A Theory of 3D Reconstruction of Heterogeneous Edge Primitives from Two Perspective Views *

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Abstract. We address the problem of 3D reconstruction of a set of heterogeneous edge primitives from two perspective views. The edge primitives that are taken into account are contour points, line segments, quadratic curves and closed curves. We illustrate the existence of analytic solutions for the 3D reconstruction of the above edge primitives, knowing the relative geometry between the two perspective views.

1 Introduction

3D computer vision is concerned with recovering the 3D structure of the observed scene from 2D projective image data. One major problem of 3D reconstruction is the precision of the obtained 3D data (see [1] and [2]). A promising direction of research is to combine or fuse 3D data obtained from different observations or by different sensors. However, simply adopting the fusion approach is not enough: an additional effort needs to be contributed at the stage of the 3D reconstruction by adopting a new strategy. For example, we think that a strategy of 3D reconstruction of heterogeneous primitives would be an interesting direction of research. The main reason behind this idea is that a real scene composed of natural or man-made objects would be characterized efficiently by a set of heterogeneous primitives, instead of uniquely using a set of 3D points or a set of 3D line segments. Therefore, the design of a 3D vision system must incorporate the processing of a set of heterogeneous primitives as a central element. In order to implement the strategy above, we must know at the stage of 3D reconstruction what kind of primitives will be recovered and how to perform such a 3D reconstruction of the primitives selected beforehand. In fact, we are interested in the 3D reconstruction of primitives relative to the boundaries of objects, i.e., the edge primitives. For the purpose of simplicity, we can roughly classify such primitives into four types which are *contour points*, *line segments*, *quadratic curves* and *closed curves*. Suppose now that a moving camera or moving stereo cameras observe a natural scene to furnish some perspective views. Then, a relevant question will be:

> *Given two perspective views with the relative geometry being knowing, how to recover the 3D information from the matched 2D primitives such as contour points, line segments, quadratic curves and closed curves ?*

2 Camera Modelling

We suppose the projection model of a camera to be a perspective one. Consider a coordinate system $OXYZ$ to be at the center of the lens of the camera, with OXY being parallel to the image plane and OZ axis being the normal to the image plane (pointing

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outside the camera). Similarly, we associate a coordinate system oxy to the image plane, with the origin being at the intersection point between OZ axis and the image plane; ox and oy axis being respectively parallel to OX, OY. If we denote $P = (X, Y, Z)$ a point in $OXYZ$ and $p = (x, y)$ the corresponding image point in oxy, by using a perspective projection model of the camera, we shall have the following relationship:

$$\begin{cases} x = f\,\frac{X}{Z} \\ y = f\,\frac{Y}{Z} \end{cases} \tag{1}$$

where f is the focal length of the camera. Without loss of generality, we can set $f = 1$.

3 Relative Geometry between Two Perspective Views

The two perspective views in question may be furnished either by a moving camera at two consecutive instants or by a pair of stereo cameras. Thus, it seems natural to represent the relative geometry between two perspective views by a rotation matrix R and a translation vector T. In the following, we shall denote (R_{v1v2}, T_{v1v2}) the relative geometry between the perspective view v_1 and the perspective view v_2. Now, if we denote $P_{v1} = (X_{v1}, Y_{v1}, Z_{v1})$ a 3D point in the camera coordinate system of the perspective view v_1 and $P_{v2} = (X_{v2}, Y_{v2}, Z_{v2})$ the same 3D point in the camera coordinate system of the perspective view v_2, then the following relation holds:

$$\begin{pmatrix} X_{v2} \\ Y_{v2} \\ Z_{v2} \end{pmatrix} = R_{v1v2} \begin{pmatrix} X_{v1} \\ Y_{v1} \\ Z_{v1} \end{pmatrix} + T_{v1v2}. \tag{2}$$

So far, we shall represent the relative geometry between two perspective views (view v_1 and view v_2) as follows:

$$\begin{cases} R_{v1v2} = (R_1, R_2, R_3)_{3\times3} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}_{3\times3} \\ T_{v1v2} = (t_x, t_y, t_z)^t_{3\times1} \end{cases} \tag{3}$$

where t means the transpose of a vector or a matrix.

