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# Automated Deduction in Geometry

Third International Workshop, ADG 2000  
Zurich, Switzerland, September 25-27, 2000  
Revised Papers



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# Preface

With a standard program committee and a pre-review process, the Third International Workshop on Automated Deduction in Geometry (ADG 2000) held in Zurich, Switzerland, September 25–27, 2000 was made more formal than the previous ADG '96 (Toulouse, September 1996) and ADG '98 (Beijing, August 1998). The workshop program featured two invited talks given by Christoph M. Hoffmann and Jürgen Bokowski, one open session talk by Wen-tsün Wu, 18 regular presentations, and 7 short communications, together with software demonstrations (see <http://calfor.lip6.fr/~wang/ADG2000/>). Some of the most recent and significant research developments on geometric deduction were reported and reviewed, and the workshop was well focused at a high scientific level.

Fifteen contributions (out of the 18 regular presentations selected by the program committee from 31 submissions) and 2 invited papers were chosen for publication in these proceedings. These papers were all formally refereed and most of them underwent a double review-revision process. We hope that this volume meets the usual standard of international conference proceedings, represents the current state of the art of ADG, and will become a valuable reference for researchers, practitioners, software engineers, educators, and students in many ADG-related areas from mathematics to CAGD and geometric modeling.

ADG 2000 was hosted by the Department of Computer Science, ETH Zurich. We thank all the individuals, in particular external referees and members of the program committee, for their help with the organization of ADG 2000 and the preparation of this volume. The next workshop ADG 2002 will take place in Linz, Austria in September 2002. The proceedings of ADG '96 and ADG '98 have been published as volumes 1360 and 1669 in the same series of Lecture Notes in Artificial Intelligence.

Jürgen Richter-Gebert  
Dongming Wang

June 2001

## Invited Speakers

**Jürgen Bokowski** (Darmstadt University of Technology, Germany)

**Christoph M. Hoffmann** (Purdue University, USA)

## Open Session Speaker

**Wen-tsün Wu** (Chinese Academy of Sciences, China)

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