An Intrinsic Coordinate System for Fingerprint Matching

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Abstract. In this paper, an intrinsic coordinate system is proposed for fingerprints. First the fingerprint is partitioned in regular regions, which are regions that contain no singular points. In each regular region, the intrinsic coordinate system is defined by the directional field. When using the intrinsic coordinates instead of pixel coordinates, minutiae are defined with respect to their position in the directional field. The resulting intrinsic minutiae coordinates can be used in a plastic distortion-invariant fingerprint matching algorithm. Plastic distortions, caused by pressing the 3-dimensional elastic fingerprint surface on a flat sensor, now deform the entire coordinate system, leaving the intrinsic minutiae coordinates unchanged. Therefore, matching algorithms with tighter tolerance margins can be applied to obtain better performance.

Keywords. Fingerprint matching, plastic distortions, flow lines, regular regions, intrinsic coordinate system.

1 Introduction

The first step in a fingerprint recognition system is to capture the print of a finger by a fingerprint sensor. In this capturing process, the 3-dimensional elastic surface of a finger is pressed on a flat sensor surface. This 3D-to-2D mapping of the finger skin introduces distortions, especially when forces are applied that are not orthogonal to the sensor surface. The effect is that the sets of *minutiae* (bifurcations and endpoints of the ridges) of two prints of the same finger no longer fit exactly. The ideal way to deal with distortions would be to invert the 3D-to-2D mapping and compare the minutiae positions in 3D. Unfortunately, there is no unique way of inverting this mapping.

Instead of modeling the distortions, most minutiae matching techniques use local similarity measures [Jia00], or allow some amount of displacement in the minutiae matching stage [Jai97]. However, decreasing the required amount of similarity not only tolerates small plastic distortions, but increases the *false acceptance rate* (FAR) as well. It is therefore reasonable to consider methods that explicitly attempt to model and eliminate the distortion. Such methods can be

3rd International Conference on AUDIO- and VIDEO-BASED BIOMETRIC PERSON AUTHENTICATION,
Halmstad, Sweden, June 6-8, 2001.

expected to be stricter in the allowed displacement during minutiae matching. As a consequence, the FAR can be decreased without increasing the *false rejection rate* (FRR).

As far as we know, the only paper that addresses plastic distortions is [Cap01]. In that paper, the physical cause of the distortions is modeled by distinguishing three distinct concentric regions in a fingerprint. In the center region, no distortions are present, since this region tightly fits to the sensor. The outer, or external, region is not distorted either, since it does not touch the sensor. The outer region may be displaced and rotated with respect to the inner region, due to the application of forces while pressing the finger at the sensor. The region in between is distorted in order to fit both regions to each other.

Experiments have shown that this model provides an accurate description of the plastic distortions in some cases. The technique has successfully been applied to the generation of many synthetic fingerprints of the same finger [Cap00]. However, the model has not yet been used in an algorithm for matching fingerprints. Accurate estimation of the distortion parameters is still a topic of research. Furthermore, the prints cannot be truly normalized by the model, since it only describes relative distortions.

In this paper, an alternative approach is proposed, using a linked multilevel description of the fingerprint. The coarse level of this description is given by the *directional field* (DF). The DF describes the orientation of the local ridge-valley structures, thus modeling the basic shape of the fingerprint. We use a high-resolution DF estimate [Baz00] that is based on the averaging of squared gradient [Kas87]. The features at the detailed level of the fingerprint are given by the minutiae. Instead of the common practice of treating the DF and the minutiae as two separate descriptions, of which one is used for classification and the other for matching, we propose a link between these two levels, by defining the *intrinsic coordinate system* of a fingerprint.

The intrinsic coordinate system of a fingerprint is defined by the DF. One of its axes runs along the ridge-valley structures, while the other is perpendicular to them. Using the intrinsic coordinate system, minutiae positions can be defined with respect to positions in the DF, instead of using the pixel coordinates as minutiae locations, thus providing a more natural representation. If a fingerprint undergoes plastic distortions, the distortions do influence the shape of the coordinate system, but the intrinsic minutiae coordinates do not change. This means that matching the intrinsic coordinates of the minutiae sets is invariant to plastic distortions.

The rest of this paper is organized as follows. First, Section 2 proposes a method to partition the fingerprint in regular regions. Then, Section 3 defines the intrinsic coordinate system, which is used in Section 4 for a the minutiae matching algorithm. Finally, Section 5 presents some preliminary results.

2 Regular Regions

In this section, a method is proposed to partition the directional field in regular regions, which are the basis for the intrinsic coordinate system that is discussed in Section 3. The first step is extraction of the *singular points* (SPs). An SP is either a *core* or a *delta*, which are discontinuities in the DF. In [Baz01], a method is proposed for robust and accurate SP extraction from the high-resolution DF, based on computing the Poincaré index using small linear filters only. That paper also describes a method for the estimation of the *orientation* of the SPs by the convolution with a reference model of the SP. The orientation is used for initializing the flow lines.

