Handling uncertainty, context, vague predicates, and partial inconsistency in possibilistic logic

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Abstract. This short paper intends to provide an introduction to possibilistic logic, a logic with weighted formulas, to its various capabilities and to its potential applications. Possibilistic logic, initially proposed in [11], see also Léa Sombé [26] for an introduction, can be viewed as an important fragment of Zadeh[32]'s possibility distribution-based theory of approximate reasoning, put in a logical form. Possibilistic logic also relies on an ordering relation reflecting the relative certainty of the formulas in the knowledge base. As it will be seen, its semantics is based on a possibility distribution which is nothing but a convenient encoding of a preference relation a la Shoham[29], between interpretations. This kind of semantics should not be confused with the similarity relation-based semantics recently proposed by Ruspini[28] for fuzzy logics which rather extends the idea of interchangeable interpretations in a coarsened universe, e.g. Fariñas del Cerro and Orlowska[17], and which corresponds to another issue.

1 Necessity-weighted formulas and their semantics

Let us first recall that a necessity measure, denoted by N, is a function from a logical (propositional or first-order) language $\mathcal{L}$ to $[0,1]$, such that $N(\top) = 1$ and $N(\bot) = 0$, where $\top$ (resp. $\bot$) denotes any tautology (resp. any contradiction), and obeying the following axiom

$$\forall p, \forall q, N(p \wedge q) = min(N(p), N(q)) \tag{1}$$

As a consequence we have $min(N(p), N(\neg p)) = 0$. However we only have $N(p \vee q) \geq max(N(p), N(q))$, since e.g. for $q = \neg p$, $N(p \vee \neg p) = N(\top) = 1$ while in case of total ignorance we may have $N(p) = N(\neg p) = 0$. A necessity measure N is the dual of a possibility measure Π, such that $\forall p, N(p) = 1 - \Pi(\neg p)$, where Π obeys the characteristic axiom $\forall p, \forall q, \Pi(p \vee q) = max(\Pi(p), \Pi(q))$ ([31]).

A possibilistic logic formula is a pair (p, α) where p is a classical first-order or propositional logic formula and α a number belonging to the semi-open real interval $(0,1]$, which estimates to what extent it is certain that p is true considering the available information we have at our disposal. More formally (p, α) is a syntactic way to code the semantic constraint $N(p) \geq \alpha$.

As suggested by the operator min in (1), and since necessity measures, as well as possibility measures, are the perfect numerical counterparts of qualitative relations aiming at modelling "at least as certain" and "at least as possible" ([13]), the numbers used for weighting the formulas have an ordinal flavor. It departs from other uncertainty-handling approaches to automated reasoning, e.g. Baldwin[1], making use of probability bounds, and thus of sum and product operations.

N.B. : In this paper, for the sake of simplicity, we only consider lower bounds of necessity measures for weighting the formulas. We can also deal with lower bounds of possibility measures, i.e. $\Pi(p) \geq \alpha$, which corresponds to a very weak form of information; see [12, 23].

Let $M(p)$ be the set of models of p. The semantics of (p, α) is represented by the fuzzy set of models $M(p, \alpha)$ defined by [5] where ω denotes an interpretation

$$\mu_{M(p,\alpha)}(\omega) = 1 \text{ if } \omega \in M(p); \quad \mu_{M(p,\alpha)}(\omega) = 1 - \alpha \text{ if } \omega \notin M(p) \tag{2}$$

In other words, the interpretations compatible with (p, α) are restricted by the above possibility distribution. The ones in $M(p)$ are considered as fully possible while the ones outside are all the more possible as α is smaller, i.e. the piece of knowledge is less certain. Note also that here we use the least specific possibility distribution $\pi = \mu_{M(p,\alpha)}$, i.e. the one with the greatest possibility degrees compatible with $\Pi(\neg p) \leq 1 - \alpha \Leftrightarrow N(p) \geq \alpha$, where the expression of a possibility measure Π in terms of a possibility distribution π is given by,

$$\forall p, \Pi(p) = sup\{\pi(\omega), \omega \in M(p)\}. \tag{3}$$

(as a consequence of the characteristic axiom of possibility measures). Indeed, any possibility distribution π satisfying the constraint $N(p) \geq \alpha$ is such that $\forall \omega, \pi(\omega) \leq \mu_{M(p,\alpha)}(\omega)$.

