

Bounded-memory Algorithms for Verification On-the-fly *

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1 Introduction

1.1 Motivation

Program verification is a branch of computer science whose business is “to prove programs correctness”. It has been studied in theoretical computer science departments for a long time but it is rarely and laboriously applied to real world problems. As a matter of fact, we must pay much more attention to practical problems like the amount of space and time needed to perform verification. Let us recall that proofs of correctness are proofs of the relative consistency between two formal specifications: those of the program, and of the properties that the program is supposed to satisfy. Such a formal proof tries to increase the confidence that a computer system will make it right when executing the program under consideration.

A considerable need for such methods appeared these last ten years in different domains, such as design of asynchronous circuits, communication protocols and distributed software in general. A lot of us accepted the challenge to design automated verification tools, and many different theories have been suggested for the automated analysis of distributed systems. There now exist elaborate methods that can verify quite subtle behaviors.

A simple method for performing automated verification is symbolic execution which is the core of most existing and planned verification systems. The practical limits of this method are the size of the state space and the time it may take to inspect all reachable states in this state space. Those quantities can dramatically rise with the problem size.

1.2 Current state-of-art

Reachability analysis is basically an exhaustive search yielding a rooted graph of global states. This technique is often called *perturbation* [22]. Starting from some specified initial state, successor states are generated and stored in the computer. The process stops when no new state (i.e. one not previously stored) can be generated. Termination is guaranteed if all the program variables (including communication channels) are bounded.

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The state graph is usually very large and for example, any protocol of practical relevance will have a state space in the order of one million states. There are two major problems when handling systems of this size: state matching (to avoid double work and to ensure termination), and state storing.

We will suppose that the memory is arranged as a balanced tree, that reachable states are numbered from 1 to R , and that states are of constant size S . The memory size M needed is then at least $R.S$. Let $C(S)$ be the time needed for the comparison of two states. The first time a state i is generated, the memory contains $i - 1$ states, thus its insertion in the tree is carried out in time at worst $C(S). \log(i)$. If d is the average degree of nodes, each node is re-generated $d - 1$ times and searched in a memory which contains at least i states. The time needed for those searches can be approximated by $(d - 1).C(S). \log(i)$. Coarsely approximating $\log(R!)$ by $R. \log(R)$, we say that the time complexity of the perturbation technique is

$$T \simeq d.C(S). \sum_{i=1}^R \log(i) \simeq d.C(S). \log(R!) \simeq d.C(S). R. \log(R)$$

If $M = 10^7$ bytes and $S = 10^2$ bytes, the size of the graphs that can actually be analysed is less than $R = 10^5$ states. If $d = 2$, $C(S) = 10^{-4}$ seconds, and trees are binary trees, the time needed is $T \simeq 6$ minutes.

In order to master the "state explosion", different works have been conducted to reduce the size of the graph [3, 21, 2, 8, 7]. Obviously, reduction must be performed during the graph generation. The other constraint is that the validity of properties to be verified must not be changed. For that reason, we do not consider simulation methods which provide only partial verification [14, 9, 23, 19, 13, 10].

1.3 Verification on-the-fly

The key idea is that, for a large class of properties, storing all the reachability graph is not mandatory. It is enough to visit all the states and/or all the transitions. A depth-first traversal of the reachability graph performs such an exhaustive search. Only the current path has to be stored but the time needed to perform a verification can be catastrophic, due to the re-generation of forgotten states.

We propose an intermediate method which offers a good compromise between time and space requirements. It is based on a depth-first traversal but uses all the available space in order to store not only the current path, but also the greatest possible number of already visited states. We will prove that bounding memory to a smaller size than the state space may not significantly increase the time complexity. Such algorithms allow us to build efficient verifiers, able to handle large graphs. This approach is often called "verification on-the-fly".

It was first proposed in terms of "on-line model-checking" by the authors in [15]. Since then, similar ideas have been advocated in [4] and [5]. [4] presents efficient algorithms to compare Büchi automata and thus proposes a new solution to the verification of temporal properties on infinite behaviors of finite state programs. [5] extends the technique to

verify on-the-fly bisimulation equivalences on transitions graphs. The core of the method is to traverse (during its generation) a kind of product of finite transitions systems. Unifying these different views would be an interesting prospect.

The remainder of the paper is organized as follows. We present in detail a class of bounded-memory algorithms that traverse exhaustively the state space of the program to be verified. Upper bounds for space and time complexities are computed and different experiments show the average behavior of our algorithms. Another part of the paper discusses applications, namely verification of safety properties and testing unboundedness of Fifo channels. We conclude with some prospects.

