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Gilles Bertrand Atsushi Imiya  
Reinhard Klette (Eds.)

# Digital and Image Geometry

Advanced Lectures



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Series Editors

Gerhard Goos, Karlsruhe University, Germany  
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Volume Editors

Gilles Bertrand  
Groupe ESIEE, Cité Descartes  
2, Bld. Blaise-Pascal, 93162 Noisy-le-Grand, France  
E-mail: bertrang@esiee.fr

Atsushi Imiya  
Chiba University, Inst. of Media and Information Technology  
Media Technology Division  
1-33 Yayoi-cho, Inage-ku, 263-8522 Chiba, Japan  
E-mail: imiya@media.imit.chiba-u.ac.jp

Reinhard Klette  
The University of Auckland, CITR Tamaki, Tamaki Campus  
Morrin Road, Glen Innes, Auckland 1005, New Zealand  
E-mail: r.klette@auckland.ac.nz

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## Preface

This edited volume addresses problems in digital image geometry and topology, a special discipline of discrete mathematics. Images or discrete objects, to be analyzed based on digital image data, need to be represented, analyzed, transformed, recovered etc. These problems have stimulated many interesting developments in theoretical fundamentals of image analysis, and this volume contains reviewing articles as well as more specialized contributions on theoretical (basic) or more applied aspects of

- *digital topology*: axiomatic foundation of digital topology (E. Domínguez and A.R. Francés), programming, algorithmic, and data structure issues on cell complexes (U. Köthe and V. Kovalevsky), cellular representations of 3-manifolds (S. Matveev), and fixed points in discrete mappings (R. Tsaur and M.B. Smyth),
- *representation of images and objects*: description of a spatial modeling tool (E. Andrès, R. Breton, and P. Lienhardt), reviews on hierarchical image representations (L. Brun and W. Kropatsch) and simplicial multi-complexes for object modeling (E. Danovaro, L. De Floriani, P. Magillo, and E. Puppo), polyhedral 3D surface representations based on combinatorial topology (Y. Kenmochi and A. Imaiya), and encodings of digital partitions (J. Žunić),
- *digital geometry*: discrete object reconstruction (A. Alpers, P. Gritzmann, and L. Thorens), Hough transform (P. Bhattacharya, A. Rosenfeld, and I. Weiss), a review on digital lines and digital convexity (U. Eckhardt), digital curvature (A. Imaiya), Hausdorff digitization (C. Ronse and M. Tajine), and cellular convexity (J. Webster),
- *multigrid convergence*: length measurements of curves in 3D digital space via minimum-length polygons (T. Bülow, R. Klette) and via digital straight segments (D. Coeurjolly, I. Debled-Rennesson, and O. Teytaud), a review on multigrid-convergent feature measurement (R. Klette), 2D curve approximation for length estimation (L. Noakes, R. Kozera, and R. Klette), and use of the relative convex hull for surface area estimation (F. Sloboda and B. Zátko),
- *shape similarity and simplification*: rotation-invariant similarity of convex polyhedra (J.B.T.M. Roerdink and H. Bekker), surface skeletons of 3D objects (S. Svensson), a review on 3D skeletonization (J-I. Toriwaki and K. Mori), iterated morphological operations (J. Van Horebeek and E. Tapias-Rodriguez), and optimal grouping of collinear line segments (P. Veelaert).

These are the five parts of the volume, each containing five or six chapters, reflecting major developments in recent digital geometry and topology.

The volume presents extended and updated versions of 27 talks given at the winterschool “Digital and Image Geometry” (December 18 - December 22, 2000, Schloss Dagstuhl, Germany). The editors thank all the reviewers for their

detailed responses and the authors for efficient collaboration in ensuring a high-quality publication. Reviewers, besides the editors, were:

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September 2001

Gilles Bertrand, Atsushi Imiya, Reinhard Klette

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