# Robust and fast computation of unbiased intensity derivatives in images

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**Abstract.** In this paper we develop high order non-biased spatial derivative operators, with subpixel accuracy. Our approach is discrete and provides a way to obtain some of the spatio-temporal parameters from an image sequence. In this paper we concentrate on spatial parameters.

## 1 Introduction

Edges are important features in an image. Detecting them in static images is now a well understood problem. In particular, an optimal edge-detector using Canny's criterion has been designed [8,7]. In subsequent studies this method has been generalized to the computation of 3D-edges [5]. This edge-detector however has not been designed to compute edge geometric and dynamic characteristics, such as curvature and velocity.

It is also well known that robust estimates of the image geometric and dynamic characteristics should be computed at points in the image with a high contrast, that is edges. Several authors, have attempted to combine an edge-detector with other operators, in order to obtain a relevant estimate of some components of the image features, or the motion field [2], but they use the same derivatives operators for both problems.

However, it is not likely that the computation of edge characteristics has to be done in the same way as edge detection, and we would like to analyse this fact in this paper.

Since edge geometric characteristics are related to the spatial derivatives of the picture intensity [2]. we have to study *how to compute "good" intensity derivatives, that is suitable to estimate edge characteristics.*

In this paper, we attempt to answer this question, and propose a way to compute image optimal intensity derivatives, in the discrete case.

## 2 Computing optimal spatial derivatives

### 2.1 Position of the problem

We consider the following two properties for a derivative filter :

- A derivative filter is *unbiased* if it outputs only the required derivative, but not lower or higher order derivatives of the signal.
- Among these filters, a derivative filter is *optimal* if it minimizes the noise present in the signal. In our case we minimize the output noise.

Please note, that we are not dealing with filters for detecting edges, here, but rather - edges having been already detected - with derivative filters to compute edge characteristics. It is thus not relevant to consider other criteria used in optimal edge detection such as localization or false edge detection [1].

In fact, spatial derivatives are often computed in order to detect edges with accuracy and robustness. Performances of edge detectors are given in term of localization and signal to noise ratio [1,8]. Although the related operators are optimal for this task, *they might* *not be suitable to compute unbiased intensity derivatives on the detected edge.* Moreover it has been pointed out [9] that an important requirement of derivative filters, in the case where one wants to use differential equations of the intensity is the preservation of the intensity derivatives, which is not the case of usual filters. However, this author limits his discussion to Gaussian filters, whereas we would like to derive a general set of optimal filters for the computation of temporal or spatial derivatives. We are first going to demonstrate some properties of such filters in the continuous or discrete case and then use an equivalent formulation in the discrete case.

### 2.2 Unbiased filters with minimum output noise

**A condition for unbiasness .**

Let us note $\odot$ the convolution product. According to our definition of unbiasness *a 1D-filter $f_r$ is an unbiased rth-order derivator* if and only if :

$$f_r(x) \odot u(x) = \frac{d^r u(x)}{dx^r}$$

for all functions $C^r$.

In particular, for $u(x) = x^n$, we have a set *necessary* conditions :

$$f_r(x) \odot x^n = n(n-1)...(n-r+1)x^{n-r} = \frac{n!}{(n-r)!}x^{n-r}$$

which is a generalization of the condition proposed by Weiss [9].

But, considering a Taylor expansion of $u(x) = \sum \frac{d^r u(x)}{dx^r}\Big|_{x=0} \frac{x^r}{r!}$ around zero, for a $C^r$ function, and using the fact that polynomials form a dense family over the set of $C^r$ functions, this enumerable set of conditions are also *sufficient*.