2 Depth-first traversal with replacement

We saw above that the main drawback of a perturbation technique is the memory size needed to perform the graph generation of real size systems. Now, there are some verifications for which a traversal of all states and transitions is enough. It is then unnecessary to store the whole graph. An algorithm performing this exhaustive traversal is a depth-first traversal in which we theoretically only need to detect cycles, provided that the memory is large enough to store the longest acyclic sequence. Unfortunately, visited states which no longer behave to the current sequence are forgotten and can be visited again in many other sequences. In the best case the number R_{gen} of generated states is R . But in the worst case R_{gen} can reach $R!.e$ for a complete graph with R states (e is the basis of natural logarithms). If the number of states in the memory is bounded by the length of the longest acyclic sequence D_{max} , the time needed to complete the traversal is in the scale

$$C(S).R.\log(D_{max}) \leq T \leq C(S).R!.e.\log(D_{max})$$

However, a depth-first traversal can significantly be improved if the whole memory amount is used [16]. Actually, since $D_{max}.S < M$, one can use the remainder of the memory to store already visited states, and then avoiding re-traversing some states. We present this technique and show with examples that it can be efficiently used to analyse real size graphs which are too large to fit in memory.

2.1 The algorithm

An algorithm performing a depth-first traversal with replacement is described above. It is very similar to a classical depth-first traversal except for the set *Visited* of already visited states and the execution phase when the memory is full.

The traversal with replacement algorithm can be used on every graph such that $D_{max}.S \leq M$. But, contrarily to the simple traversal, it is not a necessary condition for the termination because states of the longest acyclic sequences can be reached by shortest sequences. A necessary condition is $G_{max}.S \leq M$ where G_{max} is the maximal length of a geodesic with initial state S_0 (a geodesic from S to S' is an acyclic sequence from u

to v with minimum length). We have $G_{max} \leq D_{max}$ but if $G_{max}.S \leq M \leq D_{max}.S$ the algorithm may or may not terminate, depending on the order of transitions evaluations.

```

St_Stack:=nil; (* -- states of the current sequence -- *)
Tr_Stack:=nil; (* -- stack of sets of pending transitions -- *)
Visited:=∅; (* -- already visited states -- *)
push( $S_0$ ,  $St\_Stack$ ); push( $fireable(S_0)$ ,  $Tr\_Stack$ );
while  $St\_Stack \neq \emptyset$  do begin
   $S:=top(St\_Stack)$ ; (* -- current state -- *)
  if  $top(Tr\_Stack) \neq \emptyset$  then begin
     $t:=extract\_one\_elt\_of(top(Tr\_Stack))$ ; (* -- choose and remove -- *)
     $S':=succ(S,t)$ ;
    if  $S' \notin St\_Stack \cup Visited$  then begin
      if memory_full then begin (* -- replacement -- *)
         $S_{del}:=one\_state\_from(Visited)$ ;
         $Visited:=Visited - \{S_{del}\}$ ;
        end;
        push( $S'$ ,  $St\_Stack$ );
        push( $fireable(S')$ ,  $Tr\_Stack$ );
        end;
      end
    else begin (* --  $top(Tr\_Stack) = \emptyset$  -- *)
      pop( $St\_Stack$ );
      pop( $Tr\_Stack$ );
       $Visited:=Visited \cup \{S\}$ ;
      end;
    end;
  end;
end;

```

2.2 Time complexity

Let us remark that we always have $(|St_Stack| + |Visited|).S \leq M$ and the boolean variable *Memory_full* is equal to $(|St_Stack| + |Visited|).S = M$ and is a stable property. Let R_{ins} be the number of insertions of states in the memory i.e. $St_Stack \cup Visited$. The behavior of the algorithm in the case $R.S \leq M$ is almost the same as a perturbation, except for the generation order. Each state is inserted once, so $R_{ins} = R$. The time complexity is then approximately the same.

Now if $R.S > M$, R_{ins} exceeds R because an already visited state may have been forgotten. Due to the stability of the property *memory_full*, we can separate the algorithm into two phases:

- in the first phase, when $\neg memory_full$, all visited states are in $St_Stack \cup Visited$ and the algorithm behaves like a perturbation,
- in the second phase, when *memory_full*, each time a state S' is generated and not found in $St_Stack \cup Visited$, we must remove one state S_{del}

from *Visited* before pushing S' in *St_Stack*. The way this replacement is performed influences the total number of generated states R_{gen} .

