A Fuzzy Approach to Portfolio Rebalancing with Transaction Costs*

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Abstract. The fuzzy set is a powerful tool used to describe an uncertain financial environment in which not only the financial markets but also the financial managers' decisions are subject to vagueness, ambiguity or some other kind of fuzziness. Based on fuzzy decision theory, two portfolio rebalancing models with transaction costs are proposed. An example is given to illustrate that the two linear programming models based on fuzzy decisions can be used efficiently to solve portfolio rebalancing problems by using real data from the Shanghai Stock Exchange.

1 Introduction

In 1952, Markowitz [8] published his pioneering work which laid the foundation of modern portfolio analysis. It combines probability theory and optimization theory to model the behavior of economic agents under uncertainty. Konno and Yamazika [5] used the absolute deviation risk function, to replace the risk function in Markowitz's model thus formulated a mean absolute deviation portfolio optimization model. It turns out that the mean absolute deviation model maintains the nice properties of Markowitz's model and removes most of the principal difficulties in solving Markowitz's model.

Transaction cost is one of the main sources of concern to portfolio managers. Arnott and Wagner [2] found that ignoring transaction costs would result in an inefficient portfolio. Yoshimoto's emperical analysis [12] also drew the same conclusion. Due to changes of situation in financial markets and investors' preferences towards risk, most of the applications of portfolio optimization involve a revision of an existing portfolio, *i.e.*, portfolio rebalancing.

Usually, expected return and risk are two fundamental factors which investors consider. Sometimes, investors may consider other factors besides the expected

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return and risk, such as liquidity. Liquidity has been measured as the degree of probability involved in the conversion of an investment into cash without any significant loss in value. Arenas, Bilbao and Rodriguez [1] took into account three criteria: return, risk and liquidity and used a fuzzy goal programming approach to solve the portfolio selection problem.

In 1970, Bellman and Zadeh [3] proposed the fuzzy decision theory. Ramaswamy [10] presented a portfolio selection method using the fuzzy decision theory. A similar approach for portfolio selection using the fuzzy decision theory was proposed by León *et al.* [6]. Using the fuzzy decision principle, Östermark [9] proposed a dynamic portfolio management model by fuzzifying the objective and the constraints. Watada [11] presented another type of portfolio selection model using the fuzzy decision principle. The model is directly related to the mean-variance model, where the goal rate (or the satisfaction degree) for an expected return and the corresponding risk are described by logistic membership functions.

This paper is organized as follows. In Section 2, a bi-objective linear programming model for portfolio rebalancing with transaction costs is proposed. In Section 3, based on the fuzzy decision theory, two linear programming models for portfolio rebalancing with transaction costs are proposed. In Section 4, an example is given to illustrate that the two linear programming models based on fuzzy decisions can be used efficiently to solve portfolio rebalancing problems by using real data from the Shanghai Stock Exchange. A few concluding remarks are finally given in Section 5.

2 Linear Programming Model for Portfolio Rebalancing

Due to changes of situation in financial markets and investors' preferences towards risk, most of the applications of portfolio optimization involve a revision of an existing portfolio. The transaction costs associated with purchasing a new portfolio or rebalancing an existing portfolio have a significant effect on the investment strategy. Suppose an investor allocates his wealth among n securities offering random rates of returns. The investor starts with an existing portfolio and decides how to reconstruct a new portfolio.

The expected net return on the portfolio after paying transaction costs is given by

$$\sum_{j=1}^{n} r_j(x_j^0 + x_j^+ - x_j^-) - \sum_{j=1}^{n} p(x_j^+ + x_j^-) \tag{1}$$

where r_j is the expected return of security j, x_j^0 is the proportion of the security j owned by the investor before portfolio reblancing, x_j^+ is the proportion of the security j bought by the investor, x_j^- is the proportion of the security j sold by the investor during the portfolio rebalancing process and p is the rate of transaction costs.

Denote $x_j = x_j^0 + x_j^+ - x_j^-, j = 1, 2, \cdots, n$. The semi-absolute deviation of return on the portfolio $x = (x_1, x_2, \cdots, x_n)$ below the expected return over the past period t, $t = 1, 2, \cdots, T$ can be represented as

$$w_t(x) = |\min\{0, \sum_{j=1}^{n}(r_{jt} - r_j)x_j\}|. \tag{2}$$

where r_{jt} can be determined by historical or forecast data.