4 Solutions of 3D Reconstruction

4.1 3D Reconstruction of Contour Points

In an edge map, the contour points are the basic primitives. A contour chain that can not be described analytically could be considered as a set of linked contour points. So, the 3D reconstruction of non-describable contour chains will be equivalent to that of contour points. Given a pair of matched contour points: (p_{v1}, p_{v2}), we can first determine the *projecting line* which passes through the point p_{v2} and the origin of the camera coordinate system of the perspective view v_2. Then, we transform this projecting line into the camera coordinate system of the perspective view v_1. Finally, the coordinates of the corresponding 3D point can be determined by inversely projecting the contour point p_{v1} onto the transformed line. Therefore, our solution for recovering 3D contour points can be formulated by the following theorem:

Theorem 1. *A 3D point P is observed from two perspective views: the perspective view v_1 and the perspective view v_2. In the first perspective view, $P_{v1} = (X_{v1}, Y_{v1}, Z_{v1})$ represents the 3D coordinates (in the camera coordinate system) of the point P and $p_{v1} = (x_{v1}, y_{v1})$ the 2D image point of P_{v1}. In the second perspective view, $P_{v2} = (X_{v2}, Y_{v2}, Z_{v2})$ represents the 3D coordinates (in the camera coordinate system) of the same point P and $p_{v2} = (x_{v2}, y_{v2})$ the 2D image point of P_{v2}. If the relative geometry between the two perspective views is known and is represented by (3), then the 3D coordinates P_{v1} are determined by:*

$$\begin{cases} X_{v1} = \frac{x_{v1}}{\lambda_x + \lambda_y} (\lambda_x Z_x + \lambda_y Z_y). \\ Y_{v1} = \frac{y_{v1}}{\lambda_x + \lambda_y} (\lambda_x Z_x + \lambda_y Z_y). \\ Z_{v1} = \frac{1}{\lambda_x + \lambda_y} (\lambda_x Z_x + \lambda_y Z_y). \end{cases} \tag{4}$$

where:

$$\begin{cases} Z_x = \frac{x_{v2} t_z - t_x}{(x_{v1} r_{11} + y_{v1} r_{12} + r_{13}) - x_{v2} (x_{v1} r_{31} + y_{v1} r_{32} + r_{33})}. \\ Z_y = \frac{y_{v2} t_z - t_y}{(x_{v1} r_{21} + y_{v1} r_{22} + r_{23}) - y_{v2} (x_{v1} r_{31} + y_{v1} r_{32} + r_{33})}. \end{cases}$$

and (λ_x, λ_y) are two weighting coefficients.

4.2 3D Reconstruction of Line segments

The problem of 3D reconstruction of line segments has been addressed by several researchers (see [3] and [4]). In this paper, we shall develop a more simple solution with respect to the camera-centered coordinate system, knowing two perspective views. The basic idea is first to determine the *projecting plane* of a line segment in the second perspective view, then to transform this projecting plane into the first perspective view and finally to determine the 3D endpoints of the line segment by inversely projecting the corresponding 2D endpoints (in the image plane) to the transformed projecting plane in the first perspective view. In this way, we can derive a solution for the 3D reconstruction of line segments. This solution can be stated as follows:

Theorem 2. *A 3D line segment is observed from two perspective views: the perspective view v_1 and the perspective view v_2. In the second view v_2, we know the supporting line of the corresponding projected line segment (in the image plane), which is described by the equation: $a_{v2} x_{v2} + b_{v2} y_{v2} + c_{v2} = 0$. If the relative geometry between the two perspective views is known and is represented by (3), then the coordinates (X_{v1}, Y_{v1}, Z_{v1}) of a point (e.g. an endpoint) of the 3D line segment in the first perspective view are determined by the following equations:*

$$\begin{cases} X_{v1} = \frac{-(L_{v2} \bullet T_{v1v2}) x_{v1}}{(L_{v2} \bullet R_1) x_{v1} + (L_{v2} \bullet R_2) y_{v1} + (L_{v2} \bullet R_3)}. \\ Y_{v1} = \frac{-(L_{v2} \bullet T_{v1v2}) y_{v1}}{(L_{v2} \bullet R_1) x_{v1} + (L_{v2} \bullet R_2) y_{v1} + (L_{v2} \bullet R_3)}. \\ Z_{v1} = \frac{-(L_{v2} \bullet T_{v1v2})}{(L_{v2} \bullet R_1) x_{v1} + (L_{v2} \bullet R_2) y_{v1} + (L_{v2} \bullet R_3)}. \end{cases} \tag{5}$$

where $L_{v2} = (a_{v2}, b_{v2}, c_{v2})$ and (x_{v1}, y_{v1}) is the known projection of (X_{v1}, Y_{v1}, Z_{v1}) in the image plane of the first perspective view.

4.3 3D Reconstruction of Quadratic Curves

In this section, we shall show that an analytic solution exists for the 3D reconstruction of quadratic curves from two perspective views. By *quadratic curve*, we mean the curves whose projection onto an image plane can be described by an equation of quadratic form.

To determine the 3D points belonging to a 3D curve, the basic idea is first to determine the *projecting surface* of a 3D curve observed in the second perspective view, then to transform this projecting surface to the first perspective view and finally to determine the 3D points belonging to the 3D curve by inversely projecting the corresponding 2D points (in the image plane) to the transformed projecting surface in the first perspective view. If we denote $p_v = (x_v, y_v, 1)$ the homogeneous coordinates of an image point in the perspective view v, we can formulate our solution for the 3D reconstruction of quadratic curves by the following theorem:

Theorem 3. *A 3D curve is observed from two perspective views: the perspective view v_1 and the perspective view v_2. In these two views, the corresponding projected 2D curves (in image planes) can be described by equations of quadratic form. The description of the 2D curve in the second perspective view is given by: $a_{v2}\, x_{v2}^2 + b_{v2}\, y_{v2}^2 + c_{v2}\, x_{v2}\, y_{v2} + e_{v2}\, x_{v2} + f_{v2}\, y_{v2} + g_{v2} = 0$. If the relative geometry between the two perspective views is known and is represented by (3), then given a point $p_{v1} = (x_{v1}, y_{v1}, 1)$ on the 2D curve in the first perspective view, the corresponding 3D point (X_{v1}, Y_{v1}, Z_{v1}) on the 3D curve is determined by the following equations:*

$$\begin{cases} X_{v1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\, x_{v1}. \\ Y_{v1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\, y_{v1}. \\ Z_{v1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \end{cases} \tag{6}$$

where:

$$\begin{cases} A = p_{v1}^t \bullet R_{v1v2}^t \bullet Q_{v2} \bullet R_{v1v2} \bullet p_{v1}. \\ B = (p_{v1}^t \bullet R_{v1v2}^t \bullet Q_{v2} \bullet T_{v1v2} + T_{v1v2}^t \bullet Q_{v2} \bullet R_{v1v2} \bullet p_{v1}). \\ C = T_{v1v2}^t \bullet Q_{v2} \bullet T_{v1v2}. \end{cases}$$

and:

$$Q_{v2} = \begin{pmatrix} a_{v2} & c_{v2} & e_{v2} \\ 0 & b_{v2} & f_{v2} \\ 0 & 0 & g_{v2} \end{pmatrix}.$$