We also suppose that the whole memory $St_Stack \cup Visited$ is arranged as a balanced tree, which supports access, insertion and deletion operations in logarithmic worst case. The number of states in that memory is always less than M/S . Each generated state must be searched in that memory. Thus, the total time of the traversal is approximately

$$T \simeq C(S) \cdot R_{gen} \cdot \log\left(\frac{M}{S}\right)$$

Recall that for the perturbation technique, time complexity is $C(S) \cdot d \cdot R \cdot \log(R)$. If $M \leq R \cdot S$, we have $R_{gen} \simeq d \cdot R$, thus complexities are identical. If $M > R \cdot S$, a perturbation technique is no longer possible. We have $\log(M/S) < \log(R)$ thus, if R_{gen} is in the same order of magnitude that $d \cdot R$, time complexity of the depth-first traversal is close to the complexity that a perturbation would have with a memory of size $R \cdot S$.

The relation between R_{gen} and R_{ins} is almost the same that the one between $d \cdot R$ and R . So, we expect that R_{ins} will stay close to R .

The choice of a replacement strategy is then essential in such an algorithm. Several strategies have been looked at. As noticed in [11], the best one seems to be random replacement. It is easily performed and has no performance drop for particular graphs.

2.3 Experiments

The depth-first traversal with replacement has been used with different kinds of graphs. Some of them are accessibility graphs of communication protocols modelled by communicating finite state machines, and others are random graphs. The parameters of these random graphs are R_{max} a bound on the number of states and d_{max} the maximum degree of a node. They are generated in a breadth-first way. The degree of each node is chosen uniformly between 0 and d_{max} . If g is the number of already generated states, each successor of the current state has probability $1 - g/\min(2 \cdot g, R_{max})$ to be a new state. Among those random graphs, we only considered those with R close to R_{max} .

The two curves of figure 1 represent the behavior of the algorithm on a random graph when decreasing the memory size. Starting from $M_{max} = R \cdot S$, the memory size is decreased down to the minimal possible value M_{min} for which the algorithm terminates. The two bounds M_{max}/S and M_{min}/S are figured by the two dashed vertical lines.

The two first curves represent the evolution of the number of insertions R_{ins} of states in $St_Stack \cup Visited$ and the execution time. If $M = M_{max} = R \cdot S$ then $R_{ins} = R$. When M decreases, R_{ins} increases. But it increases very slowly until M comes very near from M_{min} . The number R_{ins} is then less than twice R . Finally R_{ins} explodes but the memory has been significantly reduced before explosion. The execution time T has then a similar form. For that example, with a memory size of 40% of $R \cdot S$ we have only 70% more insertion of states, which give only 50% time increase.