The expected semi-absolute deviation of the return on the portfolio $x = (x_1, x_2, \cdots, x_n)$ below the expected return can be represented as

$$w(x) = \frac{1}{T}\sum_{t=1}^{T} w_t(x) = \frac{1}{T}\sum_{t=1}^{T}|\min\{0, \sum_{j=1}^{n}(r_{jt} - r_j)x_j\}|. \tag{3}$$

Usually, the anticipation of certain levels of expected return and risk are two fundamental factors which investors consider. Sometimes, investors may wish to consider other factors besides expected return rate and risk, such as liquidity. Liquidity has been measured as the degree of probability of being able to convert an investment into cash without any significant loss in value. Generally, investors prefer greater liquidity, especially since in a bull market for securities, returns on securities with high liquidity tend to increase with time. The turnover rate of a security is the proportion of turnover volumes to tradable volumes of the security, and is a factor which may reflect the liquidity of the security. In this paper, we assume that the turnover rates of securities are modelled by possibility distributions rather than probability distributions.

Carlsson and Fullér [4] introduced the notation of crisp possibilistic mean (expected) value and crisp possibilistic variance of continuous possibility distributions, which are consistent with the extension principle. Denote the turnover rate of the security j by the trapezoidal fuzzy number $\hat{l}_j = (la_j, lb_j, \alpha_j, \beta_j)$. Then the turnover rate of the portfolio $x = (x_1, x_2, \cdots, x_n)$ is $\sum_{j=1}^{n} \hat{l}_j$. By the definition, the crisp possibilistic mean (expected) value of the turnover rate of the portfolio $x = (x_1, x_2, \cdots, x_n)$ can be represented as

$$E(\hat{l}(x)) = E(\sum_{j=1}^{n}\hat{l}_j x_j) = \sum_{j=1}^{n}(\frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6})x_j. \tag{4}$$

Assume that the investor does not invest the additional capital during the portfolio rebalancing process. We use $w(x)$ to measure the risk of the portfolio and use the crisp possibilistic mean (expected) value of the turnover rate to measure the liquidity of the portfolio. Assume the investor wants to maximize return on and minimize the risk to the portfolio after paying transaction costs. At the same time, he requires that the liquidity of the portfolio is not less than a given constant through rebalancing the existing portfolio. Based on the above discussions, the portfolio rebalancing problem is formulated as follows:

$$
(\text{P1})\begin{cases}
\max \sum\limits_{j=1}^{n} r_j(x_j^0+x_j^+-x_j^-)-\sum\limits_{j=1}^{n} p(x_j^+ + x_j^-)\\
\min \sum\limits_{t=1}^{T} \frac{|\sum\limits_{j=1}^{n}(r_{jt}-r_j)x_j|+\sum\limits_{j=1}^{n}(r_j-r_{jt})x_j}{2T}\\
s.t. \quad \sum\limits_{j=1}^{n}(\frac{la_j+lb_j}{2}+\frac{\beta_j-\alpha_j}{6})x_j \geq l,\\
\qquad \sum\limits_{j=1}^{n} x_j = 1,\\
\qquad x_j = x_j^0+x_j^+-x_j^-, j=1,2,\cdots,n,\\
\qquad 0 \leq x_j^+ \leq u_j, j=1,2,\cdots,n,\\
\qquad 0 \leq x_j^- \leq x_j^0, j=1,2,\cdots,n.
\end{cases}
$$

where l is a given constant by the investor and u_j represents the maximum proportion of the total amount of money devoted to security $j, j \in S$.