The previous conditions can be rewritten as :

$$\int f_r(t)(x-t)^n dt = \frac{n!}{(n-r)!}x^{n-r} \quad \int f_r(t) \sum_{q=0}^{n} \frac{n!}{(n-q)!q!} t^q x^{n-q} dt = \frac{n!}{(n-r)!}x^{n-r}$$
$$\sum_{q=0}^{n} x^{n-q} \frac{n!}{(n-q)!q!} \int f_r(t) t^q dt = \frac{n!}{(n-r)!}x^{n-r}$$

and these $x$-polynomial equations are verified if and only if all the coefficients are equal, that is :

$$\int f_r(t)\frac{t^q}{q!}dt = \delta_{qr} \tag{1}$$

Equations (1) are thus necessary and sufficient conditions of unbiasness. Moreover if $f_r$ is an unbiased r-order filter, $f_{r+1} = f_r'$ is an unbiased (r+1)-order filter, since :

$$\begin{aligned}
f_{r-1}(x) \odot x^{n-1} &= \int f_{r-1}(x-t)t^{n-1}dt = \frac{(n-1)!}{((n-1)-(r-1))!}x^{(n-1)-(r-1)} \\
&= \left[\frac{t^n}{n} f_{r-1}(t)\right] + \int f_{r-1}'(x-t)\frac{t^n}{n}dt = \frac{(n-1)!}{n-r}x^{n-r} \\
&= \int f_{r-1}'(x-t)\frac{t^n}{n}dt = \frac{(n-1)!}{n-r}x^{n-r} \\
&\Leftrightarrow \\
f_{r-1}'(x) \odot x^n &= \int f_{r-1}'(x-t)t^n dt = \frac{n!}{n-r}x^{n-r}
\end{aligned}$$

If equation (1) is true for all $q$, the filter will be an unbiased derivative filter. It is important to note that this condition should be verified for $q \leq r$, but also for $q > r$. If not, high-order derivatives will have a response though the filter and the output will be biased. This is the case for Canny-Deriche filters, and this is an argument to derive another set of filters.

In fact, *the only one solution to this problem is the rth-derivative of the Dirac distribution*, $\delta^r$. This is not an interesting solution because this is just the "filter" which output noise is maximal (no filtering!). However, in practice, the input signals high-order derivatives are negligible, and we can only conisder unbiasness conditions for $0 \leq q \leq Q < \infty$.

**Minimizing the output noise** In the last paragraph we have obtained a set of conditions for unbiasness. Among all filters which satisfy these conditions let us compute the best one, considering a criteria related to the noise.

The mean-squared noise response, considering a white noise of variance 1, is (see [1], for instance) : $\int f_r(t)^2 dt$, and a reasonable optimal condition is to find the filter which minimize this quantity and satisfy the constraints given by equation (1). Using the opposite of the standard Lagrange multipliers $\lambda_p$ this might be written as :

$$min_{f()} min_{\lambda_p} \frac{1}{2} \int f_r(t)^2 dt - \sum_{p=0}^{Q} \lambda_p \left[ \int f_r(t) t^p dt - p! \delta_{pr} \right]$$

From the calculus of variation, one can derive the Euler equation, which is a necessary condition and which turns out to be, with the constraints, also sufficient in our case, since we have a positive quadratic criteria with linear constraints.

The optimal filter equations (Euler equations and constraints) are then :

$$\begin{cases} f_r(t) = \sum_{p=0}^{Q} \lambda_p t^p \\ 0 \leq q \leq Q \ \int f_r(t) \frac{t^q}{q!} dt = \delta_{qr} \end{cases}$$

These equations are ***necessary conditions*** for the filter to be optimum. They yield *polynomial filters*. Functions verify these equations only if they are defined, and presently, polynomials are only define on finite supports. Thus these equations are convergent *if and only if $f_r(t)$ has a finite support*. That is we obtain optimal filters minimizing output noise, only on a finite window.

These equations have the following consequence : *the optimal derivative filter is a polynomial filter and is thus only defined on a finite window.* If not, the Euler equations are no more defined. In fact, we also studied infinite response filters, but we came with a negative answer : even if considering special families of infinite response filters (such as product of polynomial with Gaussian or exponentials) and applying the same constrainted optimum criteria, it is not possible to obtain analytic filters as an infinite series of the original basis of function, because the summation is divergent (see however section 2.5 for a discussion about sub-optimal solutions).

We thus have to work on finite windows and in this case, we can compute the values of $\lambda_p$, from a set of linear equations, since from the Euler equation and the constraints we obtain :

$$\int f_r(t) \frac{t^q}{q!} dt = \int \sum_{p=0}^{Q} \lambda_p t^p \frac{t^q}{q!} dt = \sum_{p=0}^{Q} \lambda_p \int \frac{t^{p+q}}{q!} = \delta_{qr} \tag{2}$$

for $0 \leq q \leq Q$.

