

# F.F.T. Hashing is not Collision-free

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## Abstract

*The FFT Hashing Function proposed by C.P. Schnorr [1] hashes messages of arbitrary length into a 128-bit hash value. In this paper, we show that this function is not collision free, and we give an example of two distinct 256-bit messages with the same hash value. Finding a collision (in fact a large family of colliding messages) requires approximately  $2^{23}$  partial computations of the hash function, and takes a few hours on a SUN3- workstation, and less than an hour on a SPARC-workstation.*

*A similar result discovered independently has been announced at the Asiacrypt'91 rump session by Daemen-Bosselaers-Govaerts-Vandewalle [2].*

## 1 The FFT Hashing Function

### 1.1 The Hash algorithm

Let the message be given as a bit string  $m_1 m_2 \dots m_t$  of  $t$  bit.

The message is first padded so that its length (in bits) becomes a multiple of 128. Let the padded message  $M_1 M_2 \dots M_n$  consist of  $n$  blocks  $M_1, \dots, M_n$ , each of the  $M_i$  ( $i=1, \dots, n$ ) being 128-bit long.

The algorithm uses a constant initial value  $H_0$  given in hexadecimal as

$$H_0 = 0123\ 4567\ 89ab\ cdef\ fedc\ ba98\ 7654\ 3210 \text{ in } \{0,1\}^{128}.$$

Let  $p$  be the prime  $65537 = 2^{16} + 1$ .

We will use the Fourier transform  $FT_8 : \{0, \dots, p-1\}^8 \rightarrow \{0, \dots, p-1\}^8$

$$(a_0, \dots, a_7) \rightarrow (b_0, \dots, b_7)$$

$$\text{with } b_i = \sum_{j=0}^7 2^{4ij} a_j \pmod{p}, \text{ for } i = 0, \dots, 7.$$

Algorithm for the hash function  $h$  :

INPUT :  $M_1 M_2 \dots M_n$  in  $\{0,1\}^{n \cdot 128}$  (a padded message)

DO :  $H_i = g(H_{i-1}, M_i)$  for  $i = 1, \dots, n$

OUTPUT :  $h(M) := H_n$

Algorithm for  $g : Z_p^{16} \rightarrow \{0,1\}^{8 \cdot 16}$

INPUT  $(c_0, \dots, c_{15})$  in  $\{0,1\}^{16 \cdot 16}$

1.  $(c_0, c_2, \dots, c_{14}) := FT_8(c_0, c_2, \dots, c_{14})$

2. FOR  $i = 0, \dots, 15$  DO

$$c_i := c_i + c_{i-1}c_{i-2} + c_{i-3}2^i \pmod{p}$$

(The lower indices  $i, i-1, i-2, i-3, c_{i-3}$  are taken modulo 16)

3. REPEAT steps 1 and 2

OUTPUT  $\overline{c}_i := c_i \pmod{2^{16}}$ , for  $i = 8, \dots, 15$  (an element of  $\{0,1\}^{8 \cdot 16}$ )

## 1.2 Notations

For a better clarity of our explanation, we will denote by  $c_i^0$  ( $i=0, \dots, 15$ ) the initial  $c_i$  values, and we will denote by step 3 (resp. step 4) the second pass of step 1 (resp. step 2) in the algorithm for  $g$ .

When it will be necessary to avoid any kind of slip, we will denote by  $c_i^k$  ( $i=0, \dots, 15$ ;  $k=0, \dots, 4$ ) the  $c_i$  intermediate value, after step  $k$ .

In order to simplify the expressions, we are using the following notations :

- The additions  $(x+y)$ , multiplications  $(x.y)$  and exponentiations  $(x^y)$  are implicitly made modulo  $p$ , except when the operands are lower indices.
- The  $\equiv$  symbol denotes that the right and the left terms are congruent modulo  $p$ .
- For lower indices the additions  $(i+j)$  and subtractions  $(i-j)$  are implicitly made modulo 16, and the  $\equiv$  symbol denotes that the right and the left terms are congruent modulo 16.

