6u_3, \quad \dot{x}_6 = 10^{-5}x_5 - 10^{-2}x_6$$

The measured outputs are $y_1=x_1, y_2=x_3$ and $y_3=x_5$. These equations can be written in the form (2) with matrices

$$A_o = \left[\begin{array}{cc|cc|cc} -2 & -10^3 & 1 & 0 & 1 & 0 \\ 10^{-5} & -10^{-2} & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & -2 & -10^3 & 1 & 0 \\ 0 & 0 & 10^{-5} & -10^{-2} & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & -2 & -10^{-3} \\ 0 & 0 & 0 & 0 & 10^{-5} & -10^{-2} \end{array}\right] \quad B_o^T = \left[\begin{array}{cc|cc|cc} 10^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^6 & 0 \end{array}\right] \quad C_o = \left[\begin{array}{cc|cc|cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \tag{16}$$

Prior to applying the control techniques the decoupleability of the system is checked. Since

$$D = \begin{bmatrix} c_1B \\ c_2B \\ c_3B \end{bmatrix}_0 = 10^6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for which $|D| \neq 0$, there is no inherent coupling, and so the system is directly decoupleable without the need to introduce any precompensator. To find the similarity matrix Q which transforms the system in the Luenberger form (5), we construct a matrix

$$L = \left[b_{o1},\ A_o b_{o1}, \ldots, A_o^{\sigma_1 - 1} b_{o1}, b_{02}, \ldots, A_o^{\sigma_2 - 1} b_{02}, \ldots, A_o^{\sigma_m - 1} b_{om} \right]$$

consisting of n linearity independent columns of the controllability matrix $P = \left[B_o, A_o B_o, \ldots, A_o^{n-1} B_o \right]$. Here $\sigma_1 = \sigma_2 = \sigma_3 = 2$, and so

$$L = \begin{bmatrix} 10^6 & -2\times10^6 & 0 & 10^6 & 0 & 10^6 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 10^6 & 10^6 & -2\times10^6 & 0 & 10^6 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 10^6 & 0 & 10^6 & 10^6 & -2\times10^6 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix}$$

Setting $\varepsilon_o = 0$, $\varepsilon_k = \sum_{i=1}^{k} \sigma_i$ $(k=1,2,\ldots,m)$, and letting γ_k be the ε_kth row of L^{-1}, the matrix Q is given by

$$Q = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_1 A_o^{\sigma_1 - 1} \\ \gamma_2 \\ \vdots \\ \gamma_m A_o^{\sigma_m - 1} \end{bmatrix}$$

In the present case $\varepsilon_o=0, \varepsilon_1=\sigma_1=2,\ \varepsilon_2=\sigma_1,+\sigma_2=4,\ \varepsilon_3=\sigma_1+\sigma_2+\sigma_3=6$, and

$$L^{-1} = \begin{array}{cl} \left[\begin{matrix} 10^6 & 2x10^{-1} & 0 & 10^{-1} & 0 & -10^{-1} \\ 0 & 10^{-1} & 0 & 0 & 0 & 0 \\ 0 & -10^{-1} & 10^6 & 2x10^{-1} & 0 & -10^{-1} \\ 0 & 0 & 0 & 10^{-1} & 0 & 0 \\ 0 & -10^{-1} & 0 & -10^{-1} & 10^{-6} & 2x10^{-1} \\ 0 & 0 & 0 & 0 & 0 & 10^{-1} \end{matrix}\right] & \begin{matrix} \\ \leftarrow \varepsilon_1\text{th row} \\ \\ \leftarrow \varepsilon_2\text{th row} \\ \\ \leftarrow \varepsilon_3\text{th row} \end{matrix} \end{array}$$

Hence

$$Q = \begin{bmatrix} \gamma_1 \\ \gamma_1 A_o \\ \gamma_2 \\ \gamma_2 A_o \\ \gamma_3 \\ \gamma_3 A_o \end{bmatrix} = 10^{-1} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 10^{-5} & -10^{-2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 10^{-5} & -10^{-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 10^{-5} & -10^{-2} \end{bmatrix}$$

$$Q^{-1} = 10 \begin{bmatrix} 10^3 & 10^5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^3 & 10^5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^3 & 10^5 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The matrices of the input Luenberger canonical form are found to be

$$A=QA_oQ^{-1} = \left[\begin{array}{cc|cc|cc} 0 & 1 & 0 & 0 & 0 & 0 \\ -3x10^{-2} & -2.001 & 10^{-2} & 1 & 10^{-2} & 1 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 10^{-2} & 1 & -3x10^{-2} & -2.001 & 10^{-2} & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 10^{-2} & 1 & 10^{-2} & 1 & -3x10^{-2} & -2.001 \end{array}\right] \tag{17}$$

$$B=QB_o= \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right], \quad C=C_oQ^{-1} = \left[\begin{array}{cc|cc|cc} 10^4 & 10^6 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 10^4 & 10^6 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 10^4 & 10^6 \end{array}\right]$$

