Following Corners on Curves and Surfaces in the Scale Space

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Abstract. This paper is devoted to an analytical study of extrema curvature evolution through scale-space. Our analytical study allows to get results which show that, from a qualitative point of view, corner evolution in scale-space has the same behavior for planar curves or surfaces. In particular, this analysis, performed with different corner-shape models, shows that, for a two-corner shape, two curvature maxima exist and merge at a certain scale σ_0, depending on the shape. For a two-corner grey-level surface, the evolution of the determinant of hessian (DET) shows a merging point for a certain σ_0 independently of contrast, and the evolution of Gaussian Curvature presents the same characteristic but this point evolves with contrast.

1 Introduction

Over the last few years [11], the multiscale approach for image analysis has become very popular. A lot of works have been done on curves [1, 8, 6] and on surfaces [4, 5] to propose a theory of Curvature-based Shape representation. Mokhtarian and Mackworth [7, 8] analyse the zero crossing evolution of the curvature, defining the curvature scale space image. From some results, they show that arc length evolution does not change the physical interpretation of planar curves as object boundaries.

Recently, Rattarangsi and Chin [9] have proposed a scale-space curvature extrema analysis for different corner models on planar curve. Their results show that a planar curve of a square has 4 curvature maxima for any σ. As we show below, due to the approximation of the curvature expression, their results are in contraction with the Mokhtarian's results.

In this paper, we present an analytical study of curvature extrema in scale space. This theoritical analysis is performed on exact curvature of planar curve and on Gaussian Curvature and DET maxima of surfaces. DET is the determinant of hessian, introduced by Beaudet [2] and used in [3] to detect corners.

Our analytical study, performed with different corner-shape models, allows to better understand the evolution and the behavior of these extrema in scale-space. From a qualitative point of view, our results show that corner evolution in scale-space has the same behavior for planar curves or surfaces. The study shows that, for a two-corner shape, two curvature maxima exist and merge at a certain scale σ_0, depending on the shape. These results are right for Curvature

Lecture Notes in Computer Science, Vol. 800
Jan-Olof Eklundh (Ed.)
Computer Vision – ECCV '94

on planar curves and for Gaussian Curvature and DET on surfaces. For the surface analysis with the DET, the merging point contrast independant and we compute the exact σ_0. With the Gaussian Curvature, we show that this point evoluates with contrast. The reader may find a more complete developpement in [10].

2 Planar Curves

Rattarangsi and Chin propose in [9] a method to detect corners of planar curves. The curve is smoothed at various scales, using a gaussian kernel convolution on the coordinates. Corners are detected at a low resolution and followed along the scale space to their localization at a high resolution. The definition for a corner is that it is a curvature extrema. The authors study models of isolated corner and of groups of two corners. They present the evolution of curvature extrema reported to the curvilinear abscissa of the original curve and the smoothing parameter.

Given a parametric curve $(x(s), y(s))$, Rattarangsi and Chin compute a smoothed version of it $(X(s,\sigma), Y(s,\sigma))$, and detect its curvature maxima as the zeros of $\dot{X}\dddot{Y} - \dot{Y}\dddot{X}$, where the dots represent the derivatives in s. This comes from the derivation of a simplified version of the curvature $C(X,Y) = \dot{X}\ddot{Y} - \dot{Y}\ddot{X}$. The expression is legal when s is a curvilenear abscissa for the curve, but false as soon as $\sigma > 0$.

Figure 1 revisits some of the results obtained in [9]. It shows that for a two convex corners curve, the curvature extrema merge at a finite σ, whatever the value of the corner angle is. See in particular the right-angle END model (second curve in Fig. 1), relate it to [7, 8] and oppose it to [9].

3 Surfaces

It is possible to obtain shape information on real images (grey level images) without performing a prior segmentation. This has been shown in [3], where the authors study the evolution of the DET maxima around a single corner. Our purpose in this section is to extend this result to a pattern of two corners, in a similar manner that it has been done in the planar curve case. We will study the behavior of two second-order differential measures: DET and gaussian curvature.