Equations (2) define a unique optimal unbiased r-order filter. However, if $f_r$ is this optimal unbiased r-order filter, $f_{r+1} = f_r'$ is not the optimal unbiased (r+1)-order filter, as it can be easily verified, whereas each filter has to be computed separately.

### 2.3 An equivalent parametric approach using polynomial approximation

There is another way to compute these derivatives, considering the Taylor expansion of the input as a parametric model. Writing :

$$x(t) \simeq \sum_{q=0}^{Q} x_q \frac{t^q}{q!} + \mathcal{N}oise \tag{3}$$

one can minimize :$J = \frac{1}{2} \int \left[ x(t) - \sum_{q=0}^{Q} x_q \frac{t^q}{q!} \right]^2 dt$ which is just a least-square criteria with a similar interpretation, since we minimize the variance of the residual error.

This quadratic positive criteria is minimum for $p!\frac{dJ}{dx_p} = 0$ which provides a set of linear equations in $x_q$ :

$$\int x(t) t^p \, dt = \sum_{q=0}^{Q} x_q \frac{\int t^{p+q} \, dt}{q!} \tag{4}$$

But,the quantities $x_q$ are just equal to the output of the optimal filters computed previously from $f_r() \odot x()$ at $t = 0$, then ***both approachs are equivalent***

Considering a signal with derivatives up to a given order $Q$, it is thus possible to compute unbiased estimators of these derivatives with a minimum of output noise by solving a least-square problem, as in equation (3). This result is not a surprise for someone familiar with Optimization but is crucial when implementing such filters in the discrete case, as done now.

Please note that the integration $\int \ldots dt$ is to be made over a bounded domain, in order this integration to be convergent for polynomials, but all the computations are valid for any Lebesgue integrals. In particular, this is still valid for a finite summation, a finite summation of definite integrals, etc... This will be used in the next sections.

### 2.4 Continuous implementation of unbiased filters

While, the continuous implementation of such filters is not directly usable in image processing, very helpful to study the properties and characteristics of these filters. In addition, we can compare these filters with others derivative filters, as used in edge detection.

Let us consider a finite window. For reasons of isotropy this window has to be symmetric $[-W, W]$, and it corresponds to a zero-phase non-causal filter. Moreover, changing the scale factor it is always possible to consider $W = 1$.

We compute filters for $0 \leq r \leq 3$ and $r \leq Q \leq 6$ and obtain the curves given in Fig. 2. The related output noise $\int f_r()^2$ is shown in Table 1.

| Q | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Smoother | $\frac{0.5}{W}$ | | $\frac{1.1}{W}$ | | $\frac{1.8}{W}$ | | $\frac{2.4}{W}$ | |
| First Order | | $\frac{1.5}{W^3}$ | | $\frac{9.4}{W^3}$ | | $\frac{25.0}{W^3}$ | | $\frac{65.0}{W^3}$ |
| Second Order | | | $\frac{23.0}{W^5}$ | | $\frac{280.0}{W^5}$ | | $\frac{1400.0}{W^5}$ | |
| Third Order | | | | $\frac{790.0}{W^7}$ | | $\frac{16000.0}{W^7}$ | | $\frac{120000.0}{W^7}$ |

**Table 1.** Computation of the output noise for different unbiased filters

We can make the following remarks :

- For a given window, there is a trade-off between unbiasness and output-noise limitation, as in standard filtering. The more the signal model contains high-order derivatives, and the more noise is output.
- The amount of output noise is very high as soon as the order of the model increases, especially for high-order derivatives. But it decreases very quickly with the increase of the window size. It is thus possible to tune the window size to maintain this amount of noise at a reasonable value [1].
- Contrary to usual filters the number of zero-crossing is not equal to the order of derivative but higher or equal. In particular the unbiased smoother has a number of zero-crossing equal to the order of the model as shown on Fig. 1.
- There is no simple algebraic relations between this series of polynomials.