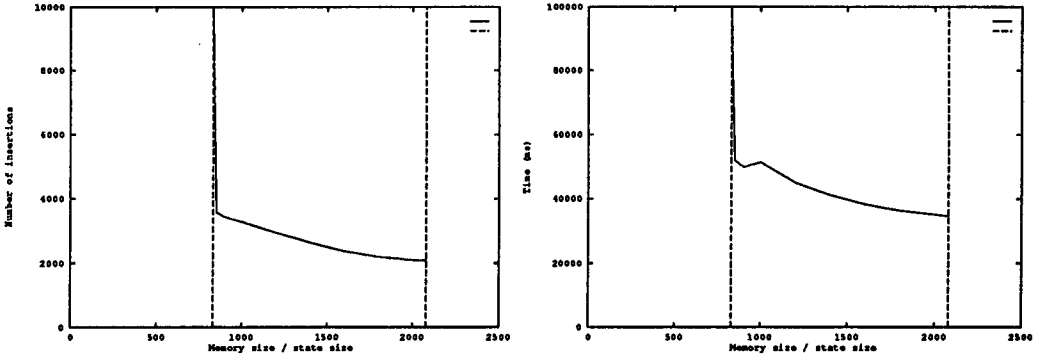


Figure 1: Number of generated states and execution time

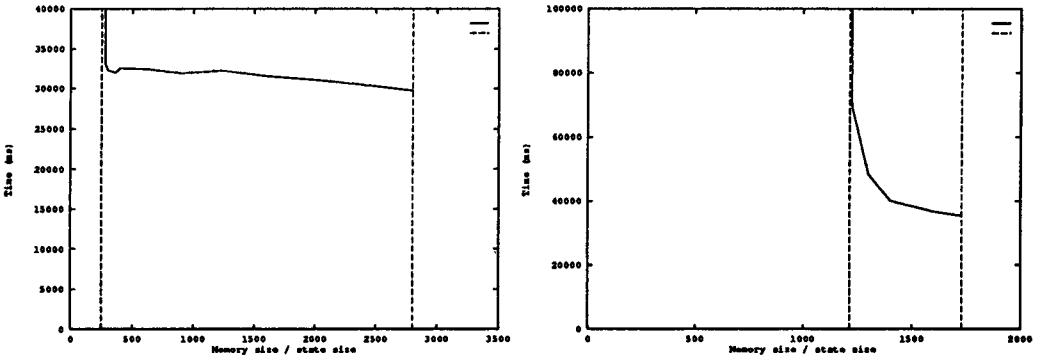


Figure 2: Execution time in extremal cases

Many examples have been tested with this traversal. They almost gave the same type of curves. But we can only decrease the memory size down to a value between $D_{max}.S$ and $G_{max}.S$. Thus when D_{max} is small with respect to R , one can reduce M very significantly, and the increase of R_{ins} and T are very slow. In the left example of figure 2, $M_{min} = M_{max}/10$, and we have an increase of only 1% of R_{ins} and 11% of T (see the left hand curve of figure 2). However, for graphs in which D_{max} is close to R as in the right hand curve of figure 2, that is when graphs are very connected (a complete graph is the worst case) results are not so good. The domain in which M/S can vary is very tight and R_{ins} and T increase very quickly.

3 Application to on-line model-checking

The first application of the depth-first traversal with replacement was introduced in [15]. The purpose was to verify that a protocol specification satisfied a property f . Properties were expressed in event based linear temporal logic (LTL) [20] and translated into finite states automata. But the idea can be generalized to other formalisms such as Büchi automata.

Let $\mathcal{S}_1 = \langle Q_1, A, T_1, q_{01} \rangle$ be the labelled transition system associated to the specification $Spec$ where Q_1 is a finite set of states, A a finite set of actions, $T \subseteq Q_1 \times A \times Q_1$ the transition relation, and q_{01} the initial state.

Suppose that a property \mathcal{P} can be expressed by a deterministic Büchi automaton $\mathcal{B} = \langle Q_2, A, T_2, q_{02}, F_2 \rangle$ where Q_2 is its finite set of states, A its set of actions, $T_2 \subseteq Q_2 \times A \times Q_2$ its transition relation, q_{02} the initial state and F_2 a set of designated states. An infinite word $a_1 \dots a_n \dots \in A^\omega$ is recognized by \mathcal{B} if and only if there exists an infinite run of \mathcal{B} : $q_{02} \xrightarrow{a_1} q_1 \dots q_{n-1} \xrightarrow{a_n} q_n \dots$ such that $q_i \in F_2$ for infinitely many i 's.

We say that $Spec$ satisfies \mathcal{P} written $Spec \models \mathcal{P}$ if and only if every infinite word labelling an infinite transition sequence of \mathcal{S}_1 is recognized by \mathcal{B} .

When the Büchi automaton is not necessarily deterministic, the usual way to verify that $Spec \models \mathcal{P}$ is to consider \mathcal{S}_1 as a Büchi automaton (its set of designated states is Q_1), make the product of \mathcal{S}_1 with the complement automaton $\overline{\mathcal{B}}$ of \mathcal{B} and check if $\mathcal{S}_1 \times \overline{\mathcal{B}}$ is empty (accepts no word). This can be done by computing the strongly connected components.

In the case of a deterministic Büchi automaton, we will show that there is a very simple algorithm which performs this verification without complementation and without computation of strongly connected components [16].

We consider \mathcal{S}_1 as a Büchi automaton with Q_1 as its set of designated states. We suppose that \mathcal{B} is complete. This can always be done by adding a new state.

The synchronous product of \mathcal{S}_1 and \mathcal{B} is $\mathcal{S} = \langle Q, A, T, q_0, F \rangle$ with:

- $Q = Q_1 \times Q_2$,
- $q_0 = (q_{01}, q_{02})$,
- $F = Q_1 \times F_2$,
- $T \subseteq Q \times A \times Q$ is defined by

$$((q_1, q_2), a, (q'_1, q'_2)) \in T \text{ if and only if } (q_1, a, q'_1) \in T_1 \text{ and } (q_2, a, q'_2) \in T_2$$

Since \mathcal{B} is complete, the infinite sequences of executable actions of \mathcal{S}_1 are exactly the words labelling the infinite runs of \mathcal{S} . And according to the definition of \mathcal{S} , $Spec \models \mathcal{P}$ if and only if every infinite run of \mathcal{S} contains infinitely many states of F . Considering \mathcal{S} as a directed graph, it is equivalent to say that every cycle of the graph contains a vertex in F . But this is equivalent to say that the sub-graph \mathcal{S}' obtained from \mathcal{S} by removing all vertices of F (and the corresponding edges) is acyclic. And \mathcal{S}' is acyclic if and only if a depth-first traversal of \mathcal{S}' doesn't detect any cycle.