The above suggest that the canonical decoupled model appropriate in the present case has the matrices

$$\hat{A} = \left[\begin{array}{cc|cc|cc} 0 & 1 & & & & \\ \hat{\alpha}_{11} & \hat{\alpha}_{12} & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \\ \hline & & 0 & 1 & & \\ \multicolumn{2}{c|}{0} & \hat{\alpha}_{21} & \hat{\alpha}_{22} & \multicolumn{2}{c}{0} \\ \hline & & & & 0 & 1 \\ \multicolumn{2}{c|}{0} & \multicolumn{2}{c|}{0} & \hat{\alpha}_{31} & \hat{\alpha}_{32} \end{array}\right], \hat{C} = \left[\begin{array}{cc|cc|cc} \hat{c}_{11} & \hat{c}_{12} & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \\ \hline \multicolumn{2}{c|}{0} & \hat{c}_{21} & \hat{c}_{22} & \multicolumn{2}{c}{0} \\ \hline \multicolumn{2}{c|}{0} & \multicolumn{2}{c|}{0} & \hat{c}_{31} & \hat{c}_{32} \end{array}\right] \quad (18)$$

$$\hat{B} = B$$

The canonical decoupling conditions are $\underline{A}^*M=\underline{M}^*\hat{A}$, $M^*\hat{B}=0$ and $CM=\hat{C}$ where $M=\left[\mu_{ij}\right]$ is a 6x6 matrix, and

$$\underline{A}^* = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad M^* = \begin{bmatrix} \mu_{11}\mu_{12}\cdots\mu_{16} \\ \mu_{31}\mu_{32}\cdots\mu_{36} \\ \mu_{51}\mu_{52}\cdots\mu_{56} \end{bmatrix}$$

Solving these conditions for $M(|M|)\neq 0)$ one obtains $M=10^{-6}\mathrm{diag}\left[\hat{c}_{12},\hat{c}_{12},\hat{c}_{22},\hat{c}_{22},\hat{c}_{32},\hat{c}_{32}\right]$ subject to the constrains $\hat{c}_{i2}=10^2\hat{c}_{i1}$ $(i=1,2,3)$. The inverse matrix M^{-1} equal to $M^{-1}=10^6\mathrm{diag}\left[\hat{c}_{12}^{-1}, \hat{c}_{12}^{-1}, \hat{c}_{22}^{-1}, \hat{c}_{22}^{-1}, \hat{c}_{32}^{-1}, \hat{c}_{32}^{-1}\right]$.

The required feedback gain matrices are given by $K=\underline{M}^{**}\hat{A}M^{-1}-\underline{A}^{**}$ and $N=\underline{M}^{**}\hat{B}$, where

$$\underline{A}^{**} = \begin{bmatrix} 3\times10^{-2} & -2.001 & 10^{-2} & -1 & 10^{-2} & 1 \\ 10^{-2} & 1 & -3\times10^{-2} & -2.001 & 10^{-2} & 1 \\ 10^{-2} & 1 & 10^{-2} & 1 & -3\times10^{-2} & -2.001 \end{bmatrix}$$

$$\underline{M}^{**} = \begin{bmatrix} \mu_{21}\cdots\mu_{26} \\ \mu_{41}\cdots\mu_{46} \\ \mu_{61}\cdots\mu_{66} \end{bmatrix} = 10^{-6}\begin{bmatrix} 0 & \hat{c}_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{c}_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{c}_{32} \end{bmatrix}$$

