Partially Ordered Constraint Optimization Problems*

Marco Gavanelli

Dip. di Ingegneria - University of Ferrara
Via Saragat, 1 - 44100 Ferrara, Italy.
mgavanelli@ing.unife.it

In Constraint Optimization Problems (COP) the objective function induces a total order on the solution set. However, in many real-life applications, more functions, possibly conflicting, should be optimized at the same time, and solutions are ranked by means of a Partial Order.

We propose the model of Partially-Ordered COP, along with a solving algorithm for it:

Definition 1. *A* Partially-ordered COP *(PCOP) is a couple $R = \langle P, f \rangle$, where $P = \langle \{X_1, \ldots, X_n\}, \{D_1, \ldots, D_n\}, C \rangle$ is a CSP and $f : D_1 \times \ldots \times D_n \mapsto S_p$, (where $\langle S_p, \preceq \rangle$ is a partially-ordered set) is a function. A* Solution *of a PCOP is a solution S of P such that $\nexists S'$ solution of P and $f(S) \prec f(S')$.*

The PCOP generalizes many real-life problems, like the Multi-Objective Problem (MOP) or CSPs that embed the concept of partial order on the solution space. E.g., in a MOP, a solution is non-dominated if a better solution (i.e., such that all the objective functions are better) does not exist. In general, there are many non-dominated solutions in a MOP.

A PCOP can be solved in a variety of ways, the most trivial is finding all the solutions of the CSP and a posteriori select only the non-dominated ones. A more efficient approach is an extension of Branch and Bound (B&B). B&B is an efficient, widely used method for solving COPs; it could be described as follows: first find a solution (typically using tree search), then add a further *unbacktrackable* constraint saying that "new solutions must be *better* than the current best". We have extended B&B by considering, instead of a single additional constraint, a set of unbacktrackable constraints that limit the next solutions to be better (in the $\preceq$ sense) than the already achieved ones. In other words, to solve a PCOP, we have to store all the solutions of the CSP that are currently believed to be non-dominated.

For example, in the MOP case, the constraints added in the B&B are $not(\overline{f(X)} \prec \overline{f(S)})$; i.e., a (tentative) possible solution X will be pruned off if an already obtained solution S is better in all the objective functions: $\forall_{i=1}^{m} f_i(X) \leq f_i(S)$.

We are currently studying very efficient methods for the propagation of the unbacktrackable constraint store. Experimental results show that our B&B extension takes about 40-50% of the time of standard methods providing the whole non-dominated frontier.

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