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Grammatical Picture Generation

A Tree-Based Approach

With 203 Figures and CD-ROM



Springer

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*To my grandparents
Herbert and Wilma Meyer
– for all the love they gave me*

Preface

Pictures are important, in everyday life as well as in art, engineering, and most branches of the natural and social sciences. About three decades ago, the observation that simple geometric processes often yield very complex geometric objects (i.e., pictures) gave rise to new branches of mathematics whose purpose was to study such processes and the resulting pictures: fractal geometry, dynamic systems, and chaos theory. More or less in parallel with this development, the increasing availability of computer desktop systems and other graphical output devices made computer scientists think about formal systems to describe sets of pictures. This led to the development of various types of picture-generating devices. This book is about such picture generators, including some of the most basic devices studied in fractal geometry.

In the computer science literature, the first devices for picture generation were proposed in the late 1960s and early 1970s, most notably the array grammars of Rosenfeld, Siromoney, and others, and the shape grammars of Gips and Stiny. About a decade later, picture generators based on string grammars were proposed, using either the chain-code interpretation of Freeman or the turtle geometry known from the LOGO programming language. Each of these lines of research has been continued ever since, and various other approaches have been proposed.

However, most books on formal mathematical aspects of picture generation are in fact books on fractal geometry and related areas written from a mathematical point of view. They present deep, interesting, and beautiful mathematics and can be recommended to anyone whose curiosity is spurred by reading the following chapters. Nevertheless, I felt that a text intended mainly for a theory-interested computer science readership ought to be written with a somewhat different focus. This book attempts to present some important types of picture generators in a unified framework that highlights their common algorithmic basis. The techniques used and the general spirit of the presentation have been greatly influenced by the theory of computation, and in particular the theory of formal languages. The text tries to illustrate all major concepts by means of examples and concentrates on theoretical ques-

tions regarding, for example, the generative power of the devices considered and their algorithmic properties.

As the book is mathematical in character, it requires a certain familiarity with formal mathematical notions and techniques. It is directed towards readers who know and understand the basic notions of the theory of computation, corresponding to a standard course in an average computer science curriculum. In particular, this includes notions and techniques from the theory of formal languages, such as regular expressions, finite automata, right-linear and context-free grammars, derivations, and the use of pumping lemmas. Readers who appreciate these concepts will, hopefully, like this book as well. However, it is neither necessary nor assumed that the reader is a specialist in formal-language theory, picture generation, or any other field.

If this book is a worthy and readable contribution to the theory of picture generation, this is to a large extent a result of the help I received. During my time at the Department of Computer Science at the University of Bremen, Hans-Jörg Kreowski introduced me to research in general and the fascinating worlds of formal languages and picture generation in particular. His example and unobtrusive guidance, support, and encouragement taught me so much more than just proving theorems. Many of the results reported in this book would not have come into existence without the relaxed and inspiring dialogue that has characterized our collaboration from the very first moment.

Quite early in my research, Joost Engelfriet's lucidly written papers stimulated my interest in tree languages and transformations, which led to the tree-based approach used in this book. Later, his friendly support and will to share his ideas with others resulted in inspiring collaborations. His painstaking proofreading of the manuscript gave rise to an amazing number of improvements and corrections. If you happen to belong to the group of people who know the quality of Joost's comments, you may be able to imagine how much they improved what is now lying in your hands. If you still do not like it, or are annoyed by the remaining mistakes, the responsibility is of course entirely mine.

Thinking back a few years, I am also very grateful to all my former colleagues at the University of Bremen, who created a stimulating research atmosphere during my time there. In particular, I wish to thank Annegret Habel for inspiring me through her work on collage grammars. Renate Klempien-Hinrichs has not only been an engaged co-author and partner in countless fruitful discussions, but also a good friend who would always find an apt remark to tease me. Furthermore, I would like to mention Sigrid Ewert, Denis Lapoire, Nils Schwabe, Stefan Taubenberger, and Carolina von Totth. Each of them has contributed ideas that, in one form or another, have found their way into this book.

Among my new colleagues and students at Umeå University, I would specifically like to thank Johanna Högberg for sharing my interest in formal-language theory and its applications. Gabriel Jonsson and Karl Azab have, as part of their masters' theses, made implementations of branching tree gram-

mars and macro tree transducers that are now part of TREEBAG. The students who attended the course *Datalogi för en aktuell tillämpning* (spring term 2003), in particular Tomas Hämälä, helped me by commenting on an early version of the manuscript and struggling with my definitions, proofs, and explanations.

The Department of Computing Science and the Faculty of Science and Technology at Umeå University provided me with the resources that made it possible to write this book.

I am, further, indebted to Arto Salomaa for his friendly support, to the staff at Springer, in particular Ingeborg Mayer and Ronan Nugent, for their pleasant cooperation during the preparation of the camera-ready version of the manuscript, and to Douglas Meekison, whose detailed copyediting I appreciated very much.

Finally, I would like to point out that there is a web page related to the book, which is going to contain a list of errata and other material. In particular, new versions of the system TREEBAG will be provided on this page whenever available. The location of the page is

<http://www.cs.umu.se/~drewes/picgen>.

I encourage all readers to report errors and send me comments and questions via e-mail using the address

drewes@cs.umu.se.

I am very much interested in every kind of suggestions and criticism!

Umeå,
October 2005

Frank Drewes

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