Hence

$$K = \left[\begin{array}{cc|cc|cc} \hat{\alpha}_{11} & \hat{\alpha}_{12} & \multicolumn{2}{c|}{0} & \multicolumn{2}{c}{0} \\ \hline \multicolumn{2}{c|}{0} & \hat{\alpha}_{21} & \hat{\alpha}_{22} & \multicolumn{2}{c}{0} \\ \hline \multicolumn{2}{c|}{0} & \multicolumn{2}{c|}{0} & \hat{\alpha}_{31} & \hat{\alpha}_{32} \end{array}\right] - A^{**}, \quad N=10^{-6}\begin{bmatrix} \hat{c}_{12} & & 0 \\ & \hat{c}_{22} & \\ 0 & & \hat{c}_{32} \end{bmatrix} \quad (19)$$

The feedback matrix gains for the original system (16) are $N_o=N$, and

$$K_o = KQ = 10^{-1}\left[\begin{array}{c|c|c|c|c|c} 10^{-5}\alpha_{12} & \alpha_{11}-10^{-2}\alpha_{12} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 10^{-5}\alpha_{22} & \alpha_{21}-10^{-2}\alpha_{22} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 10^{-5}\alpha_{32} & \alpha_{31}-10^{-2}\alpha_{32} \end{array}\right]$$

$$+\left[\begin{array}{c|c|c|c|c|c} 2.001\times10^{-6} & 0.99\times10^{-3} & -10^{-6} & 0 & -10^{-6} & 0 \\ \hline -10^{-6} & 0 & 2.001\times10^{-6} & 0.99\times10^{-3} & -10^{-6} & 0 \\ \hline -10^{-6} & 0 & -10^{-6} & 0 & 2.001\times10^{-6} & 0.99\times10^{-3} \end{array}\right]$$

The closed-loop transfer function is found to be

$$H_c(s) = C(sI-A-BK)^{-1}BN = C_o(sI-A_o-B_oK_o)^{-1}B_oN_o$$

$$= \mathrm{diag}\left[\frac{\hat{c}_{12}(s+10^{-2})}{s^2-\hat{\alpha}_{12}s-\hat{\alpha}_{11}},\ \frac{\hat{c}_{22}(s+10^{-2})}{s^2-\hat{\alpha}_{22}s-\hat{\alpha}_{21}},\ \frac{\hat{c}_{32}(s+10^{-2})}{s^2-\hat{\alpha}_{32}s-\hat{\alpha}_{31}}\right] \quad (20)$$

We observe that the resulting closed-loop system is composed by three noninteracting second-order systems. Clearly, we can control the poles and the d.c. gains of each subsystem by selecting the parameters $\hat{\alpha}_{ij}$, $i=1,2,3$, $j=1,2$, and $\hat{c}_{12}$, $i=1,2,3$ of the canonically decoupled model (18). The zeros however are fixed at $s=-10^{-2}$ and cannot be controlled by the state feedback (19). The most important advantage of this canonically decoupled system achieved is that besides the input-output decoupling one also has decoupling between states belonging to different subsystems (cores). This implies that the three subsystems composing the whole system are completely noninteracting and so one can control the output (power level) as well as the state responses of each subsystem independently.

Now, let us examine the problem of controlling the zeros. Suppose for example that each subsystem in the canonically decoupled model must have two poles at $s_{1,2}=(-3+j\sqrt{3})/2$, a zero at $s=0$, and a d.c. gain equal to $10^6/3$. This means that the desired transfer function of each subsystem is $10^6/(s^2+3s+3)$. The denominator implies that each subsystem has an undamped natural frequency $\omega_n=\sqrt{3}$ and a damping ratio $\zeta=3/2\omega_n = \sqrt{3/2}$. Now, since the zero at $s=-10^{-2}$ of each subsystem cannot be controlled by state feedback, a cascade precompensator must be added to each subsystem with a pole at $s=-10^{-2}$ and a zero at the desired position $s=0$. Hence, one obtains the equality

$$\frac{1}{s+10^{-2}}\left[\frac{\hat{c}_{i2}(s+10^{-2})}{s^2-\hat{\alpha}_{i2}s-\hat{\alpha}_{i1}}\right] = \frac{10^6}{s^2+3s+3} \qquad (i=1,2,3)$$

from which it follows that $\hat{c}_{i2}=10^6$ and a $\hat{\alpha}_{i1}=\hat{\alpha}_{i2}=-3$ (i=1,2,3). In state space form each subsystem has the equations

