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Abstract. This paper presents a theorem prover for Reiter's default logic, one of the most studied nonmonotonic logics. Our theorem prover is based on a decomposition of default logic into two main elements: we describe an extension of the resolution principle, that handles the "monotonic" aspect of the defaults, and we provide a generalization of Reiter's and Levy's algorithms for the computation of hitting sets, that takes care of the nonmonotonic part of default logic. Lastly, we describe how these two components can be separately modified in order to obtain theorem provers for other variants of default logic, notably prioritized default logic.

1 Introduction

Default reasoning occurs whenever an agent must complete the certain information he has about a domain, with some knowledge which is plausible but not infallible. Pieces of such knowledge, also called pieces of default knowledge or *defaults* for short, enable the agent to draw reliable conclusions, which may have to be retracted in presence of new information.

In many logical approaches to default reasoning, the certain information is completed with defaults that are *coherent* with one another, to generate an *extension* of the certain information. As there are, usually, several possible combinations of defaults that are coherent with one another, several extensions can be built. This notion of coherence can be the consistency in the sense of classical logic [23, 10] or of some modal logics [30, 31]. Other notions of coherence have been shown [16, 9] to underly Reiter's default logic [24] and its variants.

In order to restrict the number of extensions, it is often necessary to use priorities among the defaults, like the so-called specificity principle, widely used in taxonomy. Other kinds of priorities, studied in the context of default reasoning, include ordering relations among the defaults, representing their relative reliabilities [4, 13, 11, 6], or some notions implicitly embedded in fixed-point constructs, as it is the case in Reiter's default logic [25, 5, 8].

A growing number of studies deal with algorithmic problems related to the computation of extensions. Some of these studies are based on TMS like algorithms [14, 13]. Other methods rely on more *ad hoc* approaches [22, 28, 29, 12, 26]. In this paper, following an approach by Levy [16], we present two methods that separately handle the notion of coherence between the defaults, and that of priorities among them.

More precisely, we propose a generalization of the resolution principle to defaults. That is, we define so-called "extended clauses", that are some kind of clausal representations of defaults, and we then define some new resolution rules for these extended clauses. We can then check the coherence of a set of defaults using these resolution rules, by trying to generate an empty clause.

We then propose a generalization of Levy's [17] algorithm for the computation of sets of defaults that generate extensions. This algorithm works by "eliminating", from an initial set of defaults, as many defaults as possible in order to restore some coherence of this set. The particularity of our algorithm is that it is not specially designed for default logic. It can handle a wide class of priorities among pieces of default knowledge.

In the next section, we present Reiter's original definition of default logic, and we recall the characterization that is used in our theorem prover. In Sect. 3, we describe our extended resolution principle, together with the new type of clauses that we introduce. Sect. 4 presents our elimination algorithm, and Sect. 5 describe some modifications that can be done to these two methods in order to obtain other variants of default logic.

2 Default logic

A *default theory* is a pair (W, D), where W is a set of closed formulas of first order predicate calculus, representing the certain information, and D is a set of *defaults*, of the.form $\frac{f:g}{h}$, where f, g, h are closed formulas of the language of first order predicate calculus. The default $\frac{f:g}{h}$ is intended to mean: "if f is true, and if nothing proves that g is false, then conclude h". We suppose, in the sequel, that W is consistent. Reiter [24] defines sets of theorems that are consequences of a default theory:

Definition 1 ([24]). *A set of formulas E is an* extension *of a default theory (W, D) iff $E = \Gamma(E)$, where $\Gamma(E)$ is the smallest set that is deductively closed, contains W, and such that for all $\frac{f:g}{h} \in D$, if $f \in \Gamma(E)$ and $\neg g \notin E$, then $h \in \Gamma(E)$.*

In [19], we give another characterization of the extensions of a default theory, based on the following definitions:

Definition 2 ([19]). *Let (W, D) be a closed default theory. The set of formulas* generated *by a subset U of D, denoted by $\mathrm{Th}^{\mathrm{def}}(W \cup U)$, is the smallest set of formulas that contains W, is deductively closed (in the sense of predicate calculus), and such that for all $\frac{f:g}{h} \in U$, if $f \in \mathrm{Th}^{\mathrm{def}}(W \cup U)$ then $h \in \mathrm{Th}^{\mathrm{def}}(W \cup U)$. We will denote by Th the deduction operator of the predicate calculus.*

Definition 3 ([19]). *A* conflict *of a default theory (W, D) is a subset C of D minimal that contains a default $\frac{f:g}{h}$ such that $f \wedge \neg g \in \mathrm{Th}^{\mathrm{def}}(W \cup U)$. In the sequel, Ξ denotes the set of conflicts of a theory.*

Definition 4 ([19]). *An* elimination function *on a default theory (W, D) is a function ϕ that associates, to each conflict $C \in \Xi$, a set of pairs $(d, V) \in C \times 2^D$. A subset U of D is ϕ-preferred if it does not contain any conflict of the theory and verifies: $\forall d \in D \setminus U, \exists C \in \Xi, C \subseteq U \cup \{d\}$ and $(d, U) \in \phi(C)$*

The intended meaning of $(d, V) \in \phi(C)$ is that, in order to resolve the conflict C, one can eliminate d, as soon as the elements of V are not themselves eliminated. See [19] for a more detailled presentation of these notions.

Theorem 1 ([19]). *A set of formulas E is an extension of a default theory (W, D) iff E is generated by an ϕ^{DL}-preferred subset of D, where ϕ^{DL} is the elimination function defined by: $\phi^{\mathrm{DL}}(C) = \{(\frac{f:g}{h}, V) \in C \times 2^D \mid f \wedge \neg g \in \mathrm{Th}^{\mathrm{def}}(W \cup V)\}$.*

The next proposition shows that the extended deduction operator $\mathrm{Th}^{\mathrm{def}}$ is strongly related to the notion of conflict. It can be compared to a general result of classical logic: a formula f is classicaly entailed by a consistent set of formulas W if and only if $W \cup \{f \rightarrow \bot\}$ is inconsistent.

Proposition 1 ([20]). *Given a default theory (W, D), whose set of conflicts is empty, then $f \in \mathrm{Th}^{\mathrm{def}}(W \cup U)$ iff $\frac{f:\bot}{\bot}$ is in a conflict of the theory $W, U \cup \{\frac{f:\bot}{\bot}\}$.*

Let us now outline the major steps taken by our theorem prover. Given a default theory (W, D) and a formula f, we want to decide whether f is in at least one extension of the theory:

1. Compute the conflicts of $(W, D \cup \{\frac{f:\bot}{\bot}\})$ (including those of (W, D)).
2. Compute ϕ^{DL} on the conflicts of (W, D).
3. Compute the ϕ^{DL}-preferred subsets of D.
4. f is in the extension generated by an ϕ^{DL}-preferred subset U of D iff there is a conflict $V \cup \{\frac{f:\bot}{\bot}\}$ of $(W, D \cup \{\frac{f:\bot}{\bot}\})$ such that $V \subseteq U$.

$$
\frac{c_1 \quad \frac{\neg c_2 : c'}{c''}_d}{\frac{\neg c : c'}{c''}_d}\;(1)
\qquad
\frac{\frac{\neg c_1 : c'}{c''}_d \quad \frac{\neg c_2 : c'}{c''}_d}{\frac{\neg c : c'}{c''}_d}\;(2)
\qquad
\frac{\frac{\neg \Box : c'}{c''}_d}{c''}\;(3)
\qquad
\frac{\frac{\neg \Box : c'}{c''}_d}{\frac{\neg \Box : c'}{\top}_d}\;(4)
\qquad
\frac{c_1 \quad \frac{\neg \Box : c_2}{\top}_d}{\frac{\neg \Box : c}{\top}_d}\;(5)
\qquad
\frac{\frac{\neg \Box : c_1}{\top}_d \quad \frac{\neg \Box : c_2}{\top}_d}{\frac{\neg \Box : c}{\top}_d}\;(6)
$$

Fig. 1. Extended resolution rules (c_1 and $c2$ denote two clauses whose resolvent is c)

3 An extended resolution principle

In order to compute the conflicts of a default theory, we extend the classical resolution principle to defaults.

