# A 3D-Polar Coordinate Colour Representation Well Adapted to Image Analysis

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**Abstract.** Representations of the RGB space in terms of 3D-polar coordinates (hue, saturation and brightness) are often used in image analysis. The literature describes a large number of similar coordinate systems (HLS, HSV, etc.). We show that the reason for the existence of so many systems is a poor definition of the saturation coordinate which makes it dependent on the brightness function used, and hence poorly suited to image analysis applications. An improved saturation measurement which (1) always has small values for achromatic colours and (2) is independent of the brightness function is derived.

## 1 Introduction

Representations of the RGB space in terms of 3D-polar (or cylindrical) coordinates (hue, saturation and brightness) are often used in image analysis. Even though these representations suffer from some defects [1], they are often more intuitive than the RGB representation, and could reveal features of the image which are not visible in the RGB representation. Even though the conversion between these two systems is simply a coordinate transformation, a bewildering array of such transformations is described in the literature (HLS, HSI, HSV, etc.). In this paper, we demonstrate the cause of this colour space proliferation, and suggest a solution. In section 2, the existing transforms and their associated shortcomings are reviewed. A better saturation expression is derived in section 3. The complete transformation from the RGB coordinate system to a 3D-polar coordinate system including the suggested saturation measurement is summarised in section 4. Section 5 contains some suggested applications of the improved colour space, and the conclusion. A more detailed treatment of this subject is given in [2].

## 2 Existing Transforms

In the RGB colour space, colours are specified as vectors $(R, G, B)$ giving the amount of each red, green and blue primary stimulus in the colour. We assume

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$R, G, B \in [0,1]$ so that the valid coordinates form the cube $[0,1] \times [0,1] \times [0,1]$. These ranges can easily be expanded to any required range. We next discuss the transformations from the rectangular RGB coordinate system to 3D-polar coordinate system (section 2.1), point out the problems arising when using most of the existing transforms (section 2.2), and show the result of removing the brightness dependence of the saturation term in these transforms (section 2.3).

### 2.1 Overview of Existing Transforms

The basic idea behind the transformation from an RGB coordinate system to a 3D-polar coordinate system is described in [3]. One first places a new axis in the RGB space between the points $(0,0,0)$ and $(1,1,1)$. As this axis passes through all the grey-levels (i.e. those colours for which $R = G = B$), it is referred to as the *achromatic axis*. One then chooses a function which calculates the brightness or luminance $L(\mathbf{c})$ of a colour $\mathbf{c} = (R, G, B)$. Note that we use the standard International Commission on Illumination (CIE) definitions of brightness and luminance [4]: *Brightness* is a subjective measure of the amount of light a source appears to emit, whereas *Luminance* is radiant intensity weighted by the spectral response of the human eye. The form chosen for $L(\mathbf{c})$ determines the shape of the iso-brightness surfaces, where iso-brightness surface $l$ contains all the points satisfying $L(\mathbf{c}) = l$. These iso-brightness surfaces are then projected onto a plane perpendicular to the achromatic axis and intersecting it at the origin, referred to as the *chromatic plane* as it contains all the colour information. The *hue* and *saturation* coordinates are then respectively the angle and magnitude in a polar coordinate system within the plane centred on the origin.

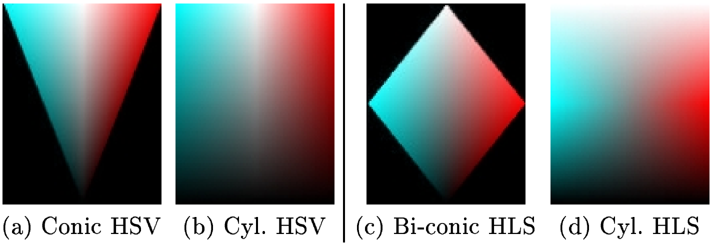
The shape of the resulting colour gamut depends on the brightness function chosen. To visualise the shape, the points of each iso-brightness surface $l$ are projected onto a chromatic plane intersecting the achromatic axis at position $l$. The sub-regions of each plane containing projected points then form the colour space gamut. We now briefly summarise two of the most commonly used 3D-polar coordinate spaces.