But we don't want to first build \mathcal{S} and then remove vertices of F . We would like to check whether that \mathcal{S}' is acyclic during a traversal of \mathcal{S} . This can be done by a traversal of \mathcal{S} which is composed of depth-first traversals of sub-graphs of \mathcal{S} in the following way. We need a set W initialized with $\{q_0\}$ which contains the roots of the depth-first traversals not yet performed. These roots are q_0 and all states of F . For each traversal initiated in a state $q_{init} \in W$, remove q_{init} from W and add the following to the traversal with replacement: if a new state $q \in F$ is reached, it is added to W and successors of q are not explored now (they will be in the traversal initiated in q) and if a cycle is detected in $q \notin F$ this simply signifies that a cycle of \mathcal{S}' is detected.

The algorithm stops when W is empty and $Spec \models \mathcal{P}$ if and only if no cycle of \mathcal{S}' is detected. This algorithm is described below:

```

Visited:= $\emptyset$ ;
W:={ $S_0$ };
while W  $\neq \emptyset$  do begin
  St_Stack:=nil;
  Tr_Stack:=nil;
  qinit:=extract_one_elt_of(W);
  push(qinit, St_Stack);
  push(fireable(qinit), Tr_Stack);
  while St_Stack  $\neq \emptyset$  do begin
    q:=top(St_Stack);
    if top(Tr_Stack)  $\neq \emptyset$  then begin
      t:=extract_one_elt_of(top(Tr_Stack));
      q':=succ(q,t);
      if q'  $\in$  St_Stack then
        if q'  $\notin F$  then ERROR
      else if q'  $\notin$  Visited then
        if q'  $\in F$  then W:=W  $\cup$  {q'}
      else begin
        if memory_full then begin
          qdel:=one_state_from(Visited);
          Visited:=Visited - {qdel};
        end;
        push(q', St_Stack);
        push(fireable(q'), Tr_Stack);
      end;
    end;
  else begin (* -- top(Tr_Stack) =  $\emptyset$  -- *)
    pop(St_Stack);
    pop(Tr_Stack);
    Visited:=Visited  $\cup$  {q};
  end;
end;
end;

```

The depth-first traversal with replacement is very efficient for the on-line model-checking of deterministic finite states and Büchi automata. It avoids constructing the complete reachability graph. Moreover, you can choose to stop as soon as an error is detected.

It appears in the literature that different kinds of verification can be performed on-the-fly, since then they are based on a depth-first traversal. We found the [5] paper, which presents a new technique to verify strong bisimulation equivalence. In the paper [4] also, the authors define a method for the verification of a temporal logic property f on a finite state program P which combines a depth-first traversal with a partial search with hashing [10]. Finally, in [17] our traversal with replacement has also been proposed for the test of unboundedness of fifo channels in some specification models such as communicating finite state machines [1], fifo-nets [18, 6] and even Estelle programs [12].

4 Conclusion and prospects

Dealing with the state space explosion problem, we have presented an alternative to the exhaustive construction of state graphs. The depth-first traversal insures an exhaustive traversal of all states and/or transitions of a reachability graph. It requires less memory since it theoretically only needs a memory large enough to store the longest acyclic sequence. In order to improve this technique, it is necessary to store some visited states. When the memory is full, visited states are randomly replaced by new states of the current sequence. We have shown that this method can significantly increase the size of the state graphs that can actually be analysed without excessively increasing the computation time.

As we saw, this method can be used for different kinds of verification. A few application examples have pointed out that it can certainly improve the verification tools in various domains such as bisimulation, Büchi acceptance, on-the-fly verification of temporal properties and test for unboundedness.

However, this technique does not solve all the problems. We still don't know the whole applicability domain of that method. For example, is it possible to verify branching time temporal logic properties with a depth-first traversal with replacement, and, if the answer is positive, is it efficient? We also know that this algorithm is not quite suited for all kinds of graphs. Perhaps an interesting problem would be to carefully study the structure of graphs for which it is well suited. We could then infer on the convenience of the method on some classes of transitions systems. Within a tool, the choose of the depth-first traversal in a particular verification could then be guided by the expected structure of graphs.

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