In case of several pieces of knowledge $(p_i, \alpha_i), i = 1, \ldots, n$, forming a knowledge base $\mathcal{K}$, in agreement with the minimal specificity principle[14], we associate the following possibility distribution, built by performing the largest conjunction operation (which is also the only idempotent one) on the membership functions $\mu_{M(p_i,\alpha_i)}$, namely,

$$\pi_{\mathcal{K}}(\omega) = min_{i=1,\ldots,n}\mu_{M(p_i,\alpha_i)}(\omega). \tag{4}$$

It can be checked that the necessity measure $N_{\mathcal{K}}$ induced from $\pi_{\mathcal{K}}$ by $N_{\mathcal{K}}(q) = 1 - sup\{\pi_{\mathcal{K}}(\omega), \omega \models \neg q\}$, where $w \models q$ means $\omega \in M(q)$, is the smallest necessity measure satisfying the constraints $N(p_i) \geq \alpha_i$. This is nothing but another formulation of the minimal specificity principle.

Let us take an example:

$$\mathcal{K} = \{(\neg p \vee q, 1), (\neg p \vee r, 0.7), (\neg q \vee r, 0.4), (p, 0.5), (q, 0.8)\}.$$

This induces the constraints,

$$\begin{array}{lll} \pi(\omega) \leq \mu_{M(\neg p \vee q,1)}(\omega) & \forall \omega \models p \wedge \neg q, \pi(\omega) = 0 & sup\{\pi(\omega), \omega \models p \wedge \neg q\} = 0 \\ \pi(\omega) \leq \mu_{M(\neg p \vee r,0.7)}(\omega) & \forall \omega \models p \wedge \neg r, \pi(\omega) \leq 0.3 & sup\{\pi(\omega), \omega \models p \wedge \neg r\} \leq 0.3 \\ \pi(\omega) \leq \mu_{M(\neg q \vee r,0.4)}(\omega) \Leftrightarrow & \forall \omega \models q \wedge \neg r, \pi(\omega) \leq 0.6 \Leftrightarrow & sup\{\pi(\omega), \omega \models q \wedge \neg r\} \leq 0.6 \\ \pi(\omega) \leq \mu_{M(p,0.5)}(\omega) & \forall \omega \models \neg p, \pi(\omega) \leq 0.5 & sup\{\pi(\omega), \omega \models \neg p\} \leq 0.5 \\ \pi(\omega) \leq \mu_{M(q,0.8)}(\omega) & \forall \omega \models \neg q, \pi(\omega) \leq 0.2 & sup\{\pi(\omega), \omega \models \neg q\} \leq 0.2 \end{array}$$

and the corresponding possibility distribution $\pi_{\mathcal{K}}$, defined by (3), is pictured on Fig 1.

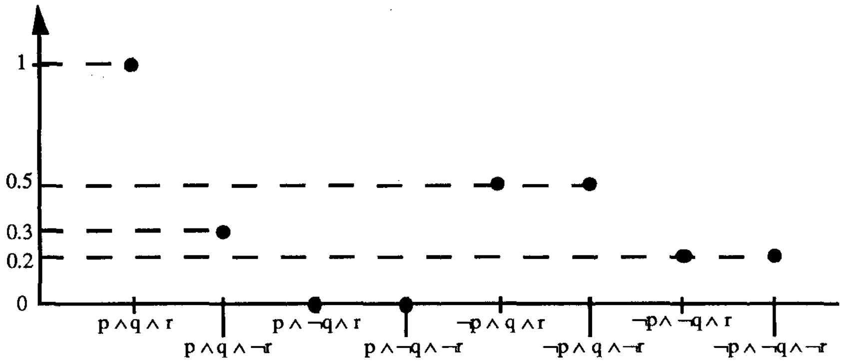


Fig. 1

The possibility distribution of Fig. 1 is normalized, i.e. $\exists \omega, \pi_{\mathcal{K}}(\omega) = 1$; in the general case, there may exist several ω such that $\pi_{\mathcal{K}}(\omega) = 1$. This means that $\mathcal{K}$ is fully consistent since there is at least one interpretation in agreement with $\mathcal{K}$ which is completely possible. More generally we define the degree of inconsistency of $\mathcal{K}$ by,

$$Inc(\mathcal{K}) = 1 - sup_{\omega} \pi_{\mathcal{K}}(\omega) \tag{5}$$

It can also be established that $Inc(\mathcal{K}) > 0 \Leftrightarrow \mathcal{K}*$ is inconsistent, where $\mathcal{K}*$ is the classical knowledge base obtained from $\mathcal{K}$ by deleting the weights.