**The HSV Model** The HSV model brightness function is

$$L_{\mathrm{HSV}}(\mathbf{c}) = \max(R, G, B) \tag{1}$$

When one visualises the corresponding colour gamut by piling up the chromatic planes as described above, one obtains a hexcone. A vertical slice through this hexcone along the achromatic axis is shown in Figure 1a. The HSV saturation expression is

$$S_{\mathrm{HSV}}(\mathbf{c}) = \begin{cases} \frac{\max(R,G,B)-\min(R,G,B)}{\max(R,G,B)} & \text{if } \max(R, G, B) \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$



(a) Conic HSV (b) Cyl. HSV (c) Bi-conic HLS (d) Cyl. HLS

**Fig. 1.** Slices through the conic and cylindrical versions of the HSV and HLS colour spaces. The brightness increases from bottom to top, and the saturation increases from the centre (achromatic axis) outwards. Colours to the right of the central achromatic axis have hues of 0°, and those to the left have hues of 180°.

**The HLS model** The brightness function used in the HLS model is

$$L_{\mathrm{HLS}}(\mathbf{c}) = \frac{\max(R,G,B) + \min(R,G,B)}{2} \tag{3}$$

The colour gamut produced by stacking the chromatic planes is a double hexcone, a cross-section of which is shown in Figure 1c. The HLS model saturation is

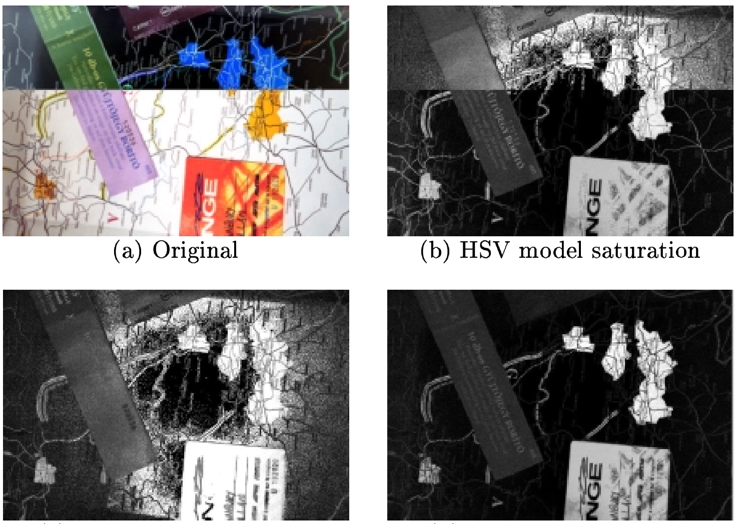
$$S_{\mathrm{HLS}} = \begin{cases} 0 & \text{if } \max(R,G,B) = \min(R,G,B) \\ \frac{\max(R,G,B)-\min(R,G,B)}{\max(R,G,B)+\min(R,G,B)} & \text{if } L_{\mathrm{HLS}} \leq \frac{1}{2} \\ \frac{\max(R,G,B)-\min(R,G,B)}{2-[\max(R,G,B)+\min(R,G,B)]} & \text{otherwise} \end{cases} \tag{4}$$

### 2.2 Problems Arising When Using These Transforms

The HSV and HLS spaces were developed during the 1970's for easy numerical specification of colours in computer graphics applications [5]. Due to the shapes of the colour gamuts, it would be easy for a user to accidentally specify a colour outside of the valid gamut. The solution of expanding the conic and double conic shapes into cylinders was therefore adopted. This is easily done by dividing the saturation values by the maximum possible values for the corresponding brightness. Slices through the cylindrically-shaped versions of the HSV and HLS spaces are shown in Figures 1b and 1d respectively. The cylindrically-shaped versions have often been carried over into image analysis and computer vision, for which they are ill-suited.