### 1.3 Preliminary remarks

The difficulty of finding collisions is related to the diffusion properties of the hashing function, i.e. the influence of a modification of an intermediate variable on the subsequent variables of the calculation.

Remark 1 (limitation on the diffusion at steps 1 and 3)

At step 1 and 3, the input values  $c_1, c_2, \dots, c_{15}$  are kept unchanged.

Remark 2 (limitation on the diffusion at steps 2 and 4)

The diffusion introduced by the  $c_{i-1}c_{i-2}$  terms in the recurrence for steps 2 and 4 can sometimes be cancelled (if one of values  $c_{i-1}$  and  $c_{i-2}$  is 0). More precisely, let  $(c_0^1, c_1^1, \dots, c_{15}^1)$  be the input to step 2 :

Proposition 1 : If for a given value  $i$  in  $\{1, \dots, 14\}$  we have  $c_{i-1}^2 = c_{i+1}^2 = 0$  and if  $c_{13}^1 \neq i$ ;  $c_{14}^1 \neq i$ ;  $c_{15}^1 \neq i$ ;  $c_j^2 \neq i$  for  $j$  in  $\{0, \dots, 12\}$ , then the impact of replacing the input value  $c_i^1$  by a new value  $c_i^1 + \Delta c_i^1$  such that  $c_i^1 + \Delta c_i^1 \equiv c_i^1$ , is limited to the output value  $c_i^2$  (that means  $c_j^2$  are not modified for  $j \neq i$ ).

Proposition 2 : If  $c_{14}^1 = c_0^2 = 0$  and if  $c_j^2 \neq 15$  for  $j$  in  $\{1, \dots, 11\}$  then the impact of replacing the input value  $c_{15}^1$  by a new value  $c_{15}^1 + \Delta c_{15}^1$  such that  $c_{15}^1 + \Delta c_{15}^1 \equiv c_{15}^1$ , is limited to the output value  $c_{15}^2$ .

Similarly, let  $(c_1^3, c_2^3, \dots, c_{15}^3)$  be the input to step 4 :

Proposition 1' : If for a given value  $i$  in  $\{1, \dots, 14\}$  we have  $c_{i-1}^4 = c_{i+1}^4 = 0$  and if  $c_{13}^3 \neq i$ ;  $c_{14}^3 \neq i$ ;  $c_{15}^3 \neq i$ ;  $c_j^4 \neq i$  for  $j$  in  $\{0, \dots, 12\}$ , then the impact of replacing the input value  $c_i^3$  by a new value  $c_i^3 + \Delta c_i^3$  such that  $c_i^3 + \Delta c_i^3 \equiv c_i^3$ , is limited to the output value  $c_i^4$ .

**Proposition 2' :** If  $c_{14}^3 = c_0^4 = 0$  and if  $c_j^4 \neq 15$  for  $j$  in  $\{1, \dots, 11\}$  then the impact of replacing the input value  $c_{15}^3$  by a new value  $c_{15}^3 + \Delta c_{15}^3$  such that  $c_{15}^3 + \Delta c_{15}^3 \equiv c_{15}^3$  is limited to the output value  $c_{15}^4$ .

## 2 Construction of two colliding messages

### 2.1 Construction of a partial collision

We first find two 128-bit blocks  $M_1$  and  $M'_1$  which hash values  $H_1 = (\overline{c}_8^4, \dots, \overline{c}_{15}^4)$  and  $H'_1 = (\overline{c}'_8^4, \dots, \overline{c}'_{15}^4)$  differ only by their right components  $\overline{c}_{15}^4$  and  $\overline{c}'_{15}^4$ . We will later refer to this property in saying that  $M_1$  and  $M'_1$  realize a partial collision.