A possibility distribution such as the one pictured in Fig. 1 is a way of encoding a preference ordering among interpretations, i.e. the kind of relation used by Shoham[29] for providing non-monotonic logics with a semantics. Indeed, it has been shown[15] that the preferential entailment $\models_\pi$, where π is short for $\pi_{\mathcal{K}}$, defined by,

$$p \models_\pi q \Leftrightarrow (\exists \omega, \omega \models_\pi p \text{ and } \forall \omega, \omega \models_\pi p \Rightarrow \omega \models q),$$

where

$$\omega \models_\pi p \Leftrightarrow \omega \models p, \Pi(p) > 0 \text{ and } \nexists \omega', \omega' \models p \text{ and } \pi(\omega) < \pi(\omega'),$$

is in complete agreement with non-monotonic consequence relations obeying the axiomatics of system P proposed by Kraus *et al.*[21]. It can be also shown that it is closely related to the notion of conditional possibility since we have the equivalences,

$$p \models_\pi q \Leftrightarrow \Pi(q|p) > \Pi(\neg q|p) \ \text{ with } \Pi(q|p) = \begin{cases} 1 & \text{if } \Pi(p) = \Pi(p \wedge q) \\ \Pi(p \wedge q) & \text{if } \Pi(p) > \Pi(p \wedge q) \end{cases}$$
$$\Leftrightarrow N(q|p) > 0 \qquad\qquad \text{with } N(q|p) = 1 - \Pi(\neg q|p)$$

$p \models_\pi q$ means that preferred models of p (as induced by π) are all models of q.

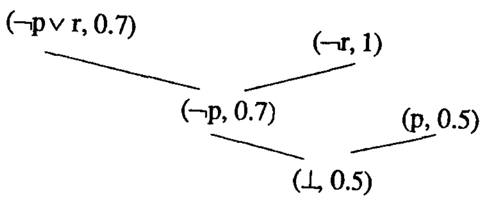
2 Resolution Principle and Combination/Projection Principle

In this section we suppose that that weighted formulas are weighted clauses; this can be done without loss of expressivity; this is mainly due to the conjunctive compositionality of necessity measure[7]. The following deduction rules[11, 12] have been proved sound and complete for refutation with respect to the above semantics; see [5] for the case of consistent knowledge bases

$$\text{resolution rule: } \frac{(\neg p \vee q, \alpha) \ \ (p \vee r, \beta)}{(q \vee r, min(\alpha, \beta))}$$

$$\text{particularization: } \frac{(\forall x \ p(x), \alpha)}{(p(a), \alpha)} \text{ (as well as more general substitutions)}$$

If we want to compute the maximal certainty degree which can be attached to a formula according to the constraints expressed by a knowledge base $\mathcal{K}$, for instance r in the above example, we add to $\mathcal{K}$ the clause(s) obtained by refuting the proposition to evaluate with a necessity degree equal to 1, here we add $(\neg r, 1)$. Then it can be shown that any lower bound obtained on $\perp$, by resolution, is a lower bound of the necessity of the proposition to evaluate. See [4] for an ordered search method which guarantees the obtaining of the greatest derivable lower bound on $\perp$. It can be shown[5, 23], that this greatest derivable lower bound on $\perp$ is nothing but $Inc(\mathcal{K} \cup \{(\neg r, 1)\})$ where r is the proposition to establish. In the example, we have the following derivation,



i.e. $N(r) \geq 0.5$ and indeed it can be checked that using the possibility distribution pictured in Fig. 1, we have $\Pi(\neg r) = sup\{\pi(\omega), \omega \models \neg r\} = 0.5$, in fact $\Pi(\neg r) \leq 0.5$ since π is the greatest possibility distribution compatible with $\mathcal{K}$, and then $N(r) = 1 - \Pi(\neg r) \geq 0.5$.