Eliminating the absolute function of the second objective function, the above problem can be transformed into the following problem:

$$
(\text{P2})\begin{cases}
\max \sum\limits_{j=1}^{n} r_j(x_j^0+x_j^+-x_j^-)-\sum\limits_{j=1}^{n} p(x_j^+ + x_j^-)\\
\min \frac{1}{T}\sum\limits_{t=1}^{T} y_t\\
s.t. \quad \sum\limits_{j=1}^{n}(\frac{la_j+lb_j}{2}+\frac{\beta_j-\alpha_j}{6})x_j \geq l,\\
\qquad y_t+\sum\limits_{j=1}^{n}(r_{jt}-r_j)x_j \geq 0, t=1,2,\cdots,T,\\
\qquad \sum\limits_{j=1}^{n} x_j = 1,\\
\qquad x_j = x_j^0+x_j^+-x_j^-, j=1,2,\cdots,n,\\
\qquad 0 \leq x_j^+ \leq u_j, j=1,2,\cdots,n,\\
\qquad 0 \leq x_j^- \leq x_j^0, j=1,2,\cdots,n.\\
\qquad y_t \geq 0, t=1,2,\cdots,T.
\end{cases}
$$

where l is a given constant by the investor.

The above problem is a bi-objective linear programming problem. One can use several algorithms of multiple objective linear programming to solve it efficiently.

3 Portfolio Rebalancing Models Based on Fuzzy Decision

In the portfolio rebalancing model proposed in above section, the return, the risk and the liquidity of the portfolio are considered. However, investor's satisfactory degree is not considered. In financial management, the knowledge and experience of an expert are very important in decision-making. Through comparing the present problem with their past experience and evaluating the whole portfolio in terms of risk and liquidity in the decision-making process, the experts may estimate the objective values concerning the expected return, the risk and the liquidity. Based on experts' knowledge, the investor may decide his levels of aspiration for the expected return, the risk and the liquidity of the portfolio.

3.1 Portfolio Rebalancing Model with Linear Membership Function

During the portfolio rebalancing process, an investor considers three factors (the expected return, the risk and the liquidity of the portfolio). Each of the factors is transformed using a membership function so as to characterize the aspiration level. In this section, the three factors are considered as the fuzzy numbers with linear membership function.

a) Membership function for the expected return on the portfolio

$$\mu_r(x) = \begin{cases} 0 & \text{if } \ E(r(x)) < r_0 \\ \frac{E(r(x))-r_0}{r_1-r_0} & \text{if } \ r_0 \leq E(r(x)) \leq r_1 \\ 1 & \text{if } \ E(r(x)) > r_1 \end{cases}$$

where r_0 represents the necessity aspiration level for the expected return on the portfolio, r_1 represents the sufficient aspiration level for the expected return of the portfolio.

b) Membership function for the risk of the portfolio

$$\mu_w(x) = \begin{cases} 1 & \text{if } \ w(x) < w_0 \\ \frac{w_1-w(x)}{w_1-w_0} & \text{if } \ w_0 \leq w(x) \leq w_1 \\ 0 & \text{if } \ w(x) > w_1 \end{cases}$$

where w_0 represents the necessity aspiration level for the risk of the portfolio, w_1 represents the sufficient aspiration level for the risk of the portfolio.

c) Membership function for the liquidity of the portfolio

$$\mu_{\hat{l}}(x) = \begin{cases} 0 & \text{if } \ E(\hat{l}(x)) < l_0 \\ \frac{E(\hat{l}(x))-l_0}{l_1-l_0} & \text{if } \ l_0 \leq E(\hat{l}(x)) \leq l_1 \\ 1 & \text{if } \ E(\hat{l}(x)) > l_1 \end{cases}$$

where l_0 represents the necessity aspiration level for the liquidity of the portfolio, l_1 represents the sufficient aspiration level for the liquidity of the portfolio.

The values of r_0, r_1, w_0, w_1, l_0 and l_1 can be given by the investor based on the experts' knowledge or past experience. According to Bellman and Zadeh's maximization principle, we can define $\lambda = \min\{\mu_r(x), \mu_w(x), \mu_{\hat{l}}(x)\}$.