To demonstrate the unsuitability of the cylindrically-shaped colour gamuts for image processing and analysis, we examine the saturations corresponding to the HSV and HLS colour spaces of the colour image in Figure 2a. This image was captured under slightly non-uniform lighting conditions, so that not all the pixels which look white have RGB coordinates of exactly $(1,1,1)$. The top part of the image was inverted by subtracting the pixel values in each channel from the maximum. The HSV saturation for this colour image is shown in Figure 2b. As expected, the white pixels in the lower region of the image have a saturation of around zero. However, due to the artificial expansion of the lower part of the HSV cone, some of the black pixels are shown as being fully saturated. This obviously contradicts the definition of saturation, which states that for achromatic pixels, the saturation should be very low. Hence, one has the undesirable situation where some of the black pixels are shown as being more highly saturated than some of the colourful pixels. The situation is worse for the HLS space, in which the expansion occurs in the upper and lower brightness regions of the double cone, resulting in spurious high saturations for both black and white pixels, as seen in Figure 2c. This demonstrates that two of the common assumption about these cylindrically-shaped models are untrue. In fact, for the HSV and HLS saturation measures:

1. Saturation is not necessarily low-valued for achromatic pixels.
2. Chromatic and brightness information is not independent. As the brightness function is used to normalise the saturation, the saturation values depend critically on the brightness function, as is visible in the large differences between Figures 2b and 2c.



(a) Original (b) HSV model saturation

(c) HLS model saturation (d) Suggested saturation

**Fig. 2.** 3D-polar coordinate components of image (a). (b) HSV model saturation. (c) HLS model saturation. (d) The suggested saturation measure.

We now give two examples of the confusion arising from using these poorly defined models. In [6], a constant saturation threshold is applied in the cylindrically-shaped HLS space to differentiate between chromatic and achromatic colours. This threshold can be represented by a vertical line on either side of the achromatic axis in Figure 1d, and it is clear that this does not give a good differentiation. This was later improved by using a hyperbola in the cylindrical HSV space

[7], which is equivalent to using a constant threshold in the conically-shaped HSV space. In [8], the assumption is made that the cylindrical HSV space is perceptually uniform when a Euclidean metric is used, but upon examining Figure 1b, one sees that a certain distance in the high brightness (top) part of the space corresponds to a far larger perceived change in colour than the same distance in the low brightness part of the space.

### 2.3 Removal of the Brightness Dependence of the Saturation

The simplest way of avoiding the disadvantages tied to the cylindrically-shaped spaces is to remove the brightness normalisation from the saturation expressions, hence reverting to the original shapes of the spaces. Removing this brightness dependence from the saturation for the HSV model is simply done by multiplying equation 2 by the brightness $L_{\mathrm{HSV}}$, giving

$$S^{\mathrm{NC}} = \max(R, G, B) - \min(R, G, B) \tag{5}$$

where the superscript 'NC' indicates that this is the non-cylindrical version. For the HLS space, removing the brightness dependence is slightly more complex due to its double-cone shape. The non-cylindrical saturation is

$$S_{\mathrm{HLS}}^{\mathrm{NC}} = S_{\mathrm{HLS}} \left[ 1 - 2 \left| \frac{1}{2} - L_{\mathrm{HLS}} \right| \right] \tag{6}$$

which after some manipulation also reduces to equation 5. We show in the next section that equation 5 can be derived in a general way.

## 3 Geometric Derivation of a Saturation Term

We geometrically derive a saturation expression which does not suffer from the disadvantages listed above. This derivation is based on the one in [3], the difference being that we avoid the steps leading to the normalisation of the saturation by the brightness. Given a vector $\mathbf{c}$ in the RGB space, we begin by considering the triangle which contains all colours with the same hue as $\mathbf{c}$, shown in Figure 3a. The vector $\mathbf{L}(\mathbf{c}) = [l(\mathbf{c}), l(\mathbf{c}), l(\mathbf{c})]$ gives the position on the achromatic axis (in RGB coordinates) of the brightness of $\mathbf{c}$. By definition, for all points $\mathbf{c}$, the lines between $\mathbf{L}(\mathbf{c})$ and $\mathbf{c}$ are parallel (iso-brightness lines). The point with the same hue as $\mathbf{c}$ lying furthest away from the achromatic axis is labelled $\mathbf{q}(\mathbf{c})$. This point necessarily lies on one of the edges of the RGB cube. Traditionally, the saturation is defined as the length of the line from $\mathbf{L}(\mathbf{c})$ to $\mathbf{c}$, divided by the length of its extension to the surface of the RGB cube. This definition, however, produces a 3D-polar coordinate space in the form of a cylinder.