Definition 5. *The* clausal form *of a default* $d = \frac{\neg CN : CN'}{CN''}$*, where* CN*,* CN' *and* CN'' *are conjunctions of clauses, is the set of defaults* $\overline{d} = \{\frac{\neg c : c'}{c''}_d \mid c \in CN, c' \in CN', c'' \in CN''\}$*. We will denote by* $\overline{D}$ *the union of the clausal forms of the set of defaults* D*, and call* extended clauses *the elements of* $\overline{D}$*.*

Figure 1 presents the "extended resolution rules", that we will use together with the classical one.

Rules (1) and (2) are used to prove the prerequisite of a default d from the clauses of W and the consequents of other defaults obtained from rule (3) (see below): we make resolutions against the negation of the prerequisite of the default only. Therefore, when the extended clause "$\frac{\neg \Box : c'}{c''}_d$" is produced, the prerequisite of this default is proved.

Once the prerequisite of d has been proved, its consequent can be used like any formula of W (rule 3). However, the consistency of its justification is checked by making resolutions on it (rules 5-6). The production, using these rules, of an extended clause of the form $\frac{\neg \Box : \Box}{\top}_d$proves a conflict.

Rule (4) is there to avoid producing again the consequent of d using rule (3) everytime a resolution is made on its justification.

The production of an empty clause, using these extended resolution rules and the classical one, shows that the falsity is a consequence, in the sense of $\mathrm{Th}^{\mathrm{def}}$, of the set of defaults. In this case, the negation of the justification of any default can also be proved, therefore it also proves a conflict.

Notice how the indexes of the extended defaults have to be taken into account in rules (2) and (6): we can only make a resolution between the prerequisites (respectively the justifications) of two extended clauses if they have the same indexes, that is, if they have been produced from the same default. We will see the effect of relaxing these conditions in Sect 5.

Theorem 2 ([20]). *Let* (W, D) *be a default theory, such that* W *is a finite set of clauses and* D *is finite and can be put under clausal form. A subset* C *of* D *is a conflict of the theory iff it is minimal such that the empty clause or an extended clause of the form* $\frac{\neg \Box : \Box}{\top}_d$ *can be produced from* $W \cup \overline{C}$ *using rules (1) to (6) and the classical resolution rule.*

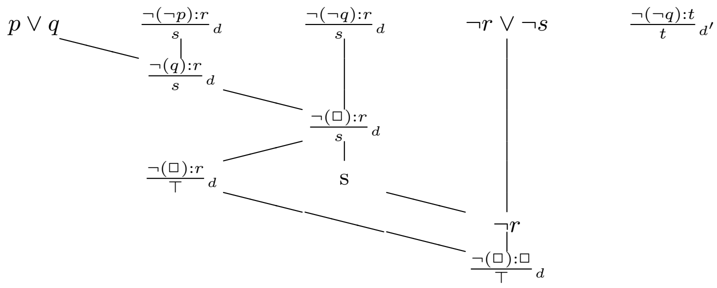

Fig. 2. Extended resolutions on Example 1

Example 1. Let $W = \{p \vee q, \neg r \vee \neg s\}$ and $D = \{d, d'\}$, where $d = \frac{p \vee q:r}{s}$ and $d' = \frac{q:t}{t}$. Figure 2 shows the extended resolutions produced to prove that $\{d\}$ is a conflict of the theory.

