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Adaptive Atmospheric Modeling

Key Techniques in Grid Generation,
Data Structures, and Numerical Operations
with Applications

With 74 Figures and 3 Tables

 Springer

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Für Katja, Laila und Janka.
Ihr seid das größte Glück in meinem Leben.

Preface

This work represents the essence of nearly 16 years of work in scientific computing for ocean and atmospheric modeling. This journey started at Alfred-Wegener-Institute for Polar and Marine Research in Bremerhaven, Germany, where – as a post graduate – I was assigned to optimize multi-grid solvers for elliptic partial differential equations evolving from ocean modeling. I am lucky and grateful that I had the chance to go all that way, to have the opportunity to explore the subject from many different angles, to have wonderful teachers and colleagues, and to have the chance to work and visit such exquisite places as the *National Center for Atmospheric Research* in Boulder, Colorado, USA, the *Frontier Research System for Global Change at the Yokohama Institute for Earth Science* (Earth Simulator) in Yokohama, Japan, the *Max-Planck-Institute for Meteorology* in Hamburg, Germany, the aforementioned *Alfred-Wegener-Institute for Polar and Marine Research* in Bremerhaven and Potsdam, Germany, the *Technische Universität München* in Garching, Germany, the *Fields Institute* in Toronto, Canada, the *Naval Research Laboratory* in Monterey, California, USA, the *Department of Informatics at the University of Bergen*, Norway and the *Centre for Mathematical Sciences at the University of Cambridge*, UK, to name only the most influential ones.

My own interest in adaptive methods arose from the exploration of finite element methods. When comparing finite elements on simple model problems with finite differences, the former method is often disregarded for computational efficiency and implementation complexity issues. However, finite element methods are much more versatile and flexible when it comes to irregular domains and locally refined meshes. Besides, finite elements are mathematically the more elegant approach. Being an aesthete, the wish for a method that fully unfolds the beauty of finite element methods was created. That was the start for the development of the adaptive semi-Lagrangian finite element method, better to be called adaptive Lagrange-Galerkin method [34]. Since then, my research was focused on adaptivity and the solution of geophysical fluid dynamics problems with advanced adaptive numerical methods. This was

during my PhD period at Alfred-Wegener-Institute (AWI), where a prototype implementation of a shallow water solver was accomplished.

With a grant from the Federal Ministry of Education and Research (BMBF), I started the development of a parallelizable adaptive mesh generation tool for oceanic and atmospheric applications in the mid 1990's. At that time there was no such tool available. This work was continued after funding ran out by an internal grant from AWI, before I decided to enlarge my scientific background by changing to Technische Universität München (TUM). The time at TUM was great in that it provided me with a lot of new knowledge in numerical analysis. I owe my teacher Folkmar Bornemann a great debt of gratitude for his patience and his clearly structured and precise way of teaching.

By now, several groups have gained a lot of experience in adaptive modeling. Yet in atmospheric sciences, the number is still limited. To fulfil my requirements of a German Habilitation, I considered to just compose some of my articles and reports for a short written document of my work in adaptive atmospheric modeling. However, thinking again, I am now convinced that taking the chance of having to write a monograph, is the best excuse for doing this a bit more carefully and summarizing what has been done in adaptive atmospheric modeling so far. I am aware that this snapshot can only be incomplete. However, the references and approaches mentioned may at least give a good starting point for research in adaptive atmospheric modeling.

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Munich,
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