To define a saturation which preserves the conical form of the space, we instead divide the distance from $\mathbf{L}(\mathbf{c})$ to $\mathbf{c}$ by the distance between $\mathbf{L}[\mathbf{q}(\mathbf{c})]$ and $\mathbf{q}(\mathbf{c})$, i.e. the longest iso-brightness line. We therefore have

$$S = \frac{\|\mathbf{L}(\mathbf{c}) - \mathbf{c}\|}{\|\mathbf{L}[\mathbf{q}(\mathbf{c})] - \mathbf{q}(\mathbf{c})\|} \tag{7}$$

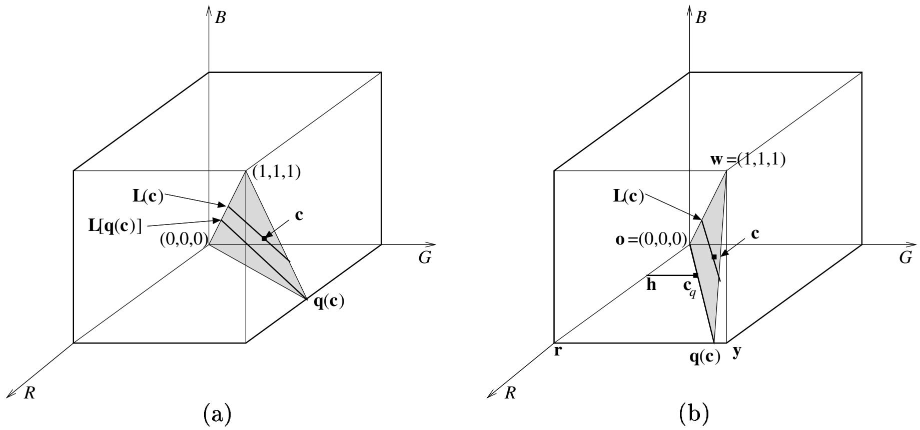



**Fig. 3.** (a) Diagram used in the derivation of a general saturation expression. The achromatic axis is the line between the points $(0, 0, 0)$ and $(1, 1, 1)$. (b) Diagram used in the derivation of the simpler saturation expression. Both diagrams show the triangle which contains all the points with the same hue as $\mathbf{c}$.

which, in addition, is independent of the brightness function. This can easily be proved by using similar triangles, as shown in [2]. This saturation calculated for Figure 2a is shown in Figure 2d. It is clear that the defects associated with the cylindrically-shaped HSV and HLS models are not present. The colourful regions always have saturation values higher than the surrounding monochromatic background. Furthermore, one would obtain the same saturation values irrespective of the brightness function used.

One can derive the saturation expression of equation 5 by choosing the iso-brightness surfaces to be parallel to the nearest side of the RGB cube which intersects the origin (this can be done due to the independence of the brightness and saturation). The brightness function producing such iso-brightness surfaces is

$$L(\mathbf{c}) = \min(R, G, B) \tag{8}$$

Initially, we consider only iso-hue triangles for which the point $\mathbf{q}(\mathbf{c})$ lies on the cube edge between $\mathbf{r}$ and $\mathbf{y}$ in Figure 3b. The brightness vector of $\mathbf{c}$ inside this region is $\mathbf{L}(\mathbf{c}) = (B, B, B)$. We project $\mathbf{c}$ onto the $B = 0$ plane resulting in point $\mathbf{c}_q$ which is the same distance from the origin as $\mathbf{L}(\mathbf{c})$ is from $\mathbf{c}$. The coordinates of $\mathbf{c}_q$ are therefore $(R - B, G - B, 0)$. We then construct the line between the point $\mathbf{h}$ indicated in the diagram and $\mathbf{c}_q$ in the $B = 0$ plane parallel to the cube edge between $\mathbf{r}$ and $\mathbf{y}$, forming two similar triangles with vertices $\mathbf{o}$, $\mathbf{h}$ and $\mathbf{c}_q$, and $\mathbf{o}$, $\mathbf{r}$ and $\mathbf{q}(\mathbf{c})$. Hence

