How Folds Cut a Scene

Patrick S. Huggins and Steven W. Zucker*

Yale University, New Haven CT 06520, USA
{huggins,zucker}@cs.yale.edu

Abstract. We consider the interactions between edges and intensity distributions in semi-open image neighborhoods surrounding them. Locally this amounts to a kind of figure-ground problem, and we analyze the case of smooth figures occluding arbitrary backgrounds. Techniques from differential topology permit a classification into what we call folds (the side of an edge from a smooth object) and cuts (the background). Intuitively, cuts arise when an arbitrary scene is "cut" from view by an occluder. The condition takes the form of transversality between an edge tangent map and a shading flow field, and examples are included.

1 Introduction

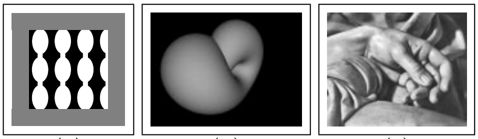
On which side of an edge is figure; and on which ground? This classical Gestalt question is thought to be locally undecidable, and ambiguous globally (Fig. 1(a)). Even perfect line drawing interpretation is combinatorially difficult (NP-complete for the simple blocks world) [13], and various heuristics, such as closure or convexity, have been suggested [7]. Nevertheless, an examination of natural images suggests that the intensity distribution in the neighborhood of edges does contain relevant information, and our goal in this paper is to show one basic way to exploit it.

The intuition is provided in Fig. 1(b). From a viewer's perspective, edges arise when the tangent plane to the object "folds" out of sight; this naturally suggests a type of "figure", which we show is both natural and commonplace. In particular, it enjoys a stable pattern of shading (with respect to the edge). But more importantly, the fold side of the edge "cuts" the background scene, which implies that the background cannot exhibit this regularity in general.

Our main contribution in this paper is to develop the difference between *folds* and *cuts* in a technical sense. We employ the techniques of differential topology to capture qualitative aspects of shape, and propose a specific mechanism for classifying folds and cuts based on the interaction between edges and the shading flow field. The result is further applicable to formalizing an earlier classification of shadow edges [1].

* Supported by AFOSR

C. Arcelli et al. (Eds.): IWVF4, LNCS 2059, pp. 323–332, 2001.



(a) (b) (c)

Fig. 1. (a) An ambiguous image. The edges lack the information present in (b), a Klein bottle. The shading illustrates the di erence between the fold , where the normal varies smoothly to the edge until it is orthogonal to the viewer, and the cut . (c) An image with pronounced folds and cuts.

2 Folds and Cuts

Figure-ground relationships are determined by the positions of surfaces in the image relative to the viewer, so we are specifically interested in edges resulting from surface geometry and viewing, which we now consider.

Consider an image ($I : Z \subset \mathbb{R}^2 \to \mathbb{R}^+$) of a smooth ($C^2$) surface $\Sigma : X \subset \mathbb{R}^2 \to Y \subset \mathbb{R}^3$; here X is the surface parameter space and Y is 'the world'. For a given viewing direction $\mathbf{V} \in \mathbb{S}^2$ (the unit sphere), the surface is projected onto the image plane by $\Pi_{\mathbf{V}} : Y \to Z \subset \mathbb{R}^2$. For simplicity, we assume that Π is orthographic projection, although this particular choice is not crucial to our reasoning. Thus the mapping from the surface domain to the image domain takes $\mathbb{R}^2$ to $\mathbb{R}^2$. See Fig. 2.

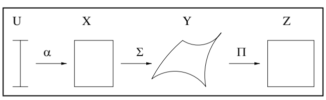


Fig. 2. The mappings referred to in the paper, from the parameter of a curve (U), to the coordinates of a surface (X), to Euclidean space (Y), to the image domain (Z).

Points in the resulting image are either regular or singular, depending on whether the Jacobian of the surface to image mapping, $d(\Pi_{\mathbf{V}} \circ \Sigma)$ is full rank or not. An important result in differential topology is the Whitney Theorem for mappings from $\mathbb{R}^2$ to $\mathbb{R}^2$ [5][10], which states that such mappings generically have only two types of singularities, folds and cusps. (By generic we mean that the singularities persist under perturbations of the mapping.)

Let $T_x[A]$ denote the tangent space of the manifold A at the point x.

Definition 1. *The* FOLD *is the singularity locus of the surface to image mapping, $\Pi_{\mathbf{V}} \circ \Sigma$, where Σ is smooth. In the case of orthographic projection the fold is the image of those points on the surface whose tangent plane contains the view direction.*

$$\gamma_{fold} = \{z_p \in Z |\ \mathbf{V} \in T_{y_p}[\Sigma(X)],\ y_p = \Sigma(x_p),\ z_p = \Pi_{\mathbf{V}}(y_p)\}$$