[1] We have, in fact, $\int f_r()^2 = o(\frac{1}{W^{2r+1}})$

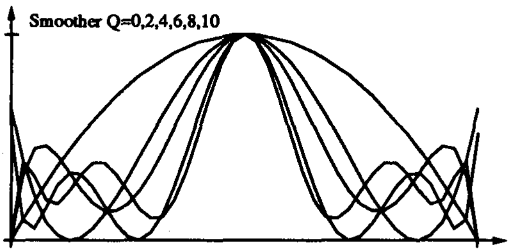


**Fig. 1.** A few examples of unbiased optimal smoothers

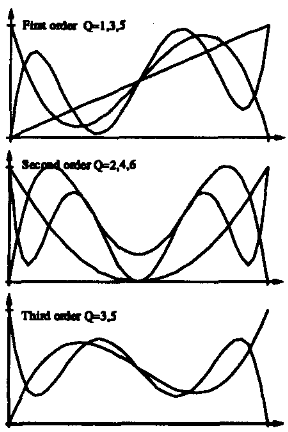


**Fig. 2.** A few examples of unbiased optimal derivators

### 2.5 What about infinite response filters ?

We can also design infinite response unbiased filters.

Consider for instance the family of filters :

$$f_r^d(t) = (\sum_{p=0}^{d} \lambda_p t^p) e^{-|\alpha t|}$$

which correspond to the set of recursively implemented digital filters (see for instance [6]), having an implementation of the form :

$$y_t = \sum_{i=0}^{p} b_i x_{t-i} - \sum_{j=1}^{q} a_j y_{t-i}$$

Applying the unbiasness condition of equation (1) to these functions leads to a finite set of linear equations :

$$\int f_r^d(t) \frac{t^q}{q!} dt = \delta_{qr} \Leftrightarrow \sum_{p=0}^{d} \lambda_p \int \frac{t^{p+q}}{q!} e^{-|\alpha t|} dt = \delta_{qr}$$

defining a affine functional subspace of finite co-dimension.

In particular for $d = Q$ there is a unique unbiased filter while if $d > Q$ we have an $(d - Q) -$ *dimensional* space of solutions. If $d > Q$, one can again choose the solution minimizing the output noise, that is the one for which :

$$\int f_r^d(t)^2 dt = \sum_{p=0}^{d} \sum_{q=0}^{d} \lambda_p \lambda_q \int t^{p+q} e^{-2|\alpha t|} dt$$

is minimum. This yields to the minimization of a quadratic positive criteria in the presence of linear constraints, having a unique solution obtained from the derivation of the related normal equations.

In order to illustrate this point, we derive these equations for $Q \leq 2$ and $d \geq 2$ for $r = 1$. And in that case we obtained : $f_1^d(t) = \beta t e^{-|\alpha t|}$ which corresponds precisely to Canny-Deriche recursive optimal derivative filters. More generally *if the signal contains derivatives up to the order of the desired derivative, usual derivative filters such as Canny-Deriche filters are unbiased filters* and can be used to estimate edge characteristics.

However, such a filter is not optimal among all infinite response operators, but only in the the small parametric family of exponential filters [2]. The problem of finding an optimal filter among all infinite response operators is an undefined problem, because the Euler equation obtained in the previous section (a necessary condition for the optimum) is undefined, as pointed out.

Since this family is dense in the functional space of derivable functions it is indeed possible to approximate any optimal filters using a combination of exponential filters, but the order $n$ might be very high, while the computation window has to be increased. Moreover, in practice, on real-time vision machines, these operators are truncated (thus biased !) and it is much more relevant to consider finite response filters.

### 2.6 An optimal approach in the discrete 2D-case

Let us now apply these results in the discrete case.

Whereas most authors derive optimal continuous filters and then use a non-optimal method to obtain a discrete version of these operators, we would like to stress the fact that *the discretization of an optimal continuous filter is not necessary the optimal discrete filter.*

In addition, the way the discretization is made depends upon a model for the sampling process. For instance, in almost all implementations [8,3], the authors make the implicit assumption that the intensity measured for one pixel is related to the true intensity by a Dirac distribution, that is, corresponds to the point value of the intensity at this point. This is not a very realistic assumption, and in our implementation we will use another model.

The key point here is that since we have obtained a formulation of the optimal filter using any Lebesgue integration over a bounded domain, then the class of obtained filters is still valid for the discrete case. Let us apply this result now.