It is has been pointed out in [7] that this procedure is in agreement with Zadeh's approach to approximate reasoning based on combination/projection of possibility distributions. This can be also checked on our example. Let us suppose that p means "$X \geq a$", q means "$X \geq b$" with $b < a$ and r means "$Y \in [c,d]$" where X and Y are two real-valued variables under consideration. Then it can be seen that the 8 interpretations correspond to,

$S(p \wedge q \wedge r) = S(p) \cap S(q) \cap S(r) = [a, +\infty) \times [c,d]$;
$S(p \wedge q \wedge \neg r) = [a, +\infty) \times \overline{[c,d]}$;
$S(p \wedge \neg q \wedge r) = \emptyset$; $S(p \wedge \neg q \wedge \neg r) = \emptyset$;
$S(\neg p \wedge q \wedge r) = [b,a) \times [c,d]$; $S(\neg p \wedge q \wedge \neg r) = [b,a) \times \overline{[c,d]}$;
$S(\neg p \wedge \neg q \wedge r) = [0,b) \times [c,d]$
$S(\neg p \wedge \neg q \wedge \neg r) = [0,b) \times \overline{[c,d]}$.

where $S(\omega)$ denotes the set of values of the pair (X,Y) corresponding to the interpretation ω. Then $\mathcal{K}$ is equivalent to a set of possibility distributions, each one corresponding to a piece of knowledge, namely,

$(\neg p \vee q, 1) : \{\pi^1_{X,Y}(u,v) = 1, \ \forall (u,v)$ because $S(p \wedge \neg q) = \emptyset$, i.e. $\neg p \vee q$ is a tautology

$$(\neg p \vee q, 0.7) : \pi^2_{X,Y}(u,v) \begin{cases} \leq 0.3, \forall (u,v) \in [b+\infty) \times \overline{[c,d]} \\ = 1 \quad \text{otherwise;} \end{cases}$$

$$(\neg q \vee r, 0.4) : \pi^3_{X,Y}(u,v) \begin{cases} \leq 0.6, \forall (u,v) \in [a,+\infty) \times \overline{[c,d]} \\ = 1 \quad \text{otherwise;} \end{cases}$$

$$(p, 0.5) : \pi^4_{X,Y}(u,v) \begin{cases} = 1 \quad \text{if } u \in [a,+\infty) \\ \leq 0.5 \text{ otherwise;} \end{cases}$$

$$(q, 0.8) : \pi^5_{X,Y}(u,v) \begin{cases} = 1 \quad \text{if } u \in [b,+\infty) \\ \leq 0.2 \text{ otherwise.} \end{cases}$$

Then it can be checked that,

$\pi_Y(v) = sup_u min(\pi^1_{X,Y}(u,v), \pi^2_{X,Y}(u,v), \pi^3_{X,Y}(u,v), \pi^4_{X,Y}(u,v), \pi^5_{X,Y}(u,v)) =_{def} sup_u \pi^*_{X,Y}(u,v)$

with

$$\pi^*_{X,Y}(u,v) \begin{cases} 1 \quad \text{if } (u,v) \in [a,+\infty) \times [c,d] \\ \leq 0.5 \text{ if } u \in [b,a) \end{cases}$$

$$\pi^*_{X,Y}(u,v) \begin{cases} \leq 0.3 \text{ if } (u,v) \in \overline{[a,+\infty) \times \overline{[c,d]}} \\ \leq 0.2 \text{ if } u \in \overline{[b,+\infty)} \end{cases}$$

Then $\pi_Y(v) = 1$ if $v \in [c,d]$; $\pi_Y(v) \leq 0.5$ if $v \notin [c,d]$ and finally $\Pi(S(\neg r)) = \Pi(\overline{[c,d]}) = sup_{v \in \overline{[c,d]}} \pi_Y(v) \leq 0.5$ and thus $N(S(r)) \geq 0.5$, i.e. we recover the result already obtained by refutation.

N.B.: Semantic evaluation methods, extending a procedure by Davis and Putnam, are also available in possibilistic logic[22].