In order to compute all the conflicts of a theory, we use [20] Besnard et al.'s *saturation by set* [2], which returns all conflicts of a set of (extended) clauses, and which is incremental: given two sets of clauses, it is possible to saturate $E \cup F$ using the result of the saturation of F. Thus, given a default theory (W, D), we will first saturate $W \cup D$ in order to compute the conflicts of the theory. Given some formula f, this result significantly simplify the saturation of $W \cup D \cup \{\frac{f:\bot}{\bot}\}$. Our implementation uses a particular resolution strategy, the *head-literal resolution*, which was proved in [3] to be complete and decidable for the class of so-called *groundable clauses*.

Lastly, let us briefly describe the computation of ϕ^{DL}: a default d can be eliminated in a context V iff its prerequisite and the negation of its justification can be proved from V. This corresponds to the production of extended clauses of the form $\frac{\neg(\Box):c}{c'}d$ and $\frac{\neg(\Box):\Box}{\top}d$ during the saturation process. Thus, the elimination function ϕ^{DL} can be computed from the result of the saturation.

4 Prioritized conflict resolution

The computation of the ϕ-preferred subsets of D, given a default theory (W, D), is done using an adaptation of Levy's algorithm for the computation "augmented HS-trees" [17]. Levy's has shown how his algorithm can be used for the computation of Reiter's default logic. We generalize below this result, by extending his notion of *exception* to a wider class of elimination functions. This result will be used in the next section, when we describe how to adapt our theorem prover to other variants of default logic.

Definition 6. *An elimination function ϕ on the set of conflicts Ξ of a default theory (W, D) is* 2-monotonic *if for all $C \in \Xi$ and $(d, V \cup V') \in D \times 2^D$, if $(d, V) \in \phi(C)$ then $(d, V \cup V') \in \phi(C)$. An* exceptions *is a pair $(d, V \cup (C \setminus \{d\}))$ where $C \in \Xi$ and $(d, V) \in \phi(C)\}$. (d, V) is* minimal *if for no exception of* *the form (d, V'), $V' \subset V$. We denote by Σ the set of minimal exceptions of a theory. Given a subset E of Σ, we denote $E_1 = \{d \mid \exists V \subseteq D, (d, V) \in E\}$ and $E_2 = \cup_{V \in \{V' \mid \exists d \in D, (d,V') \in E\}} V$.*

Theorem 3. *Let (W, D) be a default theory, and ϕ a 2-monotonic elimination function on Ξ. A subset U of D is ϕ-preferred iff there exists $E \subseteq \Sigma$ such that $U = E_1$, $E_1 \cap E_2 = \emptyset$ and $\forall C \in \Xi, C \cap E_1 \neq \emptyset$.*

Proof (sketch) 1. Suppose U is ϕ-preferred, then $\forall d \in D \setminus U, \exists C_d \in \Xi, C_d \setminus \{d\} \subseteq U$ and $(d, U) \in \phi(C_d)$. Let $V_d \subseteq U \cup (C_d \setminus \{d\})$ be minimal such that $(d, V_d) \in \Sigma$, and let $E = \{(d, V_d) \mid d \in D \setminus U\}$. Clearly $U = D \setminus E_1$. One can check that $E_1 \cap E_2 = \emptyset$ and $\forall C \in \Xi, C \cap E_1 \neq \emptyset$. For the converse, let $E \subseteq \Sigma$ s.t. $E_1 \cap E_2 = \emptyset$ and $\forall C \in \Xi, C \cap E_1 \neq \emptyset$, and let $U = D \setminus E_1$. For all $C \in \Xi$, since $E_1 \cap C \neq \emptyset$, $C \not\subseteq U$. Moreover let $d \in D \setminus U = E_1$, and let $V \subseteq D$ and $C \in \Xi$ s.t. $(d, V \cup (C \setminus \{d\})) \in E$: since $E_1 \cap E_2 = \emptyset$, $V \subseteq U$ and $C \setminus \{d\} \subseteq U$. By definition of Σ, $(d, V) \in \phi(C)$, hence $(d, U) \in \phi(C)$.