The *flow lines*, which are traced in the DF, are used to partition the fingerprint into a number of regular regions. Flow lines are curves in a fingerprint that are exactly parallel to the ridge-valley structures. However, we also use this name for curves that are exactly perpendicular to the ridge-valley structures. Flow lines are found by taking line integrals in the directional field. This is discretized by a numerical Runge-Kutta integration, giving the following expression for tracing a flow line from a start position x_i which is a complex number that represents pixel coordinates in the fingerprint:

$$x_{i+1} = x_i + \Delta_x \cdot \mathrm{mean}(DF_{x_i}, DF_{x_{i+1}}) \tag{1}$$

In this equation, Δ_x is the step size, DF_{x_i} is a unit-length complex number that indicates the orientation of the DF at x_i, $\mathrm{mean}(DF_{x_i}, DF_{x_{i+1}}) = (DF_{x_i}^2 + DF_{x_{i+1}}^2)^{1/2}$ as discussed in [Baz00] and $DF_{x_{i+1}} = DF_{x_i+\Delta_x \cdot DF_{x_i}}$. For the perpendicular flow lines, steps that are perpendicular to the DF should be taken. This method of tracing flow lines gives relatively small errors and causes circular contours to be closed.

The extracted SPs and the flow lines are used to partition the fingerprints in so called *regular regions*, which are region in which no singular points are located. In order to construct the regular regions, two sets of flow lines have to be traced. From each *core*, a curve is traced that is parallel to the ridge-valley structure, and from each *delta*, three curves are traced that are perpendicular to the ridge-valley structure. This scheme provides a partitioning in regular regions for all classes of fingerprints and is illustrated in Figure 1(a) for a "right loop". The most important property of regular regions is that the fingerprint in such a region can be warped non-linearly such that it only contains straight parallel ridge-valley structures. This is illustrated in Figure 2.

3 Intrinsic Coordinate System

The the next step is to construct the *intrinsic coordinate system* (ICS) for each regular region in the fingerprint. This coordinate system is called intrinsic since it is the coordinate system that is defined by the DF of the fingerprint itself. Using the ICS, a multi-level description of the fingerprint can be made, in which the minutiae positions (fine level) are given with respect to their relative position

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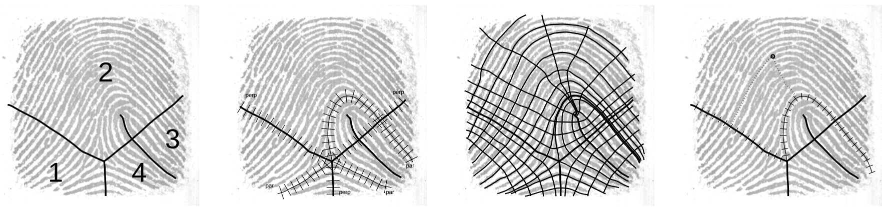

(a) Regular regions (b) Reference axes (c) ICS (d) IMCs

Fig. 1. Construction of the intrinsic coordinate system using flow lines.

Fig. 2. Some intermediate stages in the non-linear warp of regular region 2.

in the DF (coarse level). Points in the fingerprint that are on the same ridge will have the same *perpendicular coordinate*, while all points on a curve that is perpendicular to the ridges share the same *parallel coordinate*.

The intrinsic coordinate system is defined in each regular region by two *reference axes*, which is illustrated in Figure 1(b). For each regular region, the first reference axis is given by the perpendicular curve through the delta, while the second reference axis is given by the parallel line through the delta. When there are no deltas in the fingerprint, there is only one regular region. In this case, any combination of one parallel and one perpendicular flow line can be taken as reference axes.

The resulting ICS grid can be visualized by tracing curves from equally spaced positions along the reference axes, as illustrated in Figure 1(c). Although the grid is equally spaced along the reference axes, this is not the case in the rest of the fingerprint. The parallel curves may for instance diverge because a ridge that is between them bifurcates.

The *intrinsic minutiae coordinates* (IMCs) can be determined directly from the DF by means of projection of the minutiae on the intrinsic axes. The parallel coordinate of a minutia is found by tracing a perpendicular curve until it crosses a parallel reference axis. The distance along the axis of this point to the origin of the ICS gives the parallel coordinate. The perpendicular coordinate of a minutia is found by tracing a parallel curve, until it crosses a perpendicular reference

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axis. The distance along the axis of this point to the origin of the ICS gives the perpendicular coordinate. This method is illustrated in Figure 1(d).

4 Minutiae Matching in the ICS

A *minutiae matching* algorithm has to determine whether both fingerprints originate from the same finger by comparing the minutiae sets of a *primary* fingerprint (stored in a database) and a *secondary* fingerprint (provided for authentication). The goal of the algorithm is to find the mapping of the secondary to the primary minutiae set that maximizes the subset of corresponding minutiae in both sets. If the size of this common subset exceeds a certain threshold, the decision is made that the fingerprints are matching.