Our technique for finding  $M_1$  and  $M'_1$  is the following : we search  $M_1$  values such that  $c_{14}^1 = 0$ ;  $c_0^2 = 0$ ;  $c_{14}^3 = 0$ ;  $c_0^4 = 0$ . The propositions 2 and 2' suggest that for such a message  $M_1 = (c_8^0, \dots, c_{14}^0, c_{15}^0)$ ,  $M_1$  and the message  $M'_1 = (c_8^0, \dots, c_{14}^0, c_{15}^0 + 16)$  realize a partial collision with a significant probability (approximately 1/8).

There are two main steps for finding  $M_1$ .

**Step1 :** Selection of  $c_8^0, c_{10}^0, c_{12}^0$  and  $c_{14}^0$

Arbitrary (e.g. random) values are taken for  $c_{12}^0$  and  $c_{14}^0$ . The values of  $c_8^0$  and  $c_{10}^0$  are then deduced from these values by solving the following linear system :

$$\begin{cases} c_{14}^1 = 0 & (1) \\ c_0^1 = -1 & (2) \end{cases}$$

**Proposition 3 :**

If  $c_{13}^0 \equiv 14$  then  $c_{14}^1 = 0$  and  $c_0^2 = 0$  independently of the values of  $c_9^0, c_{11}^0, c_{13}^0, c_{15}^0$ .

**Proof :** This is a direct consequence of the definition of the  $g$  function.

**Step 2 :** Selection of  $c_9^0, c_{11}^0, c_{13}^0, c_{15}^0$

The values of  $c_8^0, c_{10}^0, c_{12}^0, c_{14}^0$  are taken from Step 1 .

We fix the values of  $c_{11}^0 = 0$  and  $c_{15}^0 = 0$ . An arbitrary (c.g random) value is taken for  $c_9^0$ . We first calculate the  $c_{12}^2$  and  $c_{14}^3$  values corresponding to the chosen value of  $c_9^0, c_{11}^0$  and  $c_{15}^0$  and to the temporary value  $c_{13}^0 = 14$ . Based on these preliminary calculations, we "correct" the temporary value  $c_{13}^0 = 14$  by a quantity  $\Delta c_{13}^0$ , i.e. we replace the value  $c_{13}^0 = 14$  by the value  $c_{13}^0 = 14 + \Delta c_{13}^0$ , and we leave the other input values unchanged. We denote by  $\Delta c_j^i$  ( $0 \leq i \leq 4$  ;  $0 \leq j \leq 15$ ) the corresponding variations of the intermediate variables in the  $H_1$  calculation. We select  $\Delta c_{13}^0$  in such a way that the quantity  $c_{14}^3 + \Delta c_{14}^3$  (i.e the new value of  $c_{14}^3$ ) is equal to zero with a good probability.

**Proposition 4 :** If  $c_{12}^2 \neq 0$  and  $\frac{-c_{14}^3}{2^{4.7.7} c_{12}^2} \equiv 0$  and  $c_j^2 \neq 13$  for  $1 \leq j \leq 11$  then the above values of

$c_{15}^1, c_0^2$  and the value  $\Delta c_{13}^0 = \frac{-c_{14}^3}{2^{4.7.7} c_{12}^2}$  lead to the three relations

$$\begin{cases} c_{14}^1 + \Delta c_{14}^1 = 0 & (a) \\ c_0^2 + \Delta c_0^2 = 0 & (b) \\ c_{14}^3 + \Delta c_{14}^3 = 0 & (c) \end{cases}$$

**Proof :** (a) is straightforward; (b) and (c) are direct consequences of the following relations, which result from the definition of the  $g$  function :

$$\Delta c_{j-2}^2 = 0 \text{ for } 0 \leq j \leq 12 \quad ; \quad \Delta c_{13}^2 = \Delta c_{13}^0 \quad ; \quad \Delta c_{14}^2 = c_{12}^2 \cdot \Delta c_{13}^2 \quad ; \quad \Delta c_{14}^3 = 2^{4.7.7} \cdot \Delta c_{14}^2$$

