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Uwe Schöning

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PREFACE

This monograph is a revised and extended version of a thesis submitted to the faculty of Mathematics and Computer Science at the University of Stuttgart, Germany, in partial fulfillment of the German "Habilitation" degree. It summarizes certain aspects in structural complexity theory related to the author's research and interests over the last years.

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CONTENTS

Introduction 1
1. Preliminaries 5
2. Circuit-Size Complexity 12
2.1. Circuit-Size Complexity of Boolean Functions 12
2.2. Circuit-Size Complexity of Languages 18
2.3. Generating Circuits 21
2.4. Simulation of Turing Machines by Circuits 22
2.5. P-Selective Sets 27
3. Probabilistic Algorithms 30
3.1. Monte Carlo or Las Vegas? Classifications of Probabilistic Algorithms 30
3.2. Iteration of Probabilistic Algorithms 34
3.3. Probabilistic Classes and Polynomial Size Circuits 39
4. Sparse Sets 42
4.1. Sparse Sets in NP 43
4.2. "Almost Correct" Algorithms 47
4.3. "Almost Fast" Algorithms and Complexity Cores 52
4.4. Relativization, Sparse Oracles, and Circuits 57
4.5. Self-Reducibility 60
5. The Low and High Hierarchies 64
5.1. Definitions and Elementary Results 64
5.2. Circuit-Size Complexity and the Low Hierarchy 68
5.3. Lowness of Some Special Classes 71
5.4. Lowness of BPP 73
5.5. Refinement of the Low and High Hierarchies 75
6. Oracles 81
6.1. Positive Relativizations 84
6.2. Negative Relativizations 88
6.3. Higher Levels of the Polynomial Hierarchy 90
References 93
Symbols 97
Index 98