The fuzzy portfolio rebalancing problem can be formulated as follows:

$$(\text{P3}) \begin{cases} \max \lambda \\ s.t. \ \ \mu_r(x) \geq \lambda, \\ \qquad \mu_w(x) \geq \lambda, \\ \qquad \mu_{\hat{l}}(x) \geq \lambda, \\ \qquad \sum\limits_{j=1}^{n} x_j = 1, \\ \qquad x_j = x_j^0 + x_j^+ - x_j^-, j = 1, 2, \cdots, n, \\ \qquad 0 \leq x_j^+ \leq u_j, j = 1, 2, \cdots, n, \\ \qquad 0 \leq x_j^- \leq x_j^0, j = 1, 2, \cdots, n, \\ \qquad 0 \leq \lambda \leq 1. \end{cases}$$

Furthermore, the fuzzy portfolio rebalancing problem can be rewritten as follows:

$$
\text{(P4)} \begin{cases} \max \lambda \\ s.t. \;\; \sum\limits_{j=1}^{n} r_j x_j - \sum\limits_{j=1}^{n} p(x_j^+ + x_j^-) \geq \lambda(r_1 - r_0) + r_0, \\ \qquad \frac{1}{T}\sum\limits_{t=1}^{T} y_t \leq w_1 - \lambda(w_1 - w_0), \\ \qquad \sum\limits_{j=1}^{n} (\frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6}) x_j \geq \lambda(l_1 - l_0) + l_0, \\ \qquad y_t + \sum\limits_{j=1}^{n} (r_{jt} - r_j) x_j \geq 0, t = 1, 2, \cdots, T, \\ \qquad \sum\limits_{j=1}^{n} x_j = 1, \\ \qquad x_j = x_j^0 + x_j^+ - x_j^-, j = 1, 2, \cdots, n, \\ \qquad 0 \leq x_j^+ \leq u_j, j = 1, 2, \cdots, n, \\ \qquad 0 \leq x_j^- \leq x_j^0, j = 1, 2, \cdots, n, \\ \qquad y_t \geq 0, t = 1, 2, \cdots, T, \\ \qquad 0 \leq \lambda \leq 1. \end{cases}
$$

where r_0, r_1, l_0, l_1, w_0 and w_1 are constants given by the investor based on the experts' knowledge or past experience.

The above problem is a standard linear programming problem. One can use several algorithms of linear programming to solve it efficiently, for example, the simplex method.

3.2 Portfolio Rebalancing Model with Non-linear Membership Function

Watada [11] employed a logistic function for a non-linear membership function $f(x) = \frac{1}{1+exp(-\alpha)}$. We can find that a trapezoidal membership function is an approximation from a logistic function. Therefore, the logistic function is considered much more appropriate to denote a vague goal level, which an investor considers.

Membership functions $\mu_r(x), \mu_w(x)$ and $\mu_{\hat{l}}(x)$ for the expected return, the risk and the liquidity on the portfolio are represented respectively as follows:

$$\mu_r(x) = \frac{1}{1 + exp(-\alpha_r(E(r(x)) - r_M))}, \tag{5}$$

$$\mu_w(x) = \frac{1}{1 + exp(\alpha_w(w(x) - w_M))}, \tag{6}$$

$$\mu_{\hat{l}}(x) = \frac{1}{1 + exp(-\alpha_l(E(\hat{l}(x)) - l_M))} \tag{7}$$

where α_r, α_w and α_l can be given respectively by the investor based on his own degree of satisfaction for the expected return, the level of risk and the liquidity. r_M, w_M and l_M represent the middle aspiration levels for the expected return, the level of risk and the liquidity of the portfolio respectively. The value of r_M, w_M and l_M can be gotten approximately by the values of r_0, r_1, w_0, w_1, l_0 and l_1, i.e. $r_M = \frac{r_0+r_1}{2}$, $w_M = \frac{w_0+w_1}{2}$ and $l_M = \frac{l_0+l_1}{2}$.

Remark: α_r, α_w and α_l determine respectively the shapes of membership functions $\mu_r(x), \mu_w(x)$ and $\mu_{\hat{l}}(x)$ respectively, where $\alpha_r > 0, \alpha_w > 0$ and $\alpha_l > 0$. The larger parameters α_r, α_w and α_l get, the less their vagueness becomes.

