## Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

102

### James Harold Davenport

# On the Integration of Algebraic Functions



Springer-Verlag Berlin Heidelberg New York 1981

#### **Editorial Board**

W. Brauer P. Brinch Hansen D. Gries C. Moler G. Seegmüller J. Stoer N. Wirth

#### Author

James Harold Davenport Emmanuel College Cambridge England

AMS Subject Classifications (1979): CR Subject Classifications (1979): 5.25, 5.7

#### ISBN 3-540-10290-6 Springer-Verlag Berlin Heidelberg New York ISBN 0-387-10290-6 Springer-Verlag New York Heidelberg Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to "Verwertungsgesellschaft Wort", Munich.

© by Springer-Verlag Berlin Heidelberg 1981 Printed in Germany Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr. 2145/3140-543210

#### Acknowledgements

I wish to express my gratitude to all the people who have made this work possible. The programming could not have been done without the expert advice of Drs. J.P. ffitch and A.C. Norman on their LISP system, and the advice of Prof. A.C. Hearn, Dr. A.C. Norman and Mr. R.G. Hall on the REDUCE-2 system. I am also grateful to Mrs. P.M.A. Moore for her permission to incorporate several of her changes to REDUCE-2.

I am grateful to the Director and staff of the Cambridge University Computing Service, where nearly all the work described here was performed, for their support of my apparently endless computations and for their advice, especially that of Mr M.J.T. Guy and Mr. C.E. Thompson. The implementation of the integration system at the IBM Thomas J. Watson Research Centre relied on the advice of Mr. J.E. Harry, and the MTS implementation was performed with the assistance of Mr. W.A. Dodge and Mrs. J. Caviness of Rensselaer Polytechnic Institute and Mr. M. Alexander of the University of Michigan. Many improvements to the factoriser were made by Dr. D. Dahm (Burroughs Corp.), who also pointed out several other bugs. Many problems with the LISP interface were discovered and corrected by Mr. D. Morrison (University of Utah).

I am very grateful to the many mathematicians whose advice I have sought, especially Prof. Sir Peter Swinnerton-Dyer, Dr. B.J. Birch, Prof. D.J. Lewis, Prof. S. Maclane, Dr. R.H. Risch, Prof. A. Schinzel and Prof. M.F. Singer. My original introduction to computers and algebraic geometry was performed by Dr. N.M. Stephens. The discussion in chapter 6 on special values of parameters was inspired by discussion with Dr. P.M. Neumann and the Oxford University Number Theory Seminar.

I would also like to acknowledge the help I have received from many discussions with Dr. J.P. ffitch, Prof. B.F. Caviness and the Rensselaer Polytechnic Institute seminar, Dr. R.D. Jenks and Dr. D.Y.Y. Yun of the IBM Thomas J. Watson Research Centre, and the MACSYMA group at M.I.T. (especially Mr. B.M. Trager).

I am very grateful to Professor Sir Peter Swinnerton-Dyer and Professor A. C. Hearn, who read an earlier version of this work, for their many helpful comments and suggestions. Professor M.F. Singer also read a version and made many valuable remarks. Earlier versions of this work were prepared at the Cambridge University Computing Service, and I am grateful to Dr. A.J. Herbert and Mr. M.A. Johnson for their advice on text processing. This version was prepared at the IBM Thomas J. Watson Research Centre using the Yorktown Formatting Language, and I am grateful to the text processing consultants, Miss A.M. Gruhn, Miss K.C. Keene and Mrs. C.H. Thompson, for their advice.

Finally I would like to thank my mother for her assistance, especially with the references, and Dr. A.C. Norman, my supervisor, for his continued advice and encouragement.

#### Contents

1	Introduction	. 5
2	Algebraic Computations	14
3	Coates' Algorithm	30
	Algorithm COATES	31
	Algorithm INTEGRAL_BASIS_REDUCTION	34
	Algorithm NORMAL_BASIS_REDUCTION	35
	Algorithm DIVISOR_TO_FUNCTION	43
4	Risch's Theorem	49
	Algorithm RISCH_ALGEBRAIC	54
	Algorithm FIND_ALGEBRAIC_PART	60
5	The Problem of Torsion Divisors	64
	Algorithm WEIERSTRASSFORM	68
6	Gauss-Manin Operators	76
	Algorithm FIND_ORDER_MANIN	83
7	Elliptic Integrals Concluded	92
	Algorithm LUTZNAGELL	97
	Algorithm FINITE_ORDER_ELLIPTIC	101
	Algorithm DENOMINATOR_ALGEBRAIC	104
8	Curves Over Algebraic Number Fields	106
	Algorithm MAX_POWER	107
	Algorithm BOUNDTORSION	109
	Algorithm OCCURRENCES	114
	Algorithm BOUNDTORSION	114
9	Conclusions	119
AF	PPENDICES	
1	Changes to REDUCE-2	134
	1 Printing.	134
	2 Differentiation.	135
	3 Greatest Common Divisors.	136

#### Contents

	4 Algebraics.	137
	5 Factorisation.	138
	6 Uniqueness of Algebraics.	139
2	Examples	141
	Example 1: Simple Logarithmic.	141
	Example 2: 1/SQRT((X**2-1)*(X**2-K**2)).	146
	Example 3: A Torsion Example.	146
	Example 4: Modular Curve X(7)	148
	Example 5: Chebyshev's Integral	152
	Example 6: A Nested Expression.	163
	Example 7: Logarithmically Unintegrable.	164
	Example 8: Sum of two Functions.	173
3	Algorithms for Algebraic Expressions	177
	Algorithm SQFRNORM	177
	Algorithm ALGFACTOR	178
	Algorithm ALG_FACTOR_2	179
	Algorithm PRIMITIVEELEMENT	180
	Algorithm SQUAREFREEDECOMPOSE	182
Bi	bliography	186