$$\frac{\|\mathbf{o} - \mathbf{c}_q\|}{\|\mathbf{o} - \mathbf{q}(\mathbf{c})\|} = \frac{\|\mathbf{o} - \mathbf{h}\|}{\|\mathbf{o} - \mathbf{r}\|} \tag{9}$$

The term on the left is simply the definition of saturation given by equation 7. On the right, $\|\mathbf{o}-\mathbf{r}\|=1$, and as the coordinates of $\mathbf{h}$ are $(R-B,0,0)$, $\|\mathbf{o}-\mathbf{h}\|=R-B$. Hence the saturation in this sector is $S=R-B$. By definition, in this sector, $\max(R,G,B)=R$, and $\min(R,G,B)=B$ [3]. By repeating this derivation in the other five sectors, one can show that equation 5 is valid for all points in the RGB cube.

## 4 Transformation to the IHLS Space

The complete transformation from the RGB space to the IHLS (Improved HLS) space, a 3D-polar coordinate space using the suggested saturation measure, is summarised in this section. MATLAB code implementing it is available on the author's home page. The coordinates calculated are the luminance $Y$, the saturation $S$ and the hue $H$ ($H'$ is an interim result in the calculation of $H$).

$$
\begin{aligned}
Y &= 0.2126R + 0.7152G + 0.0722B \\
S &= \max(R,G,B) - \min(R,G,B) \\
H' &= \arccos\left[\frac{R-\frac{1}{2}G-\frac{1}{2}B}{(R^2+G^2+B^2-RG-RB-BG)^{\frac{1}{2}}}\right] \\
H &= \begin{cases} 360^\circ - H' & \text{if } B > G \\ H' & \text{otherwise} \end{cases}
\end{aligned}
$$

The inverse transformation is easily derivable, as shown in [2]. Furthermore, a number of simplifications are possible, such as an approximation of the hue which avoids the use of trigonometric functions.

## 5 Conclusion

We have described the shortcomings inherent in many of the 3D-polar coordinate colour representations presently used, and have suggested a solution in the form of an improved saturation expression. The suggested saturation measure is independent of the brightness, and has the advantage of always being low-valued for achromatic colours. It can therefore replace the heuristic expressions (summarised in [9]) developed for differentiating between chromatic and achromatic colours. Tico et al. [10], for example, suggest using the standard deviation of the $R$, $G$ and $B$ coordinates in conjunction with a fuzzy membership function (containing two user-specified parameters) to calculate a weight differentiating between chromatic and achromatic colours, the basic idea being that the more colourful (higher saturated) pixels receive higher weighting in the hue histogram than the less colourful (lower saturated) ones. The saturation measurement of the IHLS space can be directly used as such a weight. This saturation measurement has also been used in the calculation of saturation-weighted hue statistics and of saturation-weighted hue histograms [11]. Furthermore, it is directly applicable to ordering colours by saturation in mathematical morphology operators [12].

Why, it may be asked, is such a colour representation space necessary? Surely it is better to use a standardised colour space such as the CIE L*a*b* space or its cylindrical coordinate version. The obvious objection to the use of the L*a*b* space is that one needs calibration information on the image which is being transformed from the RGB space, namely the colour coordinates of the source of illumination (the white point). This information is not always available for the images that are encountered in computer vision applications, especially in multimedia applications such as content-based image retrieval [13]. While it is often possible to estimate the white point [14], there could be situations in which this would result in an unacceptably long calculation time, or in which we do not wish to introduce uncertainty in the form of assumptions and estimations. This alternate and more intuitive coordinate system for representing the information that is known, namely the coordinates of the colours in the RGB space, is then extremely useful.

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