The fuzzy portfolio rebalancing problem can be formulated as follows:

$$
(\text{P5}) \begin{cases}
\max \eta \\
s.t. \quad \mu_r(x) \geq \eta, \\
\qquad \mu_w(x) \geq \eta, \\
\qquad \mu_{\hat{l}}(x) \geq \eta, \\
\qquad \sum_{j=1}^{n} x_j = 1, \\
\qquad x_j = x_j^0 + x_j^+ - x_j^-, j = 1, 2, \cdots, n, \\
\qquad 0 \leq x_j^+ \leq u_j, j = 1, 2, \cdots, n, \\
\qquad 0 \leq x_j^- \leq x_j^0, j = 1, 2, \cdots, n, \\
\qquad 0 \leq \eta \leq 1.
\end{cases}
$$

Let $\theta = \log\frac{1}{1-\eta}$, then $\eta = \frac{1}{1+exp(-\theta)}$. The logistic function is monotonously increasing, so maximizing η makes θ maximize. Therefore, the above problem may be transformed to an equivalent problem as follows:

$$
(\text{P6}) \begin{cases}
\max \theta \\
s.t. \quad \alpha_r(\sum_{j=1}^{n} r_j x_j - \sum_{j=1}^{n} p(x_j^+ + x_j^-)) - \theta \geq \alpha_r r_M, \\
\qquad \theta + \frac{\alpha_w}{T} \sum_{t=1}^{T} y_t \leq \alpha_w w_M, \\
\qquad \alpha_l \sum_{j=1}^{n} (\frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6}) x_j - \theta \geq \alpha_l l_M, \\
\qquad y_t + \sum_{j=1}^{n} (r_{jt} - r_j) x_j \geq 0, t = 1, 2, \cdots, T, \\
\qquad \sum_{j=1}^{n} x_j = 1, \\
\qquad x_j = x_j^0 + x_j^+ - x_j^-, j = 1, 2, \cdots, n, \\
\qquad 0 \leq x_j^+ \leq u_j, j = 1, 2, \cdots, n, \\
\qquad 0 \leq x_j^- \leq x_j^0, j = 1, 2, \cdots, n, \\
\qquad y_t \geq 0, t = 1, 2, \cdots, T, \\
\qquad \theta \geq 0.
\end{cases}
$$

where α_r, α_w and α_l are parameters which can be given by the investor based on his own degree of satisfaction regarding the three factors.

The above problem is also a standard linear programming problem. One can use several algorithms of linear programming to solve it efficiently, for example, the simplex method.

Remark: The non-linear membership functions of the three factors may change their shape according to the parameters α_r, α_w and α_l. Through selecting the values of these parameters, the aspiration levels of the three factors may be described accurately. On the other hand, deferent parameter values may reflect deferent investors' aspiration levels. Therefore, it is convenient for deferent investors to formulate investment strategies using the above portfolio rebalancing model with non-linear membership functions.

4 An Example

In this section, we give an example to illustrate the models for portfolio rebalancing based on fuzzy decision as proposed in this paper. We suppose that an investor wants to choose thirty different types of stocks from the Shanghai Stock Exchange for his investment.

The rate of transaction costs for stocks is 0.0055 in the two securities markets on the Chinese mainland. Assume that the investor has already owned an existing portfolio and he will not invest the additional capital during the portfolio rebalancing process. The proportions of the stocks are listed in Table 1.

Table 1. The proportions of stocks in the existing portfolio

Stock	1	2	3	4	5	6	7
Proportions	0.05	0.08	0.05	0.35	0.10	0.12	0.25

Suddenly, the financial market situation changes, and the investor needs to change his investment strategy. In the example, we assume that the upper bound of the proportions of Stock j owned by the investor is 1. Now we use the fuzzy portfolio rebalancing models in this paper to re-allocate his assets. At first, we collect historical data of the thirty kinds of stocks from January, 1999 to January, 2002. The data are downloaded from the website www.stockstar.com. Then we use one month as a period to get the historical rates of returns of thirty-six periods. Using historical data of the turnover rates of the securities, we can estimate the turnover rates of the securities as the trapezoidal fuzzy numbers.

In the following, we will give two kinds computational results according to whether the investor has a conservative or an aggressive approach.

At first, we assume that the investor has a conservative and pessimistic mind. Then the values of r_0, r_1, l_0, l_1, w_0, and w_1 which are given by the investor may be small. They are as follows: $r_0 = 0.028$, $r_1 = 0.030$, $l_0 = 0.020$, $l_1 = 0.025$, $w_0 = 0.025$ and $w_1 = 0.035$.