3 Labelled Formulas and the Handling of Vague Predicates

As pointed out in [5], the weighted clause $(\neg p \vee q, \alpha)$ is semantically equivalent to the weighted clause $(q, min(\alpha, v(p)))$ where $v(p)$ is the truth value of p, i.e. $v(p) = 1$ if p is true and $v(p) = 0$ if p is false. Indeed, for any uncertain proposition (p, α) we can write $\mu_{M(p,\alpha)}(\omega)$ under the form $max(v_\omega(p), 1 - \alpha)$, where $v_\omega(p)$ is the truth-value assigned to p by interpretation ω. Then obviously:

$$\begin{aligned}\forall \omega, \mu_{M(\neg p \vee q, \alpha)}(\omega) &= max(v_\omega(\neg p \vee q), 1 - \alpha)\\ &= max(1 - v_\omega(p), v_\omega(q), 1 - \alpha)\\ &= max(v_\omega(q), 1 - min(v_\omega(p), \alpha))\\ &= \mu_{M(q, min(v_\omega(p), \alpha))}(\omega).\end{aligned}$$

This remark is very useful for hypothetical reasoning, since by "transferring" an atom from a clause to the weight part of the formula we are introducing explicit assumptions. Indeed changing $(\neg p \vee q, \alpha)$ into $(q, min(v(p), \alpha))$ leads to state the piece of knowledge under the form "q is certain at the degree α, *provided that* p is true". Then the weight is no more just a degree but in fact a label which expresses the context in which the piece of knowledge is more or less certain.

More generally, the weight or label can be a function of logical (universally quantified) variables involved in the clause. Thus a possibilistic formula of the form $(c(x), \mu_P(x))$ expresses that for any x, one is certain that the clause $c(x)$ is true with a necessity degree greater or equal to $\mu_P(x)$ where μ_P is the membership function of a predicate. This predicate can be an ordinary predicate or a fuzzy predicate. In this latter case the possibilistic formula means "the larger $\mu_P(x)$, the more certain $c(x)$". Note that in any case the clause remains a classical clause, while the fuzzy predicate appears in the weight. For instance the rule "the younger the person, the more certain he/she is single" will be represented by $(single(x), \mu_{young}(age(x))$ where young has a membership function which has the value 1 until the legal age for marriage and then decreases. With such a clause, once instantiated, with, say $x =$ John, we need to know the age of John for computing the certainty degree. If we only have a fuzzy knowledge about John's age, modelled by a possibility distribution π, we have to change $\mu_{young}(age(John))$ which is not known, by $N_\pi(young) = inf_u\ max(\mu_{young}(u), 1 - \pi(u))$, i.e. the certainty that John is indeed definitely young given the available information about his age.

Fuzzy pieces of knowledge like "John is young" can be also modelled in possibilistic logic. Let us assume for "young" a membership function as the one described above. Then the piece of knowledge is equivalent to the family of weighted formulas making use of the non-fuzzy predicates "$\leq (age(x), a)$" expressing that $age(x) \leq a$, where the logical constant "a" ranges on the domain of attribute 'age'. The degree of necessity of "$\leq (age(John), a)$" given that "John is young" is given by,

$$N_{young}(\leq (age(John), a)) = inf_{a<u}\ 1 - \mu_{young}(u)$$

which leads to the family of possibilistic formulas,

$$(\leq (age(John), a), N_{young}(\leq (age(John), a))).$$

Similarly "John is not young" will be represented by,

$$(> (age(John), a), N_{not\_young}(> (age(John), a))).$$

Note that the resolution of these two weighted formulas leads to $(\perp, sup_a`min(N_{young}(\leq (age(John), a)), N_{not\_young}(> (age(John), a)))$ which is equal to 1/2 if μ_{young} is a continuous membership function and would be equal to 1 if 'young' were modelled in a crisp way by an interval. This is natural since the two pieces of knowledge we start with are fully contradictory only if we have a crisp understanding of the idea of "young".

Using a similar (but slightly different) interpretation of fuzzy predicates, the resolution rule has been extended in [12], to the case of weighted fuzzy propositions (which thus no longer belong to a Boolean algebra). In that case it is possible to explicitly deal with clauses like $(\neg young \vee single, \alpha)$.

4 Coping with Inconsistency

A nice feature of possibilistic logic is its capacity to cope with a partially inconsistent knowledge base $\mathcal{K}$ such that $Inc(\mathcal{K}) > 0$. Roughly speaking, the conclusions which can be obtained with a degree of uncertainty strictly higher than $Inc(\mathcal{K})$ are still meaningful since for sure only a consistent subpart of $\mathcal{K}$ (containing the most certain pieces of knowledge) is used for deducing them. Indeed in any inconsistent sub-base of $\mathcal{K}$ there is (at least) a clause with a weight less or equal to Inc(K).

