Bernd Jähne

Spatio-Temporal Image Processing

Theory and Scientific Applications

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Gerhard Goos Universität Karlsruhe Postfach 69 80 Vincenz-Priessnitz-Straße 1 D-76131 Karlsruhe, Germany Juris Hartmanis Cornell University Department of Computer Science 4130 Upson Hall Ithaca, NY 14853, USA

Author

Bernd Jähne
Physical Oceanography Research Division, Scripps Institution of Oceanography
University of California, San Diego
La Jolla, CA, 92093-0230 USA
E-mail: bjaehne@ucsd.edu

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Preface

Image sequence processing is becoming a tremendous tool to analyze spatiotemporal data in all areas of natural science. It is the key to study the dynamics of complex scientific phenomena. Methods from computer science and the field of application are merged establishing new interdisciplinary research areas. This work emerged from scientific applications and thus is an example for such an interdisciplinary approach.

It is addressed to computer scientists as well as to researchers from other fields who are applying methods of computer vision. Computer scientists will find big challenges from the applications presented here. There are other scenes to be analyzed than teddy bears, Pepsi cans, and children's block world, namely those used to investigate tough and pressing scientific problems which cannot be solved without advanced techniques from computer vision. Researchers from other fields may find the exemplary results shown mostly from environmental physics (oceanography) illuminating and helpful for applying similar methods in their own research areas. Because of its interdisciplinary nature, I tried to write this work in such a way that it is understandable for researchers with different background. I apologize to computer scientists for reiterating several topics which are already familiar to them.

I am deeply indebted to the many individuals who helped me to perform this research. I acknowledge them by tracing its history. In the early 1980s, when I worked in the Institute of Environmental Physics at the University of Heidelberg, it became obvious to me that the small-scale air-sea interaction processes I had been studying could not be adequately measured with point measuring probes. Consequently, a number of area extended measuring techniques were developed. Then I searched for techniques to extract the physically relevant data from the images and sought for colleagues with experience in digital image processing. The first contacts were established with the Institute for Applied Physics at Heidelberg University and the German Cancer Research Center in Heidelberg. I would like to thank Joseph Bille, Joachim Dengler and Markus Schmidt cordially for many eye-opening conversations and their cooperation.

At the faculty for computer science at Karlsruhe University, I learnt a great deal from the course taught by Hans-Helmut Nagel and Ralf Kories on "Algorithmic Interpretation of Image Sequences" that I attended in the summer term 1986. For stimulating discussions, I would also like to thank Joseph Bigün, David Fleet, Goesta Granlund, and Howard Schultz.

I am deeply indebted to Karl-Otto Münnich, former director of the Institute for Environmental Physics. It is due to his farsightedness and substantial support that the research group "Digital Image Processing in Environmental Physics" could develop so fruitfully at his institute. I am most grateful to my students (and former students) at Heidelberg University for their help and contribution in performing the experiments, writing software, performing data analysis, and numerous discussions about all aspects of this research. I cordially thank Peter Geißler, Horst Haußecker, Frank Hering, Werner Huber, Jochen Klinke, Hermann Lauer, Thomas Münsterer, Klaus Riemer, Stefan Waas, and Dietmar Wierzimok. I cordially thank the workshops of the Institute for Environmental Physics, Delft Hydraulics, and Scripps Institution of Oceanography for their excellent work in constructing of the various devices needed to take the image sequence data for this work. Special thanks goes to Peter Bliven of PBA Associates and Lory Rosenblatt of Mitsubishi for generously renting me expensive IR cameras for experiments at the Scripps Pier, and the Scripps and Delft wind/wave flumes. The video sequences for various motion studies in natural scenes were taken by AEON Verlag & Studio.

Teaching and research during my guest professorship at the Interdisciplinary Research Center for Scientific Computing (IWR) at Heidelberg University in the winter terms 1990/91 and 1991/92 provided much of the inspiration for this work. I cordially thank Willi Jäger, director of the IWR, for his hospitality.

A substantial fraction of the research reported here was performed in the framework of international cooperations. In a cooperation with the Institut de Mécanique Statistique de la Turbulence (IMST, Fred Ramamonijarisoa), image sequences of small-scale water surface waves taken in the wind/wave flume of IMST. Most substantial were several experimental campaigns conducted in the huge wind/wave flume of Delft Hydraulics in Delft, The Netherlands from 1987 to 1990. Some of these experiments were part of the VIERS-1 project (1987-1992). VIERS-1 is a Dutch acronym for Preparation and Interpretation of the first European Remote Sensing (ERS1) satellite data. This joint Dutch-German research project aimed at a better understanding of the physical mechanisms of the microwave backscatter from the ocean surface. Within the American-German SAXON-FPN project, another experiment was performed in the Delft flume and a new optical technique for the measurement of small-scale ocean surface waves was used for the first time at sea. Testing of the instruments took place at the Scripps Pier, California, in summer 1990 and 1991, while the first true sea measurements were carried out at the Noordwijk research platform in the North Sea off the Dutch coast in November 1990. Further research reported here and funded by the National Science Foundation is devoted to a better understanding of air-sea gas transfer and long-wave/short wave interaction. The latter project is being performed in cooperation with Ken Melville. I gratefully acknowledge financial support from the European Community (twinning contract ERBST2*CT000451, large installation project at Delft Hydraulics), the National Science Foundation (OCE8911224, OCE9115994, OCE9217002), and the Office of Naval Research (N00014-89-J-3222).

Preface

Last, but not least, I would like to thank Hans Burkhardt cordially for his interest in my research and the opportunity and substantial help to submit my habilitation thesis in Applied Computer Science to the Forschungsschwerpunkt "Informations- und Kommunikationstechnik" at the Technical University of Hamburg-Harburg in October 1991. This monograph is a revised and extended version of the habilitation thesis. The habilitation colloquium took place on October 29, 1992. For proofreading of the final manuscript, I cordially thank Horst Haußecker, Jochen Klinke, Christian Wolf, Sven Weber, and Christhard Beringer.

La Jolla, California, September 1993

Bernd Jähne

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