A CHARACTERIZATION OF IRREDUCIBLE SETS MODULO LEFT-LINEAR TERM REWRITING SYSTEMS BY TREE AUTOMATA

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J. H. Gallier and R. V. Book showed that the set of irreducible terms modulo a left-linear term rewriting system can be recognized by a deterministic bottom-up tree automaton. However, this device is too general to characterize irreducible sets: there are recognizable term sets which cannot be the set of irreducible terms modulo any term rewriting system. We introduce the class of one-state deterministic top-down tree automata with prefix look-ahead, which exactly recognizes the class of irreducible terms modulo left-linear term rewriting systems.

A one-state deterministic top-down tree automaton with prefix look--ahead is a system $A=(\Sigma,q,P)$, where

(a) Σ is a ranked alphabet,

(b) q is the state of A,

(c) P is a finite set of rules of the form

 $\langle q\sigma + (q, \ldots, q); L \rangle$ with $m \ge 0$, $\sigma \in \Sigma_m$, where $L = tT_{\Sigma} = \{t(t_1, \ldots, t_n) | t_i \in T_{\Sigma} \text{ for } l \le i \le n\}$ for some linear tree $t \in T_{\Sigma}(X_n)$. Moreover, for any different rules $\langle q\sigma + (q, \ldots, q); L_1 \rangle$ and $\langle q\sigma + (q, \ldots, q), L_2 \rangle$, $L_1 \cap L_2 = \emptyset$ holds.

The tree language recognized by A is $L(\bar{A}) = \{t \in T_{\Sigma} | q(t)^{\ddagger}t\}$, where $\stackrel{\ddagger}{\Rightarrow}$ is the reflexive-transitive closure of $\stackrel{\Rightarrow}{\Rightarrow}$ defined as follows: for $u, v \in T_{\Sigma}(qT_{\Sigma})$, $u \Rightarrow v$ if and only if

(i) $u=c(q(\sigma(t_1,\ldots,t_m)))$ for some $c\in T_{\Sigma}(\{x_1\})$,

 x_1 occurs once in c, m ≥ 0 , $\sigma \in \Sigma_m$ and $t_1, \ldots, t_m \in T_{\Sigma}$,

(ii) $v=c(\sigma(q(t_1),...,q(t_m))),$

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(iii) the rule \langle q_\sigma + (q, \ldots, q); L \rangle is in P,
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(iv) $\sigma(t_1,\ldots,t_m) \in L$.

Our main result is the following theorem.

Theorem: For any set T of terms, T is the set of irreducible terms with respect to a left-linear term rewriting system if and only if T can be recognized by a one-state deterministic top-down tree automaton with prefix look-ahead.