For instance, let us consider the knowledge base $\mathcal{K}$ previously introduced to which we add the clause $(\neg p \vee \neg q, 0.2)$; let $\mathcal{K}' = \mathcal{K} \cup \{(\neg p \vee \neg q, 0.2)\}$. Clearly we have $\pi_{\mathcal{K}'}(p \wedge q \wedge r) = 0.8$ now (instead of $\pi_{\mathcal{K}}(p \wedge q \wedge r) = 1$ in Fig. 1), and $Inc(\mathcal{K}') = 0.2$. A minimal inconsistent sub-base of $\mathcal{K}'$ is $\{(\neg p \vee \neg q, 0.2), (p, 0.5), (q, 0.8)\}$. From $\mathcal{K}'$ we can still deduce $(r, 0.5)$ or $(p, 0.5)$ using only consistent parts of $\mathcal{K}'$ since $0.5 > Inc(\mathcal{K}')$. Suppose now that we add $(\neg p \vee \neg q, 0.6)$ instead of $(\neg p \vee \neg q, 0.2)$. Let $\mathcal{K}'' = \mathcal{K} \cup \{(\neg p \vee \neg q, 0.6)\}$. Then $Inc(\mathcal{K}'') = 0.5$, and now we can deduce $(\neg p, 0.6)$ by refutation from $\mathcal{K}''$ using the consistent part of $\mathcal{K}'' \cup \{(\neg p \vee \neg q, 0.6), (q, 0.8)\}$, since $0.6 > Inc(\mathcal{K}'')$ while $(p, 0.5)$ is no longer considered as an allowed deduction since $0.5 = Inc(\mathcal{K}'')$. Thus a non-monotonic reasoning process is at work in possibilistic logic when partial inconsistency is introduced in the knowledge base. In Lang *et al.*[23] an inconsistency-tolerant semantics is proposed, adding an "absurd interpretation" on which the possibility distribution attached to the knowledge base is normalized. Using this semantics the soundness and completeness results for refutation that we have in the consistent case can still be shown to hold. Moreover it has been established in [15] that we have $N_{\mathcal{K}}(q|p) > 0$ if and only if it is possible to deduce (q, β) from $\mathcal{K} \cup \{(p, 1)\}$ with $\beta > Inc(\mathcal{K} \cup \{(p, 1)\})$, where $N_{\mathcal{K}}$ is the necessity measure defined from $\pi_{\mathcal{K}}$. If we define $p \mid\!\sim q$ by $N_{\mathcal{K}}(q|p) > 0$, then $\mid\!\sim$ is a non-monotonic consequence relation [15]; this illustrates the close relation that exists between non-monotonic reasoning and belief revision [27], in the possibilistic framework. The reader is referred to [16] for a detailed analysis of belief revision in possibility theory. Besides, the problem of recovering consistency in a partially inconsistent knowledge base $\mathcal{K}$ by building maximal consistent sub-bases (obtained by deleting suitable pieces of knowledge in $\mathcal{K}$) is discussed in [9]. Note that such a syntactic approach is not necessarily equivalent to a treatment based on a semantic representation such as $\pi_{\mathcal{K}}$, since syntactically distinct knowledge bases $\mathcal{K}$ and $\mathcal{K}'$ may be such that $\pi_{\mathcal{K}} = \pi_{\mathcal{K}'}$.