We compute the ϕ-preferred subsets of D by constructing a binary tree, whose nodes are labelled by pairs of the form (χ, Υ), where $\chi \subseteq 2^D$ and $\Upsilon \subseteq D$: χ is a set of conflicts that remain to be solved, and Υ is a set of defaults that cannot be eliminated any more, because they justify previous eliminations. The root is labelled by $(\Xi, \emptyset)$, and we consider an enumeration of Σ. Consider a new exception (d, V) in this enumeration. For each leaf of the tree labeled by (χ, Υ), if $d \notin \Upsilon$, and if $\emptyset \notin \{C \setminus V \mid C \in \chi, d \notin C\} = \chi'$, build two edges. The first one, labelled by d, leads to a new leaf labelled by $(\chi', \Upsilon \cup V)$. The second one is not labelled, and leads to a leaf labelled by (χ, Υ). Then U is an ϕ-preferred subsets of D iff U is the complementary in D of the set of labels of the edges on a path from the root to a leaf labelled by a pair of the form $(\emptyset, \Upsilon)$.

This tree can be pruned by a depth-first search. Let U be the ϕ-preferred subset of D corresponding to a leaf labelled by $(\emptyset, \Upsilon)$. Consider a node labelled by (χ', Υ'), such that the set L of labels of the edges from the root to this node, joined with $\cup_{C \in \chi'} C$, is included in $D \setminus U$: if this branch leads to a new solution U', it will be the complementary of a set included in $L \cup (\cup_{C \in \chi'} C)$. Since ϕ-preferred subsets of D are *maximal* such that they do not contain any conflict of the theory, we cannot obtain a new solution in the branches below this node.

5 Other variants of default logic

A major interest of this decomposition of the theorem prover into two completely independent tasks is that it enables us to adapt it to several other variants of default logic. In this section, we describe three such modifications.

Strong regularity Whereas Reiter's default logic requires that the justifications of the defaults used to generate an extension are separately consistent with the extension, several variants put a stronger condition: they require that the justifications of all the defaults together are consistent with the extensions [5, 27]. Froidevaux and Mengin [9] formalize it with a definition similar to the following one:

$$\frac{\frac{\neg\Box:c_1}{\top}\,d_1 \quad \frac{\neg\Box:c_2}{\top}\,d_2}{\frac{\neg\Box:c}{\top}\,d_1}\ (6') \qquad \frac{\frac{\neg c_1:c_1'}{c_1''}\,d_1 \quad \frac{\neg c_2:c_2'}{c_2''}\,d_2}{\frac{\neg c:c_1'}{c_1''\vee c_2''}\,d_1}\ (2') \qquad \frac{\frac{\neg c_1:c_1'}{c_1''}\,d_1 \quad \frac{\neg c_2:c_2'}{c_2''}\,d_2}{\frac{\neg c:c_2'}{c_1''\vee c_2''}\,d_2}\ (2'')$$

Fig. 3. Modified extended resolution rules

Definition 7 ([9]). *A set of defaults D is* strongly regular *w.r.t. a set of formulas W if* $\mathrm{Th}^{\mathrm{def}}(W \cup D)$ *is consistent with* $\{g \mid \frac{f:g}{h} \in D\}$.

We can modify the definitions of conflict and of irregularity accordingly. In order for our extended resolution procedure to compute these irregularities, we simply have to replace rule (6) with rule (6') above,where c is the resolvent of c_1 and c_2. So we are now allowed to make resolutions between clauses that have been produced with the justifications of several defaults. If this leads to some extended clause $\frac{\neg\Box:\Box}{\top}$, then it means that the set of defaults is strongly irregular.