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_i = \begin{bmatrix} 0 & 1 \\ \hat{\alpha}_{i1} & \hat{\alpha}_{i2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i,\ y_i=\begin{bmatrix} \hat{c}_{i1}, 10^2\hat{c}_{i1} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_i$$

and each cascade precompensator the equations $\dot{\bar{x}}_i = -10^{-2}\bar{x}_i+\bar{u}_i$, $u_i=\bar{x}_i$. Hence the state equations of each precompensated canonically decoupled subsystem are

$$\frac{d\tilde{x}_i}{dt} = \left[\begin{array}{cc:c} 0 & 1 & 0 \\ \hat{\alpha}_{i1} & \hat{\alpha}_{i2} & 1 \\ \hline 0 & 0 & -10^{-2} \end{array}\right]\tilde{x}_i + \left[\begin{array}{c} 0 \\ 0 \\ \hline 1 \end{array}\right]\bar{u}_i,\ \tilde{x}_i = \left[\begin{array}{c} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_i \\ \hline \bar{x}_i \end{array}\right]$$

$$y_i = \left[\begin{array}{cc:c} \hat{c}_{i1}, & 10^2\hat{c}_{i1} & 0 \end{array}\right]\tilde{x}_i$$

It is easy to verify that the transfer function of the preceding subsystem is $y_i(s)/\bar{u}_i(s)=\hat{c}_{i2}/(s^2-\hat{\alpha}_{i2}s-\hat{\alpha}_{i1})$ as desired.

<u>Example 2</u>

The second example is a two coupled-core reactor system with control rod dynamics:

$$\begin{aligned} &\dot{x}_1 = -\frac{D}{\tau}x_1-\frac{\alpha n_o}{\tau}x_2+\frac{D}{\tau}x_3+\frac{n_o}{\tau}x_5,\ \dot{x}_2=kx_1-mx_2,\ \dot{x}_5=-\Theta x_5+\mu u_1 \\ &\dot{x}_3 = \frac{D}{\tau}x_1-\frac{D}{\tau}x_3-\frac{\alpha n_o}{\tau}x_4+\frac{n_o}{\tau}x_6,\ \dot{x}_4 = kx_3-mx_4,\ \dot{x}_6=-\Theta x_6+\mu u_2 \\ &y_1 = x_1,\ \ y_2 = x_3 \end{aligned} \tag{21}$$

where $\Theta=10\ \mathrm{sec}^{-1}$, $\mu=1$ and the remaining parameters have the same values as in example 1. Introducing the parameter values one finds that the matrices A_o,B_o and C_o of the state space description are

$$A_o=\begin{bmatrix} 1 & -10^3 & 1 & 0 & 10^6 & 0 \\ 10^{-5} & -10^{-2} & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -10^3 & 0 & 10^6 \\ 0 & 0 & 10^{-5} & -10^{-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix},\quad B_o^T=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\quad C_o=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

The system has no inherent coupling since

$$D = \begin{bmatrix} c_1AB \\ c_2AB \end{bmatrix}_o = 10^6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |D| \neq 0$$

The similarity matrix Q and its inverse are found to be

$$Q = \begin{bmatrix} 0 & 10^{-1} & 0 & 0 & 0 & 0 \\ 10^{-6} & -10^{-3} & 0 & 0 & 0 & 0 \\ -1.01\text{x}10^{-6} & -0.99\text{x}10^{-3} & 10^{-6} & 0 & 1 & 0 \\ 0 & 0 & 0 & 10^{-1} & 0 & 0 \\ 0 & 0 & 10^{-6} & -10^{-3} & 0 & 0 \\ 10^{-6} & 0 & -1.01\text{x}10^{-6} & -0.99\text{x}10^{-3} & 0 & 1 \end{bmatrix}$$

$$Q^{-1} = \begin{bmatrix} 10^4 & 10^6 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^4 & 10^6 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 2\text{x}10^{-2} & 1.01 & 1 & -10^{-2} & -1 & 0 \\ -10^{-2} & -1 & 0 & 2\text{x}10^{-2} & 1.01 & 1 \end{bmatrix}$$

and the input-Luenberger form of (22) is found to be

$$A = QA_o^{-1} = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -2\text{x}10^{-1} & -10.12 & -11.01 & 10^{-1} & 10.01 & 1 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 10^{-1} & 10.01 & 1 & -2\text{x}10^{-1} & -10.12 & -11.01 \end{array}\right]$$

$$B^T = (QB_o)^T = \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right], \quad C = C_oQ^{-1} = \left[\begin{array}{ccc|ccc} 10^4 & 10^6 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 10^4 & 10^6 & 0 \end{array}\right] \tag{23}$$