Considering the three factors (the return, the risk and liquidity) as fuzzy numbers with trapezoidal membership function, we get a portfolio rebalancing strategy by solving (P4). The membership grade λ, the obtained risk, the obtained return and obtained liquidity are listed in Table 2.

Table 2. Membership grade λ, obtained risk, obtained return and obtained liquidity when $r_0 = 0.028$, $r_1 = 0.030$, $l_0 = 0.020$, $l_1 = 0.025$, $w_0 = 0.025$ and $w_1 = 0.035$.

λ	obtained risk	obtained return	obtained liquidity
0.835	0.0266	0.0297	0.0301

Considering the three factors (the return, the risk and liquidity) as fuzzy numbers with non-linear membership function, we get a portfolio rebalancing strategy by solving (P6).

In the example, we give three deferent values of parameters α_r, α_w and α_l. The membership grade η, the obtained risk, the obtained return and obtained liquidity are listed in Table 3.

Table 3. Membership grade η, obtained risk, obtained return and obtained liquidity when $r_M = 0.029$, $w_M = 0.030$ and $l_M = 0.0225$.

η	θ	α_r	α_w	α_l	obtained risk	obtained return	obtained liquidity
0.811	1.454	600	800	600	0.0282	0.0314	0.0304
0.806	1.425	500	1000	500	0.0286	0.0319	0.0303
0.785	1.295	400	1200	400	0.0289	0.0322	0.0302

Secondly, we assume that the investor has an aggressive and optimistic mind. Then the values of r_0, r_1, l_0, l_1, w_0, and w_1 which are given by the investor are big. They are as follows: $r_0 = 0.028$, $r_1 = 0.036$, $l_0 = 0.021$, $l_1 = 0.031$, $w_0 = 0.032$ and $w_1 = 0.036$.

Considering the three factors (the return, the risk and liquidity) as fuzzy numbers with trapezoidal membership function, we get a portfolio rebalancing strategy by solving (P4). The membership grade λ, the obtained risk, the obtained return and obtained liquidity are listed in Table 4.

Table 4. Membership grade λ, obtained risk, obtained return and obtained liquidity when $r_0 = 0.028$, $r_1 = 0.036$, $l_0 = 0.021$, $l_1 = 0.031$, $w_0 = 0.032$ and $w_1 = 0.036$.

λ	obtained risk	obtained return	obtained liquidity
0.890	0.0324	0.0351	0.0298

Considering the three factors (the return, the risk and liquidity) as fuzzy numbers with non-linear membership function, we get a portfolio rebalancing strategy by solving (P6).

In the example, we give three deferent values of parameters α_r, α_w and α_l. The membership grade η, the obtained risk, the obtained return and obtained liquidity are listed in Table 5.

Table 5. Membership grade η, obtained risk, obtained return and obtained liquidity when $r_M = 0.032$, $w_M = 0.034$ and $l_M = 0.026$.

η	θ	α_r	α_w	α_l	obtained risk	obtained return	obtained liquidity
0.849	1.726	600	800	600	0.0318	0.0349	0.0295
0.836	1.630	500	1000	500	0.0324	0.0353	0.0293
0.802	1.396	400	1200	400	0.0328	0.0355	0.0295

From the above results, we can find that we get the different portfolio rebalancing strategies by solving (P6) in which the different values of the parameters (α_r, α_w and α_l) are given. Through choosing the values of the parameters α_r, α_w and α_l according to the investor's frame of mind, the investor may get a favorite portfolio rebalancing strategy. The portfolio rebanlancing model with the non-linear membership function is much more convenient than the one with the linear membership function.

5 Conclusion

Considering the expected return, the risk and liquidity, a linear programming model for portfolio rebalancing with transaction costs is proposed. Based on fuzzy decision theory, two fuzzy portfolio rebalancing models with transaction costs are proposed. An example is given to illustrate that the two linear programming models based on fuzzy decision-making can be used efficiently to solve portfolio rebalancing problems by using real data from the Shanghai Stock Exchange. The computation results show that the portfolio rebanlancing model with the non-linear membership function is much more convenient than the one with the linear membership function. The portfolio rebalaning model with non-linear membership function can generate a favorite portfolio rebalancing strategy according to the investor's satisfactory degree.

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