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André Thayse

Boolean Calculus of
Differences



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W. Brauer · P. Brinch Hansen · D. Gries · C. Moler · G. Seegmüller
J. Stoer · N. Wirth

Author

André Thayse
Philips Research Laboratory
av. Van Becelaere, 2
B-1170 Brussels
Belgium

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FOREWORD

by Sheldon B. Akers

The development of switching circuit theory over the past three decades has mirrored the varying concerns of the logic designers who have had to confront the many problems presented by constantly changing circuit technologies. All too often, yesterday's elegant solution has been rendered obsolete by today's technological breakthrough. It is not surprising, therefore, that the accepted techniques and procedures of present day switching circuit theory too often tend to stand as distinct entities rather than as part of a cohesive whole.

Accordingly, it is a great pleasure to be able to recommend a book which not only provides a much needed historical perspective to these many developments but, even more importantly, does so within the framework of a single comprehensive structure. Starting with the basic definitions of Boolean algebra and the Boolean difference, the author carefully and systematically develops and extends these concepts to subsume such diverse areas as two-level minimization, hazard detection,unate functions, fault diagnosis, functional decomposition, and many others. A significant part of this theory derives directly from previous work by the author and his colleagues at the Philips Research Laboratory.

The elegance of the underlying theory, together with its breadth of coverage and the clarity of the author's presentation, is destined to make this book a classic in its field.

Syracuse, New York, U.S.A.

Sheldon B. Akers.

Contents.

Preface of Prof. Sheldon B. Akers

1. Introduction	1
2. Canonical expansions of Boolean functions	5
2.1. General Boolean notations	5
2.1.1. Definitions and elementary properties	5
2.1.2. Boolean algebra	6
2.1.3. Boolean ring	7
2.1.4. The two element Boolean algebra ; Boolean functions	9
2.1.5. Well-formed Boolean expression ; well-formed Galoisian expression	9
2.2. Partial expansions with respect to one variable	10
2.2.1. Lagrange expansion and redundant expansion	10
2.2.2. Hybrid expansions	12
2.2.3. Galoisian expansions	12
2.2.4. Matrix expression of the canonical expansions	13
2.2.5. Equivalent forms for canonical expansions	13
2.3. Complete expansions	14
2.3.1. Lagrange expansions	14
2.3.2. The extended state vector	15
2.3.3. Redundant canonical expansions	16
2.3.4. Prime implicants and prime implicates	17
2.3.5. Galoisian expansions	18
3. Differences of Boolean functions	19
3.1. Simple differences	19
3.1.1. Definitions	19
3.1.2. Functional properties	20
3.1.3. Relations between the differences	21
3.1.4. Relations between the function and its differences	22
3.1.5. Relations between differences for a fixed operation	22
3.1.6. Table of differences	23
3.1.7. Lattice of differences	25
3.1.8. The Boolean difference $\Delta^A f / \Delta x$: properties and bibliographical notes	28
3.1.9. The meet difference $p^A f / p x$: properties and bibliographical notes	29
3.1.10. The join difference $q^A f / q x$: properties and bibliographical notes	31
3.1.11. Properties of the envelopes	32
3.1.12. Envelopes and Galoisian expansions	33
3.1.13. The oriented differences : properties and bibliographical notes	34
3.1.14. Example	35

3.2. Multiple differences	36
3.2.1. Definitions and functional properties	36
3.2.2. Expansions and properties of the function $T_{\underline{x}_0}^A f / T_{\underline{x}_0}$	38
3.2.3. The function δ	39
3.2.4. Generalization of theorem 3.1.3.	41
3.2.5. Generalization of theorem 3.1.4.	42
3.2.6. Functional properties of the meet and join differences	44
3.2.7. Properties of the function δ	45
3.2.8. Applications in switching theory	46
3.2.9. The Boolean differences and the Galoisian expansions	48
3.2.10. The Boolean differences and the circuit optimization	50
3.2.11. The sensitivities	50
3.2.12. Theorem	52
3.2.13. Meaning of the sensitivity function	53
3.2.14. Meaning of the function $Sf / S_{\underline{x}_0} \oplus \delta f / \delta \underline{x}_0$	
3.2.15. Theorem	56
3.2.16. Generalization of theorem 3.1.11.	56
3.2.17. Generalization of theorem 3.1.12.	57
3.2.18. Application of the concept of envelope in switching theory	57
3.2.19. Continuation of the example 3.1.14.	58
4. Application to switching theory	62
4.1. Introduction	62
4.2. Universal algorithms	62
4.2.1. Algorithms grounded on the extended vector	62
4.2.1.1. Formulas and theorems	62
4.2.1.2. The algebra derived from the general law	65
4.2.1.3. Continuation of the example 3.1.14.	65
4.2.2. Algorithm grounded on the generalized consensus	69
4.2.2.1. Introduction	69
4.2.2.2. The generalized consensus with respect to the law $\uparrow$	73
4.2.2.3. Continuation of example 3.1.14.	76
4.2.2.4. Obtention of the matrix $\underline{\psi}^{(\uparrow)}(f)$ from a diagonal matrix	78
4.3. Particular algorithms related to circuit analysis synthesis methods	79
4.3.1. Synthesis of two-level circuits using AND- and OR-gates	79
4.3.1.1. Problem statement	79
4.3.1.2. Algorithm	80
4.3.1.3. Theorem	81
4.3.1.4. Algorithm	82
4.3.1.5. Computation method for obtaining $pf / p_{\underline{x}}^k$ and $qf / q_{\underline{x}}^k$	82
4.3.1.6. Algorithm	83

4.3.1.7. Example	84
4.3.1.8. Algorithm grounded on the use of the extended vector	93
4.3.2. Synthesis of three-level networks using AND- and OR-gates	96
4.3.2.1. Problem statement	96
4.3.2.2. Algorithm	96
4.3.2.3. Some further considerations related to logical design	97
4.3.3. Two-level networks using AND and EX-OR gates	102
4.3.3.1. Problem statement	102
4.3.3.2. Algorithms grounded on the use of differential operators	103
4.3.3.3. Algorithms grounded on the use of the Taylor expansions	105
4.3.3.4. Algorithm grounded on the Kronecker matrix product	107
4.4. Analysis of combinatorial networks	108
4.4.1. Problem statement	108
4.4.2. Hazard detection	108
4.4.2.1. Problem statement	108
4.4.2.2. Algorithms	110
4.4.2.3. Example	111
4.4.3. Fault detection	111
4.4.3.1. Problem statement	111
4.4.3.2. Computation of test functions for simple faults	114
4.4.3.3. Computation of test functions for multiple faults	114
4.4.3.4. Algorithm and examples	116
4.4.3.5. Continuation of example 4.2.1.7.	120
4.5. Detection of functional properties	120
4.5.1. Detection of the decomposition	120
4.5.1.1. Problem statement	120
4.5.1.2. Decomposition classification	121
4.5.1.3. The results of Ashenurst and Curtis	122
4.5.1.4. Fundamental theorems for simple decompositions	123
4.5.1.5. Algorithm for disjunctive decomposition detection	127
4.5.2. Detection of symmetry	128
4.5.3. Detection of $\underline{A}$ -degeneracy	129
4.5.4. Other applications	130
References	131
Author index	140
Subject index	142
List of symbols	144