Pulsed neural networks and perceptive grouping

Dominique DEROU and Laurent HERAULT

LETI-(CEA-Technologies Avancées)
DSYS/SCSI-CENG-17 rue des Martyrs
F38054-Grenoble Cedex 9-France
derou@dsys.ceng.cea.fr – herault@dsys.ceng.cea.fr

Abstract. Tracking elementary features and coherently grouping them is an important problem in computer vision and a real challenging feature extraction problem. Perceptual grouping techniques can be applied to some feature tracking problems. Such an approach is presented in this paper. Moreover we show how a perceptual grouping problem can be expressed as a global optimization problem. In order to solve it, we devise an original neural network, called pulsed neural network. The specific application concerned here is particle tracking velocimetry in fluid mechanics.

1 Introduction

The particle tracking velocimetry technique deals with recording on a single image, at n different instances in time, positions of small tracers particles following a fluid flow and illuminated by a sheet of light. It aims to determine each particle velocity vector, made of n different spots. We suggest an approach using perceptual grouping notions, a global optimization formulation and an original neural network. Our algorithm consists of two distinct processing steps:

1. Extraction of potential features from the original image, by using metric constraints imposed by the image acquisition process, and determination of coefficients of mutual consistency and incompatibility between potential features, by use of perceptual grouping notions and physical properties of the phenomena.
2. Extraction of a set of features satisfying each constraint in terms of global consistency, through a global optimization problem and a pulsed neural algorithm.

Particle tracking is one of the simplest and most powerful methods of quantitative visualization. Some reviews on particle tracking velocimetry (PTV) describe the principles and applications of many types of PTV ([1], [5], [2]), and present several drawbacks (high sensitivity to noise, low speed of computation, degraded results). We attack the PTV problem as a perceptual grouping problem. Several papers express some feature grouping problem as combinatorial optimization problems and try to minimize a global cost function including local

Lecture Notes in Computer Science, Vol. 800
Jan-Olof Eklundh (Ed.)
Computer Vision – ECCV '94

constraints ([8], [7], [9], [4], [6]). We present in this paper a new neural approach able to minimize such a cost function while satisfying all the constraints.

2 A combinatorial optimization formulation for the particle tracking problem

2.1 Extraction of potential features

First, points identified as possible particle spots are extracted from the original numerical image. The corresponding points set is used to generate potential features. Each feature represents a potential particle trajectory (a vector). Generation of such potential features uses some *a priori* knowledge related to the experimental acquisition process. At this step, erroneous trajectories can be selected. Our aim is to label each feature as "good" if it corresponds to a real particle trajectory, or "erroneous" otherwise.

2.2 Notion of consistency and incompatibility between features

Our method makes use of perceptual grouping notions from Gestalt theory (similarity and proximity laws) and physical properties of the fluid (viscosity, speed, Reynolds number), so as to define consistency and incompatibility coefficients between any two potential trajectories. A consistency coefficient must be the higher as the feature pair is consistent with the fluid motion in its local environment. The incompatibility coefficient is binary and indicates a strict incompatibility between features. Our goal is to extract and quantify, in both coefficients, the features ability to induce a continuity feeling beyond their physical limits. The consistency coefficient is defined by:

$$\forall j \in V(i),\ q_{ij} = (1 - \frac{\Gamma(\Theta_i, \Theta_j)}{\pi/2}) + (\exp - \frac{d_{ij}^2}{2.\sigma_d^2}) \tag{1}$$

where Θ_i and Θ_j are orientations of features i and j regard to an axis of the image, $V(i)$ is a neighbourhood of the feature i, d_{ij} is the distance between features i and j, σ_d is a fraction of the standard deviation of all the distances over the image and :

$$\Gamma(b,a) = \begin{cases} \mid a-b \mid & \text{if } \mid a-b \mid \leq \frac{\pi}{2} \\ \mid a-b-\pi \mid & \text{if } \frac{\pi}{2} < a-b \leq \pi \\ \mid a-b+\pi \mid & \text{if } -\pi \leq a-b < -\frac{\pi}{2} \end{cases}$$

The incompatibility coefficient is defined by:

$$\bar{c}_{ij} = \max\,(\bar{c}_{ij}^{\,direction}, \bar{c}_{ij}^{\,common\ point}) \tag{2}$$

where: $\bar{c}_{ij}^{\,direction} = \begin{cases} 0 & \text{if } \mathbf{i.j} \geq 0 \\ 1 & \text{if } \mathbf{i.j} < 0 \end{cases}$ $\qquad \bar{c}_{ij}^{\,common\ point} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ share a common point} \\ 0 & \text{otherwise} \end{cases}$

