Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

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Large Sparse Numerical Optimization

Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

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CR Subject Classifications (1982): G.1.6, G.1.3

ISBN 3-540-12914-6 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-12914-6 Springer-Verlag New York Heidelberg Berlin Tokyo

Library of Congress Cataloging in Publication Data. Coleman, Thomas F. (Thomas Frederick), 1950- Large sparse numerical optimization. (Lecture notes in computer science; 165) 1. Mathematical optimization-Data processing. 2. Sparse matrices-Data processing. I. Title. II. Series.
QA402.5.C543 1984 519 84-5300
ISBN 0-387-12914-6 (U.S.)

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2145/3140-543210

PREFACE

In the spring of 1983 I taught a graduate course entitled 'Large Sparse Numerical Optimization'. Each week I prepared a set of lecture notes to reflect the material being covered. This manuscript is the result. The purpose of the course was to discuss recent developments in the area in order to prepare a student to pursue research. I hope this manuscript will prove useful to others with a similar purpose in mind.

My class was comprised of graduate students from applied mathematics, computer science, and operations research. All students had strong mathematical backgrounds and were familiar with computational methods in mathematics. In addition, most students had some familiarity with numerical optimization, optimization theory and graph-theoretic concepts (or at least I assumed they did). The students were required to do all exercises listed at the end of each chapter. In addition, each student either presented a paper in the area (- I supplied a number of possibilities) or worked on one of the research problems listed in the notes. It was a fun course !

The development of algorithms for large sparse numerical optimization is currently a very active area of research in numerical analysis. The adaptation of efficient methods to the large sparse setting is proving to be a difficult and challenging task. Apparently, it is often impossible to preserve sparsity and attain other *desirable properties* simultaneously: algorithms must achieve a delicate compromise. An exciting facet of problems in this area is that a full understanding of all algorithmic considerations requires ideas and concepts from many disciplines: eg. linear algebra, real analysis, data structures, graph theory, optimization theory, numerical methods, ... One of my goals in the design of this course was to emphasize this rich diversity of thought.

Though my personal research is mostly concerned with nonlinear optimization, I thought it best to start the course with a fairly detailed discussion of large sparse linear problems. The reasons are obvious: Firstly, they represent important optimization problems in their own right - there are still many open questions. Secondly, nonlinear problems are often solved via a sequence of linear approximations and therefore it is imperative that the complexity of the 'easy' subproblems be fully appreciated. Chapter 1 is devoted to large square systems of linear equations, Chapter 2 discusses overdetermined linear systems and Chapter 3 deals with large linear programs. The flavour of the first three chapters is somewhat different from the remainder of the notes: Their primary purpose to to provide some linear background to the more research oriented nonlinear chapters. Hence ideas are covered more quickly, almost in the style of a survey. (Nevertheless, I do point out research directions and interesting possibilities from time to time.)

The heart of the course and this manuscript is represented by Chapters 4 and 5. They are concerned with unconstrained nonlinear problems: equations, least squares and unconstrained optimization problems are all discussed with

respect to algorithms, theoretical developments and current software. The material is presented unevenly with emphasis on topics and ideas of particular interest to me. I do not apologize for this: These notes are meant to reflect a personal and timely view of recent and current research in a lively field. They are not meant to be a tidy and complete presentation of a mature subject.

Finally, in Chapter 6, some of the issues concerning large quadratic programs are examined. This chapter is even less complete than the others - the intent is to give the reader a taste of the complexity and nature of large scale QP problems. I hope to teach a sequel course, in the near future, that deals with large QP's in greater depth.

I would like to thank Cornell University and the Department of Computer Science in particular for the opportunity to devote my time to such a course. Thanks also to Charlie Van Loan for encouraging me to prepare the lecture notes, to Mike Todd for enduring my lectures and keeping me on my toes, and to Jorge Moré for reading a preliminary draft and suggesting improvements (some of which I implemented). I am grateful to the department's VAX 780, IMAGEN laser printer, and TROFF for helping me prepare these notes 'on the fly'. Finally, a heap of love and thanks goes to Marianne for supporting me through a most demanding term.

January 1984

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