We performed a large number  $n_1$  of trials of step 1. For each trial of step 1, we made a large number  $n_2$  of trials of step 2. The success probability of step 2, i.e the probability that the trial of a  $c_9^0$  value leads to a message such that (a), (b) and (c) are realized is slightly less than 1/16 (since the strongest

condition in proposition 2 is :  $\frac{-c_{14}^3}{2^{4.4.7} \cdot 2^{c_{12}}} \equiv 0$ ). Therefore the probability that a step 2 trial leads to a message

$M_1$  such that  $c_{14}^1 = c_0^2 = c_{14}^3 = c_0^4 = 0$  is slightly less than  $1/16 \cdot 2^{-16} = 2^{-20}$ .

Moreover, the probability that such a message  $M_1$  leads to a partial collision is basically the probability that none of the  $c_{i-3} \bmod 16$  indices occurring in the calculation of  $c_0^2$  to  $c_{15}^2$  and  $c_0^4$  to  $c_{15}^4$  takes the value 15, which is close to  $1/8$ . So, in summary, approximatively  $2^{23}$  partial computations of the  $g$  function were necessary to obtain a suitable message  $M_1 = (c_8^0, \dots, c_{14}^0, c_{15}^0)$ , such that  $M_1$  and the message  $M'_1 = (c_8^0, \dots, c_{14}^0, c_{15}^0 + 16)$  lead to partially colliding hash values  $H_1 = (\bar{c}_8^4, \dots, \bar{c}_{15}^4)$  and  $H'_1 = (\bar{c}_8^4, \dots, \bar{c}_{15}^4 + 16)$ .

## 2.2 Construction of a full collision using a partial collision

We now show how to find a 128-bit message  $M_2 = (c_8^0, \dots, c_{15}^0)$  such that the previously obtained hash values  $H_1$  and  $H'_1$  (denoted in this section by  $(c_0^0, \dots, c_7^0)$  and  $(c'_1{}^0, \dots, c'_6{}^0, c'_7{}^0) = (c_1^0, \dots, c_6^0, c_7^0 + 16)$ ) respectively lead to the same hash value  $H_2$  (when combined with  $M_2$ ):  $g(H_1, M_2) = g(H'_1, M_2)$ .

Our technique for finding  $M_2$  is quite similar to the one used for finding  $M_1$  and  $M'_1$ . Let us denote by  $c_j^i$  (resp  $c'_j{}^i$ ) ( $0 \leq i \leq 4$ ,  $0 \leq j \leq 15$ ) the intermediate variables of the calculations of  $g(H_1, M_2)$  (resp  $g(H'_1, M_2)$ ).

We search  $M_2$  values such that  $c_6^2 = c_8^2 = c_6^4 = c_8^4 = 0$ . The propositions 1 and 1' suggest that the probability that the 16-uples  $(c_0^4, \dots, c_{15}^4)$  and  $(c'_0{}^4, \dots, c'_{15}{}^4)$  differ only by their components  $c_7^4$  and  $c'_7{}^4$  which implies that the probability to have  $g(H_1, M_2) = g(H'_1, M_2)$  is quite substantial, approximatively  $1/8$ .

There are two main steps for the search of  $M_2$  :

**Step 1 :** Selection of  $c_8^0, c_{10}^0, c_{12}^0, c_{14}^0, c_9^0$ .

An arbitrary (c.g random) value is taken for  $c_{14}^0$ . The values of  $c_8^0, c_{10}^0, c_{12}^0$  are deduced from  $c_{14}^0$  by solving the following linear system :

$$\begin{cases} c_{14}^1 = 0 & (3) \\ c_0^1 = -1 & (4) \\ c_8^1 = -2^8 & (5) \end{cases}$$

A preliminary calculation, where  $c_9^0, c_{11}^0$  and  $c_{15}^0$  are set to the temporary value 0 and  $c_{13}^0$  is set to the temporary value 14, is made. The obtained value of  $c_6^2$ , denoted by  $\delta$ , is kept.