4.4 3D Reconstruction of Closed Planar Curves

A solution for the 3D reconstruction of closed curves can be derived by using a *planarity* constraint, i.e. the closed curves to be recovered in a 3D space being planar (that means that a closed curve can be supported by a plane). Therefore, given two perspective views of a closed curve, our strategy will consist of first trying to estimate the supporting plane of a closed curve in the first perspective view and then of determining the 3D points of the closed curve by inversely projecting the points of the corresponding 2D curve (in the image plane) to the estimated supporting plane. At the first step, we shall make use of Theorem 1. Below is the development of our solution for the 3D reconstruction of closed planar curves:

Let $O_{v1} = \{(X_{v1}^i, Y_{v1}^i, Z_{v1}^i),\ i = 1, 2, 3, \dots, n\}$ be a set of n 3D points belonging to a closed curve C in the first perspective view and $I_{v1} = \{(x_{v1}^i, y_{v1}^i),\ i = 1, 2, 3, \dots, n\}$ be a set of n corresponding image points of O_{v1}. Due to the visibility of a closed curve detected in an image plane, its supporting plane can not pass through the origin of the camera coordinate system. Thus, we can describe a supporting plane by an equation of the form: $a\,X + b\,Y + c\,Z = 1$.

Based on the assumption that the observed closed curve C is planar, so, a 3D point $(X_{v1}^i, Y_{v1}^i, Z_{v1}^i)$ on C must satisfy the equation of its supporting plane, that is:

$$a\,X_{v1}^i + b\,Y_{v1}^i + c\,Z_{v1}^i = 1. \tag{7}$$

By applying (1) to the above equation, we obtain:

$$a\,x^i_{v1} + b\,y^i_{v1} + c = \frac{1}{Z^i_{v1}}. \tag{8}$$

where Z^i_{v1} will be calculated by (4) of Theorem 1.

(8) is a linear equation of the unknown variables (a, b, c). To solve it, we need at least three non-collinear points in order to obtain an unique solution. In practice, there will be more than three points on a closed curve. As for the closed curve C, if we define:

$$A_{n\times 3} = \begin{pmatrix} x^1_{v1}\, y^1_{v1} & 1 \\ x^2_{v1}\, y^2_{v1} & 1 \\ \ldots\ \ldots & \ldots \\ x^n_{v1}\, y^n_{v1} & 1 \end{pmatrix};\ B_{n\times 1} = \begin{pmatrix} 1/Z^1_{v1} \\ 1/Z^2_{v1} \\ \ldots \\ 1/Z^n_{v1} \end{pmatrix};\ W_{3\times 1} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}. \tag{9}$$

then a linear system will be established as follows:

$$A \bullet W = B. \tag{10}$$

To estimate the unknown vector W, we use a least-squares technique. So, the solution for (a, b, c) can be obtained by the following calculation:

$$W = (A^t \bullet A)^{-1} \bullet (A^t \bullet B). \tag{11}$$

Knowing the supporting plane determined by (a, b, c), the 3D points of the closed curve C can be calculated as follows (by combining (1) and (8)):

$$\begin{cases} X^i_{v1} = \frac{x^i_{v1}}{a\,x^i_{v1}+b\,y^i_{v1}+c}. \\ Y^i_{v1} = \frac{y^i_{v1}}{a\,x^i_{v1}+b\,y^i_{v1}+c}. \\ Z^i_{v1} = \frac{1}{a\,x^i_{v1}+b\,y^i_{v1}+c} \end{cases} \quad i = 1, 2, \ldots, n. \tag{12}$$

5 Conclusions

We have addressed the problem of 3D reconstruction of heterogeneous edge primitives by using two perspective views. With respect to the edge primitives such as contour points, line segments, quadratic curves and closed curves, the existence of (analytic) solutions has been illustrated. An advantage of our work is that the proposed solutions are derived by reasoning in the discrete space of time. Consequently, they are directly applicable to the situation where a set of discrete perspective views (or a sequence of discrete digital images) are available.

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