The matrices (23) suggest the following canonically decoupled Luenberger model

$$A=\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & 0 & \\ \hat{a}_{11} & \hat{a}_{12} & \hat{a}_{13} & & & \\ \hline & & & 0 & 1 & 0 \\ & 0 & & 0 & 0 & 1 \\ & & & \hat{a}_{21} & \hat{a}_{22} & \hat{a}_{23} \end{array}\right] \quad \hat{C}=\left[\begin{array}{ccc|ccc} \hat{c}_{11} & 10^2\hat{c}_{11} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \hat{c}_{21} & 10^2\hat{c}_{21} & 0 \end{array}\right]$$

$$\hat{B}=B,\ \hat{c}_{12}=10^2\hat{c}_{11},\ \hat{c}_{22}=10^2\hat{c}_{21}$$

In the present case the decoupling conditions give $M=10^{-4}\text{diag}\left[\hat{c}_{11},\hat{c}_{11},\hat{c}_{11},\hat{c}_{21},\hat{c}_{21},\hat{c}_{21}\right]$ and the decoupling matrix gain pair $\{K,N\}$ is found to be

$$K=\left[\begin{array}{ccc|ccc} \hat{a}_{11} & \hat{a}_{12} & \hat{a}_{13} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \hat{a}_{21} & \hat{a}_{22} & \hat{a}_{23} \end{array}\right]$$

$$-\left[\begin{array}{ccc|ccc} -2\times10^{-1}, & -10.12, & -11.01 & 10^{-1}, & 10.01, & 1 \\ \hline 10^{-1}, & 10.01, & 1 & -2\times10^{-1} & -10.12, & -11.01 \end{array}\right]$$

$$N=10^{-4}\begin{bmatrix} \hat{c}_{11} & 0 \\ 0 & \hat{c}_{21} \end{bmatrix}$$

Finally, the closed-loop transfer matrix function is

$$H_c(s)=C(sI-A-BK)^{-1}BN=\text{diag}\left[\frac{\hat{c}_{12}(s+10^{-2})}{s^3-\hat{a}_{13}s^2-\hat{a}_{12}s-\hat{a}_{11}},\ \frac{\hat{c}_{22}(s+10^{-2})}{s^3-\hat{a}_{23}s^2-\hat{a}_{22}s-\hat{a}_{21}}\right]$$

which can be treated for pole, d.c. gain, and zero control as in Example 1.

5. CONCLUSIONS

The technique presented in this paper is applicable to coupled nuclear reactor systems which, owing to the large number of states of each core and the large number of cores possible, belong to the class of large multivariable systems. Spatially-distributed-core reactors can be treated using this technique by subdividing the core into a number of coupled subcores. Actually, the results of this paper constitute the first part of a work aiming to apply the state feedback approach to complete power reactor systems [20]. This will first require an extension of the method to systems with time delays in the state and/or the control variables. An extension of the method for treating the inputs and outputs in groups is also under investigation. When the system involves some states which are not accessible to direct measurements one must use output feedback or generate the unknown state from the measured states upon which they are dependent. This problem is essentially open. A general FORTRAN program which will provide the solution for systems with large matrices is being developed. The method is general and can be used not only in reactor systems but in all cases where simultaneous input-output and state variable decoupling, pole control, d.c. gain control, and zero control is desired.

It is noted that one may reverse the order of steps 3 and 4 in the algorithm of sec. 3.3, i.e. introduce the precompensator prior to decoupling, by defining the matrices of the precompensated input-Luenberger canonical system under control as

$$A' = \left[\begin{array}{c|c} A & 0 \\ \hline 0 & 0 \end{array}\right], \quad B' = \left[\begin{array}{c|c} B & 0 \\ \hline 0 & 1 \end{array}\right], \quad C' = \left[\begin{array}{c|c} C & C^{11} \\ \hline C^{21} & C^{22} \end{array}\right]$$

where the elements of C^{11}, C^{21}, C^{22}, except of the requirement to be selected such that the system has not inherent coupling, are otherwise arbitrary. Of course in this case there is again the possibility of loosing or introducing undesired zeros, and so the algorithm must be preferred in the order step 1, step 2, step 3, step 4.

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