In the final solution, following binary constraints must exist:

$$\forall i,j,\ p_i = p_j = 1 \Longrightarrow \bar{c}_{ij} = 0$$

2.3 A combinatorial optimization statement

A consistency matrix $Q = (q_{ij})_{(i,j)\in<1,N>^2}$, with $q_{ij} \in [0,1]$ and an incompatibility matrix $\bar{C} = (\bar{c}_{ij})_{(i,j)\in<1,N>^2}$, with $\bar{c}_{ij} \in \{0,1\}$ have been generated. At this step, we intend to select all real features corresponding to particle trajectories. The solution of the particle tracking problem is consequently a subset of the potential trajectories, in which all "erroneous trajectories" have been suppressed. Those "erroneous trajectories" are features which display a low consistency with their local environment or which do not satisfy the constraints inherent in the problem. As a consequence, the problem can be expressed as finding a subset of potential features maximizing a quality function representing a global consistency measure of the solution, and satisfying all binary constraints. In other words, if $\mathbf{p} = (p_1, \ldots, p_N)$ is a vector in which $p_i = 1$ if the trajectory i represents a real particle, or $p_i = 0$ if it is an erroneous one, the problem can be expressed by: finding $\mathbf{p}$ such that $\{p_i \mid p_i = 1\}$ is an independent subset of $\bar{C}$ and such that $\mathbf{p}$ maximizes $E(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} q_{ij}\cdot p_i \cdot p_j$. We will devise a neural method to solve this global optimization problem, using a new neural network called pulsed neural network.

3 Pulsed neural network

3.1 Formulation as a maximum independent set problem

Let us define a graph $\bar{G}$, in which each vertex is a potential feature and whose adjacency matrix $A_{\bar{G}} = (\bar{g}_{ij})_{(i,j)\in<1,N>^2}$ is defined by $\bar{g}_{ij} = \bar{c}_{ij} + \overline{B(q_{ij})} \in \{0,1\}$, where $B(q_{ij})$ is the result of binarization of the consistency coefficient. In [3], we demonstrate that, in this case, the particle tracking problem reduces to the search of the largest independent set of graph $\bar{G}$.

Our artificial neural network consists of N recursive neurons which are auto-connected and potentially fully inter-connected with symmetric synaptic weights. A neuron is ascribed to each feature. We denote p_i the binary output of neuron i and u_i its potential. A neuron belonging to the final solution will have its output equal to 1 in the final state vector. The gain function used, for each neuron, is the McCulloch-Pitts one. Initially, all ouputs are set equal to 0 and all inputs are randomly generated with negative values.

Definition 1. Let the following functions defined by:

$$deg : <1, N> \longrightarrow \mathbb{N}$$
$$x \mapsto card\{j \in <1, N> \mid \bar{g}_{xj} = 0\}$$

$$h : \mathbb{R} \rightarrow \{0,1\} \qquad x \mapsto \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases}$$

$$l : \mathbb{R} \times \{0,1\}^N \times \mathbb{N} \rightarrow \{0,1\} \qquad (x, \mathbf{p}, t) \mapsto \begin{cases} 1 & \text{if } x < R(\mathbf{p}, t) \\ 0 & \text{otherwise} \end{cases}$$

where $R(\mathbf{p}, t)$ is the size of the largest independent set found at t.

Theorem 2. *The neural network with feed-back and pulsations, whose evolution equation for each neuron i is:*

$$\frac{du_i}{dt}(t+1) = p_i(t).\left[-1 + h(\sum_{j\in V(i)\setminus\{i\}} \bar{g}_{ij}.p_j(t))\right] +$$
$$(1-p_i(t)).\left[h(\sum_{j\in V(i)\setminus\{i\}} \bar{g}_{ij}.p_j(t)) + \delta(\frac{d\mathbf{u}}{dt},t).l(deg(i),\mathbf{p},t)\right]$$

converges, in asynchronous running mode, between two pulsation phases, towards a feasible solution (proof in [3]). $\delta(\frac{d\mathbf{u}}{dt},t)$ *returns* 0 *while the network has not converged* ($\exists i \mid \frac{du_i}{dt}(t) \neq 0$)*, and returns* 1 *during several complete updatings of the network as soon as the network has converged. This defines a pulsation.*

The two first terms of the evolution equation ensure that the solution satisfies all binary constraints, i.e. that the solution is an independent set of $\bar{G}$. The third term, called pulsation term, enables the network to leave a local minimum and converge towards a new feasible solution. This term is only used during the pulsation phase, after the network has converged. During a pulsation phase, all neurons, whose degree in $\bar{G}$ is smaller than the size of the largest independent set previously found and which are inactivated are excited. If no neuron has a degree higher than the maximum independent set found size, then the solution can not be improved (stopping criteria). The final solution is the largest solution among the feasible solutions proposed by the network. We notice that this network is powerful to solve a maximum clique problem.