Lastly, another way of dealing with inconsistency might be to allow for paraconsistent pieces of knowledge. Roughly speaking, the idea of paraconsistency, first introduced by da Costa[3], is to say that we have a paraconsistent knowledge about p if we both want to state p and to state $\neg p$. It corresponds to the situation where we have conflicting information about p. In a paraconsistent logic we do no want to have every formula q deducible as soon as the knowledge base contains p and $\neg p$ (as it is the case in classical logic). The idea of paraconsistency is "local" by contrast with the usual view of inconsistency which considers the knowledge base in a global way. In possibilistic logic it would correspond to have p in $\mathcal{K}$ with both $N(p) \geq \alpha > 0$ and $N(\neg p) \geq \beta > 0$. The situation may be perhaps better understood if we consider Zadeh's combination/projection point of view first. Indeed let us suppose that, for some variable X (representing the value of some attribute), our knowledge is represented by a *non-normalized* possibility distribution π_X. Then $\forall A \subseteq domain(X)$, we have $min(N_X(A), N_X(\overline{A})) > 0$ where the necessity measure N_X is based on π_X. When combining this piece of knowledge π_X with other possibility distributions π^i, it can be easily seen that the resulting possibility distribution π will be such that its height $h(\pi) =_{def} sup_u\ \pi(u) = h(\pi_X)$ iff $h(\pi_i) \geq h(\pi_X)$, $\forall i$. In other words, *if used* in a reasoning process with other non-paraconsistent pieces of knowledge, π_X will affect the result by "denormalizing" the resulting possibility distribution, thus leading to a paraconsistent conclusion. This could be handled in a syntactic way in possibilistic logic as suggested by the following simple example of the modus ponens,

$$\frac{\begin{array}{l} N(\neg p \vee q) \geq \alpha > 0 \\ N(p) \geq \beta > 0;\ \ N(\neg p) \geq \gamma > 0 \end{array}}{N(q) \geq min(\beta, max(\alpha, \gamma))\ \ ; N(\neg q) \geq min(\beta, \gamma).}$$

Indeed, we have $(\neg p \vee q) \wedge p \models q$, so $N(q) \geq N((\neg p \vee q) \wedge p) = min(N(\neg p \vee q), N(p)) \geq min(\alpha, \beta)$, and also $p \wedge \neg p \models q$, so $N(q) \geq min(N(p), N(\neg p)) \geq min(\beta, \gamma)$. Then $N(q) \geq max(min(\alpha, \beta), min(\beta, \gamma))$. The situation is pictured in Fig. 2

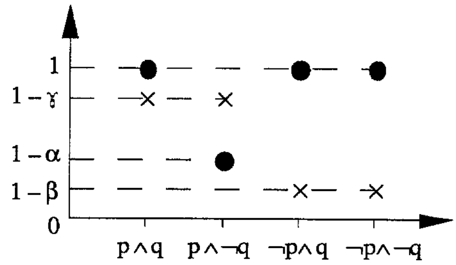


Fig. 2

It can be checked that $min(N(p), N(\neg p)) = min(N(q), N(\neg q)) \geq min(\beta, \gamma) > 0$, which expresses that the degree of paraconsistency is propagated to the conclusion. If $\gamma = 0$, a particular case of the resolution principle is recovered. This suggests the following way of dealing with paraconsistent knowledge: i) keep the paraconsistent pairs $N(p) \geq \alpha, N(\neg p) \geq \beta$ (with degree of paraconsistency $min(\alpha, \beta)$) separate from the remaining part of the knowledge base which is supposed to be consistent; ii) use the paraconsistent knowledge only, when it is impossible to produce a strictly positive lower bound on the necessity degree of a proposition of interest by using the consistent part of the knowledge base only. Then any conclusion which will be produced will have a degree of paraconsistency equal to the maximum of the degrees of paraconsistency of the paraconsistent pairs involved in the production of this conclusion. The idea to add paraconsistent knowledge only when necessary, to the consistent part of the knowledge base is a bit similar to the building of maximal consistent sub-bases[9]. The development of these ideas is left for further research.

5 Concluding Remarks: Potential Applications

First steps towards possibilistic logic programming can be found in [8]. The use of possibilistic logic as a programming language is all the more of interest that min-max discrete optimization problems (and more generally systems of possibly incompatible, prioritized constraints) can be expressed (and then solved) in possibilistic logic[23]. Applications to hypothetical reasoning for diagnosis purposes or for finding "optimal" maximal consistent sub-bases of an inconsistent possibilistic knowledge base are discussed in [6, 10] where possibilistic Assumption-based Truth Maintenance Systems are developed. Other developments of ideas very close to possibilistic logic can be found in Froidevaux and Grossetête[18] and in Chatalic and Froidevaux[2]. See also Jackson[20] for the computation of possibilistic prime implicants and their use in abduction. Besides these, Larsen and Nonfjall[25], Yager and Larsen[30] have used possibilistic logic in validation of knowledge bases.

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