In this section, an alternative minutiae matching method that makes use of the ICS is proposed. As a consequence of the definition of the ICS, the IMCs only change in some simple and well-defined ways. Since distortions do not affect the ordering of the IMCs of neighboring minutiae, dealing with distortions amounts to independent 1-dimensional non-linear dynamic warps [Rab93] in 2 directions. This is a huge reduction in the number of degrees of freedom and computational complexity, compared to a 2-dimensional non-linear dynamic warp.

In the ICS, minutiae matching reduces to finding the warp function that maximizes the number of minutiae that fit exactly. Once the minutiae sets have been ordered along one intrinsic coordinate, the problem is to find the largest subset of both minutiae sets in which the ordering in the other intrinsic coordinate exactly corresponds. This problem can be solved by dynamic programming as described below. Usually, the next step would be to determine the warp function that interpolates the points that were found. However, since we are only interested in the number of matching minutiae, this step does not have to be performed.

The set of minutiae of the primary fingerprint is given by $\mathbf{a} = [a_1, \ldots, a_m]$. A minutia a_i of this set is described by its parallel coordinate $x(a_i)$ and perpendicular coordinate $y(a_i)$. The set is sorted by $x(a_i)$. In the same way, $\mathbf{b}$ describes the n minutiae of the secondary fingerprint. The problem is to find the longest series $[(a_{i_1}, b_{j_1}), \ldots, (a_{i_k}, b_{j_k})]$ with $1 \leq i_1 < \cdots < i_k \leq m$ and $1 \leq j_1 < \cdots < j_k \leq n$ such that a criterion of local similarity is satisfied:

$$a_{i_k} - a_{i_{k-1}} \approx b_{j_k} - b_{j_{k-1}} \tag{2}$$

Consider the partial solution $\lambda(i,j)$, representing the longest series of minutiae pairs that has (a_i, b_j) as the last pair in the series. This means that $\lambda(i,j)$ can be constructed recursively by adding (a_i, b_j) to the longest of all series $\lambda(k,l)$, with $k < i$ and $l < j$, behind which the pair fits:

$$\lambda(i,j) = \lambda(k,l) + (a_i, b_j) \tag{3}$$

with k and l chosen to maximize the length of $\lambda(i,j)$. Using the starting conditions $\lambda(1,j) = [(a_1, b_j)]$ and $\lambda(i,1) = [(a_i, b_1)]$, the final solution is the longest of all calculated partial solutions $\lambda(i,j)$.

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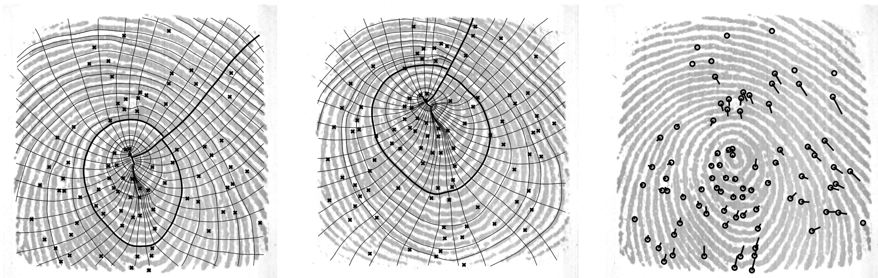
5 Preliminary Results

Although the plastic distortion-invariant minutiae matching algorithm is not yet operational, this section is meant to show the potentials of our algorithm. In order to make the matching result only dependent on the ability to deal with plastic distortions, and not on the minutiae extraction algorithm, the minutiae were extracted by human inspection. After cross-validation of both sets, 77 corresponding minutiae were in the two prints of the same finger that are shown in Figure 3(a) and 3(b).



(a) Fingerprint 1 (b) Fingerprint 2 (c) Superimposed

Fig. 3. Minutiae sets located in the ICSs of two fingerprints of the same finger.

In Figure 3(c), the sets have been aligned as well as possible, using only translation, rotation and scaling; matching minutiae are connected by a line. The figure clearly shows that, although the two minutiae sets fit exactly at the center of the print, they do not fit very well in the rest of the fingerprint. Under these distortions, the displacements required to tolerate corresponding minutiae is larger than the distance between some neighboring minutiae. Therefore, tolerance of small displacements is not a solution in this case. For a reasonable tolerance, only 24 matching minutiae pairs are found, out of 77 true matching minutiae pairs.

When the ICS is used, the perpendicular ordering is preserved since the parallel lines exactly follow the ridge-valley structures. However, the plastic distortions can influence the parallel ordering. Especially the parallel coordinate of minutiae near the core are less reliable. Nevertheless, over 70 correctly ordered minutiae pairs can be found using the ICS.

6 Conclusions

In this paper, the intrinsic coordinate system of a fingerprint is presented. It is defined by the directional field of the fingerprint itself. The locations of minutiae

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are insensitive to plastic distortions if they are given in this intrinsic coordinate system. It is shown how minutiae-based matching is drastically simplified when minutiae are characterized by means of their intrinsic coordinates.

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