3.2 Formulation as a global optimization problem with constraints

The problem is presented in section 2.3. The evolution equation is defined so that to maximize $E(\mathbf{p})$, while satisfying all binary constraints. We propose a synchronous by bloc running mode. A neuron, whose state is inactivated ($p_i = 0$) just before its updating, must be shifted to $p_i = 1$ if it contributes towards a quality function increase. Furthermore, so as to evolve in the space of independent set of the incompatibility graph, we impose that, if an inactivated neuron i such that $p_i(t) = 0$ is updated to $p_i(t+1) = 1$, then all the activated neurons j incompatible with it ($\bar{c}_{ij} = 1$ and $p_j(t) = 1$) are updated to $p_j(t+1) = 0$.

$$\begin{array}{ll} \text{Before the transformations} & p_i(t) = 0 \\ \text{After the transformations} & \begin{cases} p_i(t+1) = 1 \\ \forall j \mid (p_j(t) = 1 \text{ and } \bar{c}_{ij} = 1);\ p_j(t+1) = 0 \end{cases} \end{array} \tag{3}$$

The expression of the quality function variation associated to the previous transformations, when neuron i is considered at time $(t+1)$, is:

$$\Delta E_i(t+1) = -2 \sum_{\{s\mid\bar{c}_{is}=1\}} \sum_{j\neq s\ ,\ j\neq i} q_{js}.p_j(t).p_s(t)$$
$$+2 \sum_{j\neq s\ ,\ j\neq i} q_{ij}.p_j(t) - \sum_{\{s\mid\bar{c}_{is}=1\}} p_s(t) + 1 \tag{4}$$

Definition 3. Let $k(\mathbf{p}, i)$ be a function that randomly selects, among all components p_i of $\mathbf{p}$ such that $p_i(t) = 1$, x components and returns ∞ if i belongs to this set of components, and 0 otherwise: $k(\mathbf{p}, i) = \infty$ and so $p_i(t+1) = 0$.

Theorem 4. *The neural network with feed-back, pulsations and Potts neurons, whose evolution equation of neuron i is:*

$$\begin{cases} \frac{du_i}{dt}(t+1) = (1 - p_i(t)).max(0, \Delta E_i(t+1)) - p_i(t).\delta(\frac{d\mathbf{u}}{dt}, t).k(\mathbf{p}, i) \\ \forall j \neq i \ , \ \frac{du_j}{dt}(t+1) = -\bar{c}_{ij}.(1 - p_i(t)).max(0, \Delta E_i(t+1)) \end{cases} \tag{5}$$

converges, between two pulsation phases, towards a feasible solution.(cf. [3]).

The network, conducted by the first term of the evolution equation, converges to a feasible solution. The second term enables it to be pulsed in another starting point and find another solution. The solution maximizing $E(\mathbf{p})$ is finally chosen.

4 Experimental results and discussion

The algorithm described in the previous section has been applied to many images of fluid mechanics. Figs. 1 and 2 respectively show an original image and the final result proposed with the second algorithm. The computationnal time required for the total processing is, in this example, 2s870ms on a standard SUN SPARC 10. Our algorithm is far quicker than all classical velocimetry methods. The results proposed testify to the great visual quality of the algorithm. Regardless of the image tested, the algorithm recognizes at least 95% of the particles seeded in the fluid.

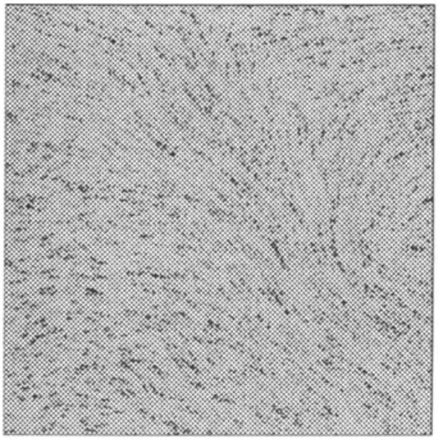

Fig. 1. A particle tracking image of 800*800 pixels.

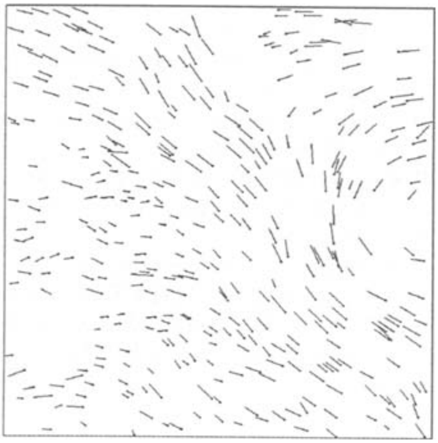

Fig. 2. The result of applying our neural algorithm

In this paper, we have proposed a new paradigm for the feature grouping problem, with special emphasis on the problem of particle tracking. First and foremost, we suggest a mathematical encoding of the problem, which takes into account metric constraints specific to the problem, perceptual properties of the image and physics properties of the phenomena. Second, we propos a new statement of the particle tracking problem as a global optimization problem. Endly, in order to solve this combinatorial optimization problem, we devise original neural networks, named pulsed neural networks. The advantages of these new neural networks are:

- They need no coefficient. Accordingly it has a completely black-box behaviour from a user points of view.
- When the network has converged ($\forall i,\ \frac{du_i}{dt} = 0$), all constraints are necessarily satisfied.
- The iterations number necessary to converge is much smaller than most of other methods.

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