**Proposition 5 :** If  $c_8^0, c_{10}^0, c_{12}^0, c_{14}^0$  are solutions of (3), (4), (5) and if in addition the values  $c_9^0 = p-\delta, c_{11}^0 = 0, c_{13}^0 = 14, c_{15}^0 = 0$  lead to intermediate values such that :  $c_1^2 \bmod 16$  is not in  $\{9,11,13,15\}$ ;  $c_2^2 \bmod 16$  is not in  $\{9,11,13,15\}$ ;  $c_3^2 \equiv 9 \bmod 16$ ;  $c_4^2 \bmod 16$  is not in  $\{9,11,13,15\}$ ;  $c_5^2 \bmod 16$  is in  $\{0,6,14\}$ , then if we fix the value  $c_9^0 = p-\delta$ , for any value of  $c_{13}^0 \equiv 14$  and for any value of  $c_{15}^0$  such that  $c_{15}^0 \equiv 0$  we have :

$$c_{14}^1 = 0 ; \quad c_0^2 = 0 ; \quad c_6^2 = 0 ; \quad c_8^2 = 0 .$$

**Proof :** The proof of this proposition is easy. Finding the  $c_8^0, c_{10}^0, c_{12}^0, c_{14}^0$  and  $c_9^0$  values satisfying the conditions of the above proposition is quite easy, and requires the trial of a few hundreds  $c_{14}^0$  values.

**Step 2 :** Selection of  $c_{11}^0, c_{13}^0, c_{15}^0$

The values of  $c_8^0, c_{10}^0, c_{12}^0, c_{14}^0, c_9^0$  are taken from Step 1 ; these values are assumed to realize the conditions of the above proposition.

An arbitrary (c.g random) value is taken for  $c_{11}^0$ . A preliminary calculation is made, using the selected  $c_{11}^0$  value and the temporary values  $c_{13}^0 = 14; c_{15}^0 = 0$ . The corresponding values of  $c_{12}^2$  and  $c_8^3$  are kept.

Based on these preliminary calculations, we "correct" the temporary value of  $c_{13}^0$  by a quantity  $\Delta c_{13}^0$  and we also consider new values  $c_{15}^0 + \Delta c_{15}^0$  for  $c_{15}^0$ . The variation  $\Delta c_{13}^0$  is selected in such a way that for any  $\Delta c_{15}^0$  value satisfying  $\Delta c_{15}^0 \equiv 0$ , the new value  $c_8^3 + \Delta c_8^3$  of  $c_8^3$  is equal to  $-2^8$  with a substantial probability.

**Proposition 6 :** If  $c_{12}^2 \neq 0$  and  $\frac{-2^8 - c_8^3}{2^{4.4.7} \cdot 2^{c_{12}^2}} \equiv 0$  and  $c_j^2 \bmod 16$  is not in  $\{13, 15\}$  for  $1 \leq j \leq 11$  then for

any variation  $\Delta c_{15}^0 \equiv 0$  on  $c_{15}^0$  such that  $c_{15}^2 + \Delta c_{15}^0 < p$  and  $c_{15}^4 + \Delta c_{15}^0 < p$ , the variation

$\Delta c_{13}^0 = \frac{-2^8 - c_8^3}{2^{4.4.7} \cdot 2^{c_{12}^2}}$  on the  $c_{13}^0$  value leads to the following new values :

$$c_{14}^1 + \Delta c_{14}^1 = 0 ; \quad c_0^2 + \Delta c_0^2 = 0 ; \quad c_6^2 + \Delta c_6^2 = 0 ; \quad c_8^2 + \Delta c_8^2 = 0 ; \quad c_8^3 + \Delta c_8^3 = -2^8 .$$