The model. Our model for a pair of corners in a surface is the elevation of a right-angle END model. Given an arbitrary *contrast* A, the surface is defined as a function of the plane coordinates (x, y) to be $z = A$ when x and y are inside the right-angle END, and to be $z = 0$ when x and y are outside the right-angle END. More precisely:

$$\begin{cases} z = A & \text{if } x \in [-1 \ldots 1] \text{ and } y \in]-\infty \ldots 0] \\ z = 0 & \text{otherwise} \end{cases}$$

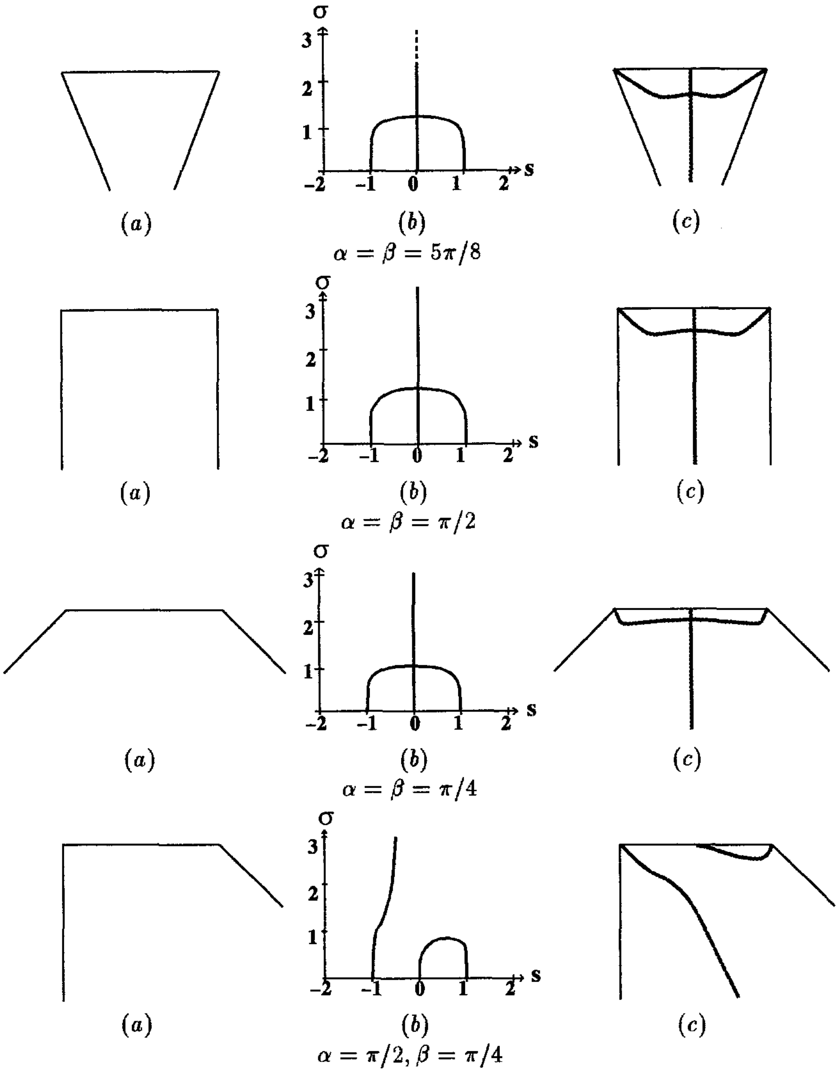


Fig. 1. Evolution of curvature for a few two-corners curves

The surface is smoothed by a convolution with a two-dimensional gaussian kernel of parameter σ.

DET maxima. The DET is defined on a surface $S(x, y)$ as follows:

$$DET = \frac{\partial^2 S}{\partial x^2}\frac{\partial^2 S}{\partial y^2} - \left(\frac{\partial^2 S}{\partial x \partial y}\right)^2$$

It is easy to see that the locations of the DET maxima do not depend on the surface contrast: multiplying S by a constant factor k does not affect the solutions of the equation $\frac{\partial DET}{\partial x} = \frac{\partial DET}{\partial y} = 0$. This property makes the DET an interesting tool in means of shape analysis. For a single corner, the DET presents a single maximum which stays on the angle bisector, which is close to the corner when σ is small, and which goes to infinity as σ tends to infinity as it has been shown in [3].

For two corners, the behavior is the following:

- the DET presents two extrema which are close to the two corners when σ is small;
- as σ increases, but is still small enough for the interaction between the two corners to be negligeable, the two extrema go far away from the corners, following the two angle bisectors;
- when σ reaches a certain value σ_0^D, the two DET maxima merge in a single maxima;
- for each value of $\sigma \geq \sigma_0^D$, the DET presents a single maximum which goes to infinity along the symmetry axis as σ tends to infinity.

