

Zhong Li

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Fuzzy Chaotic Systems

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# Fuzzy Chaotic Systems

Modeling, Control, and Applications

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To Juan and Yifan

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## Preface

Bringing together the two seemingly unrelated concepts, *fuzzy logic* and *chaos theory*, is primarily motivated by the concept of *soft computing* (SC), initiated by Lotfi A. Zadeh, the founder of fuzzy set theory. The principal constituents of SC are fuzzy logic (FL), neural network theory (NN) and probabilistic reasoning (PR), with the latter subsuming parts of belief networks, genetic algorithms, chaos theory and learning theory. What is important to note is that SC is not a melange of FL, NN and PR. Rather, it is an integration in which each of the partners contributes a distinct methodology for addressing problems in their common domain. In this perspective, the principal contributions of FL, NN and PR are complementary rather than competitive.

SC differs from conventional (hard) computing in that it is tolerant of imprecision, uncertainty and partial truth. In effect, the role model for soft computing is the human mind. From the general SC concept, we extract FL and chaos theory as the object of this book to study their relationships or interactions.

Over the past few decades, fuzzy systems technology and chaos theory have received ever increasing research interests from, respectively, systems and control engineers, theoretical and experimental physicists, applied mathematicians, physiologists, and other communities of researchers. Especially, as one of the emerging information processing technologies, fuzzy systems technology has achieved widespread applications around the globe in many industries and technical fields, ranging from control, automation, and artificial intelligence (AI) to image/signal processing and pattern recognition. On the other hand, in engineering systems chaos theory has evolved from being simply a curious phenomenon to one with real, practical significance and utilization. We are now standing at the threshold of major advances in the control and synchronization of chaos with new applications across a broad range of engineering disciplines, where chaos control promises to have a major impact on novel time- and energy-critical engineering applications.

Notably, studies on fuzzy systems and chaos theory have been carrying out separately. Although there have been some efforts on exploring the inter-

actions between fuzzy logic and chaos theory, it is still far away from fully understanding their mutual relationships. Intuitively, fuzzy logic resembles human reasoning in its use of approximate information and uncertainty to generate decisions, and chaotic dynamics could be a fundamental reason for human brain to produce and process massive information instantly. It is believed that they have a close relationship in human reasoning and information processing, and it has a great potential to combine fuzzy systems with chaos theory for future scientific research and engineering applications.

This book does not intend to – in fact, is not able to – give an in-depth explanation of the interactions or intrinsic relationships between fuzzy logic and chaos theory, but tries to provide some heuristic research achievements and insightful ideas to attract more attention on the topic. Although this book may raise more questions than it can provide answers, We hope that it nevertheless contains seeds for future brooming research.

More precisely, this book is written in the following way: it starts with the fundamental concepts of fuzzy logic and fuzzy control, chaos theory and chaos control, as well as the definition of chaos on the metric space of fuzzy sets, followed by fuzzy modeling and (adaptive) fuzzy control of chaotic systems, all based on both Mamdani fuzzy models and Takagi-Sugeno (TS) fuzzy models; then, it discusses some other topics, such as synchronization, anti-control of chaos, intelligent digital redesign, all for TS fuzzy systems, and spatiotemporal chaos and synchronization in complex fuzzy systems; finally, it ends with a practical application example of fuzzy-chaos-based cryptography.

I am very grateful to Prof. Wolfgang A. Halang and Prof. Guanrong Chen for their long-term support and friendship in various aspects of my work and life, without which this book would not have been completed. Special thanks are given to Dr. Hojae Lee for providing me with the materials presented in Chapters 6, 9, and 12. Thanks also go to the following individuals who provided some original figures or helpful comments: Oscar Calvo, Federico Cuesta, Patrick Grim, Z.H. Guan, K.-Y. Lian, Domenico M. Porto, M. La Rosa, Hua O. Wang, and H.B. Zhang. I am very appreciative of the discussions and collaborations with Peter Kloeden, Zongyuan Mao, Ping Ren, Bo Zhang, Yaobin Mao, Wenbo Liu, Shujun Li, Martin Skambraks, Jutta Düring, Ping Li, and Hong Li. I owe my deepest thanks to my parents for their fostering and education. Finally, I wish to express my appreciation to Prof. Janusz Kacprzyk for recommending this book to the Springer series, Studies in Fuzziness and Soft Computing, and the editorial and production staff of Springer-Verlag in Heidelberg for their fine work in producing this new monograph.

Hagen, Germany  
February 2006

*Zhong Li*

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