We performed a number  $n_1$  of trials of step 1. For each successful trial of step 1, we made a large number  $n_2$  of trials of  $c_{11}^0$  values at step 2. For those  $c_{11}^0$  values satisfying the conditions of the above proposition, we made a large number  $n_3$  of trials of new  $c_{15}^0$  values such that  $\Delta c_{15}^0 \equiv 0$ . The probability that the trial of a new  $\Delta c_{15}^0$  value leads to intermediate variables satisfying the four equations  $c_6^2=0; c_8^2=0; c_6^4=0; c_8^4=0$  is basically the probability that randomly tried  $c_6^4$  and  $c_5^4$  values satisfy  $c_6^4 = 0$  and  $c_5^4 \equiv 6$ ; the order of magnitude of this probability is therefore  $2^{-20}$ .

Moreover, the probability that a message  $M_2$  satisfying the four equations  $c_6^2=0; c_8^2=0; c_6^4=0; c_8^4=0$  leads to a full collision  $g(H_1, M_2) = g(H_1, M_2)$  is basically the probability that none of the  $c_{i-3} \bmod 16$  indices occurring in the calculation of  $c_0^2$  to  $c_{15}^2$  and of  $c_0^4$  to  $c_{15}^4$  takes the value 15, which is close to  $1/8$ . So in summary approximatively  $2^{23}$  partial computations of the  $g$  function are necessary to obtain a message  $M_2$  giving a full collision.



### 2.3 Implementation details

The above attack method was implemented using a non-optimized Pascal program. The search for a collision took a few hours on a SUN3 workstation and less than an hour on a SPARC workstation. We provide in annex the detail of the intermediate calculations for two colliding messages  $M_1M_2$  and  $M'_1M_2$ , of two 128-bit blocks each.

Note that for many other values  $M'_1$  of the form  $(e_0^0, \dots, e_{15}^0 + k.16)$  ( $k$  : an integer) of the first 128-bit block, the message  $M'_1M_2$  leads to the same hash value as  $M_1M_2$  : the observed phenomenon is in fact a multiple collision.

### 3 Conclusions

The attack described in this paper takes advantage of the two following weaknesses of the FFT-Hashing algorithm :

- the influence of the term  $c_{i-3}$  in the recurrence  $c_i := c_i + c_{i-1}e_{i-2} + c_{i-3} + 2^i \pmod{p}$  on the

security of the algorithm is rather negative (see for example the method to obtain  $c_6^2 = 0$  (or  $c_8^2 = 0$ ) at step 1 of Section 2.2).

- as mentioned in Section 1.3, the diffusion introduced by the four steps of the algorithm is quite limited. In particular, the  $FT_8$  Fourier transform acts only on half of the intermediate values  $(e_0, \dots, e_{15})$ , namely the 8 values  $e_0, e_2, \dots, e_{14}$ .

This suggests that quite simple modifications might result in a substantial improvement of the security of the FFT-Hashing algorithm.

### 4 Acknowledgements

The authors are grateful to Jacques BURGER (SEPT PEM, 42 rue des Coutures, BP 6243, 14066 CAEN, France) for the Sparc implementation as well as useful discussions.

### 5 References

- [1] : C.P. SCHNORR : FFT-Hashing : An Efficient Cryptographic Hash Function; July 15, 1991  
(This paper was presented at the rump session of the CRYPTO'91 Conference, Santa Barbara, August, 11-15, 1991)
- [2] : DAEMEN - BOSSELAERS - GOVAERTS - VANDEWALLE : Announcement made at the rump session of the ASIACRYPT '91 Conference, Fujiyoshida, Japan, November 11-14, 1991)

## ANNEX

FIRST MESSAGE M - H1 M2      with

M1 = F95A 807A 26A    0 440 365E    0 0  
M2 = 1537 5202 3284 358 501C 959E 6D6B 75E0

calculation of H1 :