Figure 2 plots the DET maxima in the (x, y) plane as σ varies. We know from [3]

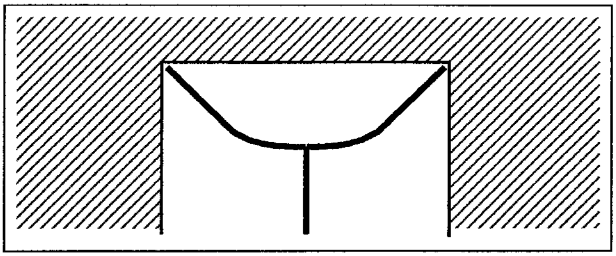


Fig. 2. DET maxima in (x, y) plane

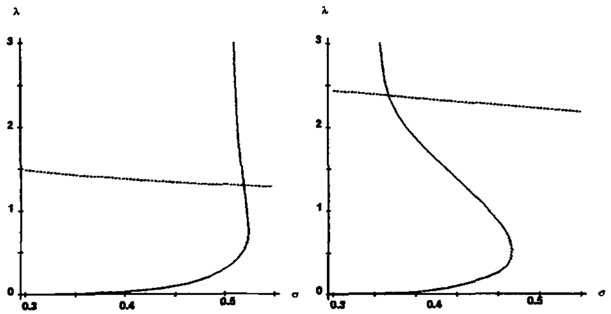
what happends precily around the corners at low σ values, as the corners may be considered independent. We show in [10] that the merging point $(x_0^D, y_0^D, \sigma_0^D)$ can be found as the unique solution of Equ. 1.

$$\begin{cases} x & = 0 \\ y & \leq 0 \\ \dfrac{\partial^2 DET}{\partial x^2} & = 0 \\ \dfrac{\partial DET}{\partial y} & = 0 \end{cases} \tag{1}$$

Gaussian Curvature maxima. The gaussian curvature is defined on a surface $S(x, y)$ as follows:

$$CURV = DET / \left(1 + \frac{\partial S}{\partial x}^2 + \frac{\partial S}{\partial y}^2\right)^{\frac{3}{2}}$$

The curvature extrema behavior is qualitatively the same as the DET extrema behavior. When $\sigma < \sigma_0^C$, the curvature presents two extrema. When $\sigma \geq \sigma_0^C$, the two curvature extrema merge in one extrema. The method we have used to find out the value of σ_0^C is the same as for the DET: we detect the merging point as the point in the y-axis where the second derivative in x of the curvature and first derivative in y of the curvature simultaneously nullify. As the numerator of the gaussian curvature is a power of the expression $1 + (\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2$, the derivatives of the gaussian curvature depend on the surface parameter A. Thus, the location of the curvature maxima depend on the surface contrast. Let us just give plots of the implicit curves defined by $\frac{\partial CURV}{\partial y} = 0$ and $\frac{\partial^2 CURV}{\partial x^2} = 0$ for two arbitray values of A (fig. 3).



$A = 1$ $A = 10$

Fig. 3. The merging point in a $(\sigma, \lambda = -y/\sigma)$ system of coordinates. The plain curve represents the second derivate in x of the curvature and the dotted curve represents the first derivative in y of the curvature.

4 Conclusions

This paper presents an analytical study of the evolution through the scale space of the corners location on planar curves and on elevation surfaces. Corners are defined on planar curves as curvature maxima, and on elevation surfaces as DET maxima and gaussian curvature maxima. The work on planar curves revisits the analysis of Rattarangsi and Chin in [9], and shows that in a shape, a pair of two adjacent and convex corners merge to a single one as scale increase, whatever the values of the corner angles are.

This work is extended to the case of the elevation to a surface of a right-angle END model. Corners are detected as DET maxima or as gaussian curvature maxima. The analysis shows that the behavior of corners on surfaces is, from a qualitative point of view, the same as for the planar curves. We are able to compute the exact location of the merging point in the case of the DET analysis, and we present examples that show that the merging point depends on the surface contrast in the case of the gaussian curvature analysis.

The two differential measures we have used, the DET and the gaussian curvature, present similarities and dissimilarities. The qualitative behavior of the corners is the same in the two cases. But the quantitative results are quite different. In particular, DET maxima measurements are contrast independent, while gaussian maxima measurements are not. This tends to show that the DET measurements may be used for surfaces for which just the planar projection is of interest, and the Gaussian Curvature measurements should be used for surfaces where the nature of elevation is also interesting.

This paper is a contribution for a proposal of a shape formalism, shape in terms of planar curve or shape in picture, directly driven by corners through the scale space.

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