In the previous section we have shown that optimal estimators of the intensity derivatives should be computed on a bounded domain, and we are going to consider here a squared window of $N \times N$ pixels in the picture, from $(0,0)$ to $(N-1, N-1)$. We would like to obtain an estimate of the derivatives around the middle point $(\frac{N}{2}, \frac{N}{2})$.

This is straightforward if we use the equivalent parametric approach obtained in section 2.3.

[2] The same parametric approach could have been developed using Gaussian kernels.

Generalizing the previous approach to 2D-data we can use the following model of the intensity, a Taylor expansion, the origin being at $(\frac{N}{2}, \frac{N}{2})$ :

$$I(x,y) = Io+I_x x+I_y y+\frac{I_{xx}}{2}x^2+\frac{I_{yy}}{2}y^2+I_{xy}xy+\frac{I_{xxx}}{6}x^3+\frac{I_{xxy}}{2}x^2y+\frac{I_{xyy}}{2}xy^2+\frac{I_{yyy}}{6}y^3+etc...$$

where the development is not made up to the order of derivative to be computed, but *up to the order of derivative the signal is supposed to contain.*

Let us now modelize the fact that the intensity obtained for one pixel is related to the image irradiance over its surface. We consider rectangular pixels, with homogeneous surfaces, and no gap between two pixels. Since, one pixel of a CCD camera integrates the light received on its surface, this means that a realistic model for the intensity measured for a pixel $(i,j)$ is, under the previous assumptions :

$$\begin{aligned} I_{ij} &= \int_i^{i+1}\int_j^{j+1} I(x,y)dxdy \\ &= IoP_0(i) + I_x P_1(i) + I_y P_1(j) + I_{xx}P_2(i) + I_{xy}P_1(i)P_1(j) + I_{yy}P_2(j) + \cdots \end{aligned}$$

where $P_k(i) = \int_i^{i+1} \frac{x^k}{k!}dx = \frac{1}{(k+1)!}\sum_{p=0}^{k} C_{k+1}^{p} i^p$.

Now, the related least-square problem is

$$J = \frac{1}{2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}[I_{ij}-(IoP_0(i)+I_xP_1(i)+I_yP_1(j)+I_{xx}P_2(i)+I_{xy}P_1(i)P_1(j)+I_{yy}P_2(j)+\cdots)]^2$$

and its resolution provides optimal estimates of the intensity derivatives $\{Io, I_x, I_y, I_{xx}, I_{xy}, I_{yy}, \cdots\}$ in function of the intensity values $I_{ij}$ in the $N \times N$ window.

In other words we obtain the intensity derivatives as a linear combination of the intensity values $I_{ij}$, as for usual finite response digital filters.

For a $5 \times 5$ or $7 \times 7$window, for instance, and for a intensity model taken up to the fourth order one have convolutions given in Fig3.

This approach is very similar to what was proposed by Haralick [3], and **we call these filters Haralick-like filters**. In both methods the filters depends upon two integers : (1) the size of the window, (2) the order of expansion of the model. In both methods, we obtain polynomial linear filters. However it has been shown [4] that Haralick filters reduce to Prewitt filters, while our filters do not correspond to already existing filters. The key point, which is - we think - the main improvement, is to consider the intensity at one pixel not as the simple value at that location, but as the integral of the intensity over the pixel surface, which is closer to reality.

Contrary to Haralick original filters these filters are not all separable, however this not a drawback because separable filters are only useful when the whole image is processed. In our case we only compute the derivatives in a small area along edges, and for that reason efficiency is not as much an issue[3]

## 2.7 Conclusion

We have designed a new class of unbiased optimal filters dedicated to the computation of intensity derivatives, as required for the computation of edge characteristics. Because these filters are computed though a simple least-square minimization problem, we have been capable to implement these operators in the discrete case, taking the CCD subpixel mechanisms into account.

These filters are dedicated to the computation of edge characteristics, they are well implemented in finite windows, and correspond to unbiased derivators with minimum output noise. They do not correspond to optimal filters for edge detection.