-----  
H0 = 123 4567 89AB CDEF FEDC BA98 7654 3210  
M1 = F95A 807A 26A    0 440 365E    0 0  
  
step 1: 10000 4567 4F72 CDEF B84C BA98 D98A 3210  
FB30 807A F62E    0 3677 365E    0 0  
  
step 2:    0 4569 4F76 1DD1 6CEA F49C 1D89 7D13  
ADDC 156 5AFE CD52 158A 4626 B81B  
  
step 1: CFA9 4569 2466 1DD1 2F1A F49C F307 7D13  
B305 156 3057 CD52 5A7 158A    0 B81B  
  
step 2:    0 456B F1BC 91E1 64F8 F6D2 FB99 A787  
7DCA CDE2 4508 3BE5 8F64 E23C 988A 5BE6  
  
H1 = 7DCA CDE2 4508 3BE5 8F64 E23C 988A 5BE6

calculation of H2 :

-----  
H1 = 7DCA CDE2 4508 3BE5 8F64 E23C 988A 5BE6  
M2 = 1537 5202 3284 358 501C 959E 6D6B 75E0  
  
step 1: 10000 CDE2 C5BE 3BE5 3E13 E23C 418A 5BE6  
FF01 5202 9B04 358 EF0 959E    0 75E0  
  
step 2:    0 CDE4 C5C2 17A9 6501 6370    0 2A59  
0 5402 F306 99A5 8BB5 9A6E 3BEF 73A9  
  
step 1: E26B CDE4 8B79 17A9 E6CC 6370 E7C2 2A59  
FF01 5402 CD5 99A5 37CB 9A6E 7FE2 73A9  
  
step 2: 5551 E84C 4E20 EA99 C82F 9886    0 9872  
0 AB53 5EF5 27D8 9554 995 983F 89CF  
  
H2 =    0 AB53 5EF5 27D8 9554 995 983F 89CF

HASHED MESSAGE : 0 AB53 5EF5 27D8 9554 995 983F 89CF

SECOND MESSAGE M - H1 M2      with

M1 = F95A 807A 26A    0 440 365E    0 10  
M2 = 1537 5202 3284 358 501C 959E 6D6B 75E0

calculation of H1 :

-----  
H0 = 123 4567 89AB CDEF FEDC BA90 7654 3210  
M1 = F95A 807A 26A    0 440 365E    0 10  
  
step 1: 10000 4567 4F72 CDEF B84C BA98 D98A 3210  
FB30 807A F62E    0 3677 365E    0 10  
  
step 2:    0 4569 4F76 1DD1 6CEA F49C 1D89 7D13  
ADDC 156 5AFE CD52 158A 4626 B81B  
  
step 1: CFA9 4569 2466 1DD1 2F1A F49C F3D7 7D13  
B305 156 3057 CD52 5A7 158A    0 B81B  
  
step 2:    0 456B F1BC 91E1 64F8 F6D2 FB99 A787  
7DCA CDE2 4508 3BE5 8F64 E23C 988A 5BE6  
  
H1 = 7DCA CDE2 4508 3BE5 8F64 E23C 988A 5BE6

calculation of H2 :

-----  
H1 = 7DCA CDE2 4508 3BE5 8F64 E23C 988A 5BE6  
M2 = 1537 5202 3284 358 501C 959E 6D6B 75E0  
  
step 1: 10000 CDE2 C5BE 3BE5 3E13 E23C 418A 5BE6  
FF01 5202 9B04 358 EF0 959E    0 75E0  
  
step 2:    0 CDE4 C5C2 17A9 6501 6370    0 2A59  
0 5402 F306 99A5 8BB5 9A6E 3BEF 73A9  
  
step 1: E26B CDE4 8B79 17A9 E6CC 6370 E7C2 2A59  
FF01 5402 CD5 99A5 37CB 9A6E 7FE2 73A9  
  
step 2: 5551 E84C 4E20 EA99 C82F 9886    0 9882  
0 AB53 5EF5 27D8 9554 995 983F 89CF  
  
H2 =    0 AB53 5EF5 27D8 9554 995 983F 89CF

HASHED MESSAGE : 0 AB53 5EF5 27D8 9554 995 983F 89CF