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90

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Using Sophisticated Models in Resolution Theorem Proving

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ABSTRACT

Two related topics are treated dealing with methods for mechanical theorem proving in the first order predicate calculus using the resolution system of J. A. Robinson.

The first topic deals with a new strategy, called Hereditary Lock Resolution (HLR) which is a refinement of the original resolution inference rule. HLR is composed of two interacting refinements. One is a modification of the locking refinement (a syntactic refinement) of Boyer, and the other is a strengthening of the model strategy (a semantic refinement) of Luckham. Previously known strategies combining syntactic and semantic components either used a weaker syntactic strategy than lock resolution, or used weaker semantic notions, or were incomplete (i.e. unable to prove some theorems). HLR is complete and sound (i.e. never constructs fallacious proofs). HLR generates a search space involving clauses, as does ordinary resolution, but each clause has attached to it an additional data structure which contains information about the deduction leading to that clause. This data structure is called an FSL (False Substitution List) and consists of a set of literals all of which must be falsifiable according to some model (which initially can be chosen arbitrarily). The FSL mechanism is applicable to other semantic refinements of resolution besides HLR, and this is illustrated specifically for the case of the model strategy of Luckham.

The second topic concerns the specification and use of models in resolution inference systems. The usual requirement in semantic refinements of resolution has been that the model used must be a Herbrand interpretation, which is an abstractly defined way of considering models. However, in pragmatic situations where implemented procedures must utilize models, Herbrand interpretations which capture the relevant structure of the domain to be modeled usually are both difficult to find and computationally costly to use. We take the position that the essence of the difficulty is that Herbrand interpretations require the specification of details which are mostly irrelevant to the theorem proving task, and that the way out of this difficulty is to develop a theory of models which are based on incomplete specification. The key to doing this is to focus on the interface between a semantic refinement of resolution (e.g. HLR) and the model. This interface is simple and is adequately summarized by the notion of a semantic function, which is a function mapping logical sentences into the values "true" or "false". Once this is done a simple theory of incompletely specified models can be developed which defines semantic functions with the appropriate properties. The completeness of HLR and other semantic refinements can be demonstrated using these semantic functions instead of the usual Herbrand interpretations.

PREFACE

This work is a slightly edited version of the authors Ph.D. thesis, which was originally entitled "HL-Resolution: A Semantic Refinement of Resolution and a Theory of Model Specification". Thanks are due to several people who had a constructive influence on the completion of this work.

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CONTENTS

Abstract..... ii

Preface.....iii

List of Figures..... vi

Chapter 1

1.0 Introduction..... 1

1.1 Overview..... 4

1.2 Conventions and Definitions..... 8

Chapter 2

2.0 Hereditary Lock Resolution..... 12

2.1 Examples of HL-Resolution..... 13

2.2 Combining LR and TMS: The Combination Issue..... 34

2.3 TMS and History Tracing..... 43

2.4 Formal Definition of HL-Resolution..... 53

2.5 Soundness of HL-Resolution..... 77

Chapter 3

3.0 Completeness of HL-Resolution..... 81

3.1 Completeness Theorems..... 81

3.2 Herbrand Model Completeness Proof..... 84

3.3 Completeness of the Inference Rule HLR - t.v.d.....103

3.4 HLR False Permissive Completeness Proof.....110

Chapter 4

4.0 Models.....114
4.1 Notation and Preliminary Definitions.....118
4.2 Theoretical Construction
of Sound Semantic Functions.....127
4.3 Example III -- A Simple Model Structure.....143
4.4 Pragmatic Construction
of Sound Semantic Functions.....149
4.5 Example IV --- A Sophisticated Model.....164
4.6 General Discussion of Models.....193

Chapter 5

5.0 Discussion of HLR.....207
5.1 Refinement Strength of HLR.....207
5.2 Overhead of HLR.....215
5.3 Experimental Results -- Domain of Applicability....219
5.4 Compatibility with other Strategies.....222
5.5 Areas for Future Research.....227

Bibliography.....229
Table of Symbol Conventions.....234
Index of Definitions.....237

LIST OF FIGURES

Figure 1--1 A TMS Search..... 5

Figure 2--1 Example I..... 19

Figure 2--2 Example II..... 25

Figure 2--3 A TMS + FSL Search..... 45

Figure 2--4 An Infeasible Deduction..... 45

Figure 4--1 Relationship of Herbrand Interpretations and Model Schemes in Different Languages.....126

"There are no solved problems; there are only problems that are more or less solved."

H. Poincare