

# Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

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Varol Akman

Unobstructed Shortest Paths  
in Polyhedral Environments

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Dedicated to my parents and my brother

For me there is only the traveling on paths that have heart, on any path that may have heart. There I travel, and the only worthwhile challenge is to traverse its full length. And there I travel, looking, looking, breathlessly.

- DON JUAN

Namely, because the shape of the whole universe is most perfect and, in fact, designed by the wisest creator, nothing in all of the world will occur in which no maximum or minimum rule is somehow shining forth.

- LEONHARD EULER

Whenever the local endpoint of the unknown quantity describes a straight line or a circle, a plane locus results, and when it describes a parabola, hyperbola, or ellipse, a solid locus results.

- PIERRE DE FERMAT

Is the three-dimensional shortest path problem NP-complete? For one, it does not seem to be in NP at all, because of the difficulty with the skew lines mentioned before. The difficulty is reminiscent of a similar one with the Euclidean traveling salesman problem and the precision required in evaluating tours, although the present situation is far more complex, and furthermore, there is no obvious discretization to help avoid the issue.

- CHRISTOS PAPADIMITRIOU

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## Foreword

The study of minimum paths on or around polyhedra in Euclidean 3-space is of growing importance in robotics. This work presents new algorithms based on extensions of the Voronoi diagram. Since experience with new algorithms is also important, this work also describes a workbench to allow experimentation.

This book is based on the Ph.D. research of my former student, Varol Akman, who graduated from the Electrical, Computer, and Systems Engineering Dept. of Rensselaer Polytechnic Institute in August 1985.

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