[3] Anyway, separable filters are quicker than general filters if and only if they are used on a whole image not a few set of points

$$Ix = \frac{1}{28224}\begin{bmatrix} 1115 & 380 & -61 & -208 & -61 & 380 & 1115 \\ -610 & -1100 & -1394 & -1492 & -1394 & -1100 & -610 \\ -711 & -956 & -1103 & -1152 & -1103 & -956 & -711 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 711 & 956 & 1103 & 1152 & 1103 & 956 & 711 \\ 610 & 1100 & 1394 & 1492 & 1394 & 1100 & 610 \\ -1115 & -380 & 61 & 208 & 61 & -380 & -1115 \end{bmatrix} \quad Iy = Ix^T$$

$$Ixx = \frac{1}{294}\begin{bmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ -4 & -4 & -4 & -4 & -4 & -4 & -4 \\ -3 & -3 & -3 & -3 & -3 & -3 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{bmatrix} \quad Ixy = \frac{1}{784}\begin{bmatrix} 9 & 6 & 3 & 0 & -3 & -6 & -9 \\ 6 & 4 & 2 & 0 & -2 & -4 & -6 \\ 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ -6 & -4 & -2 & 0 & 2 & 4 & 6 \\ -9 & -6 & -3 & 0 & 3 & 6 & 9 \end{bmatrix}$$

$$Iyy = Ixx^T$$

$$Ixxx = \frac{1}{42}\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \quad Ixxy = \frac{1}{1176}\begin{bmatrix} -15 & -10 & -5 & 0 & 5 & 10 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 6 & 3 & 0 & -3 & -6 & -9 \\ 12 & 8 & 4 & 0 & -4 & -8 & -12 \\ 9 & 6 & 3 & 0 & -3 & -6 & -9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -15 & -10 & -5 & 0 & 5 & 10 & 15 \end{bmatrix}$$

$$Ixyy = Ixxy^T \quad Iyyy = Ixxx^T$$

**Fig. 3.** Some Haralick-like $5 \times 5$ and $7 \times 7$ improved filters

## 3 Experimental result : computing edge curvature

In order to illustrate the previous developments we have experimented our operators for the computation of edge curvature. Under reasonable assumptions, the edge curvature can be computed as :$\kappa = \frac{2\mathcal{I}_{xy}\mathcal{I}_x\mathcal{I}_y - \mathcal{I}_{xx}\mathcal{I}_y\mathcal{I}_y - \mathcal{I}_{yy}\mathcal{I}_x\mathcal{I}_x}{\mathcal{I}_x^2+\mathcal{I}_y^2{}^{\frac{3}{2}}}$.

We have used noisy synthetic pictures, containing horizontal, vertical or oblique edges with step and roof intensity profiles.

Noise has been added both to the intensity (typically 5 % of the intensity range) and to the edge location (typically 1 pixel). Noise on the intensity will be denoted "I-Noise", its unit being in percentage of the intensity range, while noise on the edge location will be denoted "P-Noise", its unit being in pixels.

We have computed the curvature for non-rectilinear edges, either circular or elliptic. The curvature range is between 0 for a rectilinear edge and 1, since a curve with a curvature higher than 1 will be inside a pixel. We have computed the curvature along an edge, and have compared the results with the expected values. Results are plotted in Fig.4, the expected values being a dashed curve.

We have also computed the curvature for different circles, in the presence of noise, and evaluated the error on this estimation. Results are shown in Table 2. The results are the radius of curvature, the inverse of the curvature expressed in pixels. The circle radius was of 100 pixels.

Although the error is almost 10 %, it appears that for important edge localization errors, the edge curvature is simply not computable. This is due to the fact we use a

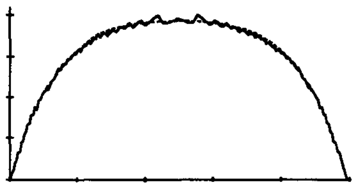

**Fig. 4.** Computation of the curvature along an elliptic edge

| I-Noise | 2% | 5% | 10% | 0 | 0 | 0 |
|---|---|---|---|---|---|---|
| P-Noise | 0 | 0 | 0 | 0.5 | 1 | 2 |
| Error (in pixel) | 2.1 | 6.0 | 10.4 | 6.0 | 12.2 | huge |

**Table 2.** Computation of the curvature at different level of noise

$5 \times 5$ window, and that our model is only locally valid. In the last case, the second order derivatives are used at the border of the neighbourhood and are no more valid.

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