Reasoning by cases It is well-known that Reiter's default logic does not enable one to reason by cases using defaults: from the theory defined by $W = \{a \vee b\}$ and $D = \{\frac{a:c}{c}, \frac{b:d}{d}\}$ it is not possible to conclude $c \vee d$. Several authors have propose to modify Reiter's definition of the extension in order to allow such reasoning by cases. The idea is that one must be able to fire defaults "by sets", that is, to conclude the disjunction of the consequences of some defaults once the disjuction of their prerequisite has been proved. In order to adapt our theorem prover to reasoning by cases, we simply have to replace rule (2) with rules (2') and (2") abobe, where c is the resolvent of c_1 and c_2. As an example, it is possible to deduce the empty clause, using rules (1), (3) to (6), the above rule and the classical resolution rule, from $W \cup D \cup \{\frac{c:\Box}{\Box}, \frac{d:\Box}{\Box}\}$, where the latter set contains the extended clausal form of $\frac{c \vee d:\Box}{\Box}$.

Moinard [21] describes how this modification of default logic leads to some unwanted contrapositions of the defaults. He proposes to remedy to this problem by strenghtening the consistency condition on the justifications of the defaults, and the condition that allows one to eliminate a default. It would be interesting to specify the notion of irregularity as well as the elimination function that correspond to Moinard's definitions.

Prioritized default logic Several authors have proposed to add to default logic the possibility to define an ordering among the defaults, such that $d_1 \leq d_2$ means that the default d_1 is less reliable than the default d_2. The idea is that if d_1 and d_2 are conflicting, then one must use d_2 to generate an extension rather that d_1. In [19], we have proposed to associate, to such an ordering, the elimination function $\min_{\leq}$, such that $\min_{\leq}(C) = \{(d, V) \in C \times 2^D \mid \forall d' \in C, d' \not< d\}$. The meaning of this elimination function is that in order to resolve a conflict, one must eliminate one of its minimal (for $\leq$) elements. In order to adapt our theorem prover to the use of such a relation among the defaults, we simply have to apply the elimination algorithm to the set of conflicts of a theory, considering the elimination function $\min_{\leq} \circ\phi^{\mathrm{DL}}$: $\min_{\leq} \circ\phi^{\mathrm{DL}}(C) = \{(d, V) \in \phi^{\mathrm{DL}}(C) \mid \forall d' \in C, d' \not< d\}$.

6 Conclusion

We have described here the two main elements of a theorem prover for default logic. The first one generalizes the resolution principle to the (monotonic) deductions that can be made using defaults. The main interest of our presentation, by means of the introduction of special resolution rules, is that it is completely independent from any resolution strategy. We have implemented a theorem prover for default logic that uses theses rules combined to a resolution strategy by Bossu and Siegel [3], the resolution on head literals. But other resolution strategies could have been used. It is important to note that the resolution steps obtained using rules (1) to (6) correspond to resolution steps that would be obtained using the classical rule only on a clausal form of $W \cup \{\neg f, g, h \mid \frac{f:g}{h} \in D\}$. Consequently, any resolution strategy that is complete and that terminates on some particular class of clauses can be generalized to an extended resolution strategy that terminates and is complete on the corresponding class of extended clauses.

In comparison to many other theorem provers for default logic, we do not simply use some classical theorem prover to compute proofs for the prerequisites of the defaults or counter-arguments against their justifications. We provide, together with the extension of the resolution principle, a kind of "deduction theorem" that permits to make the link between the notion of conflicting defaults and that of monotonic deduction with defaults. Notice also that we do not need the introduction of any new propositional variable, as it is the case for the theorem provers that use some kind of traduction of the defaults into classical logic. This results in a greater clarity of the resolution steps, and simpler implementation. We have also described how simple modifications of some of the extended resolution rules can lead to other variants of default logic, that strengthen the consistency condition on the justification of the defaults or allow one to reason by cases.

Whereas the first part of our theorem prover is specific to default logic, the second one, that is, the elimination algorithm, is completely independent from any particular nonmonotonic logic. Levy [17] gives a detailed comparison between his algorithm and TMS-based algorithms used to compute extensions of default logic. As the algorithm that we have presented in this paper is not fundamentally different from Levy's one, this comparison still holds for our algorithm.

The main difference between Levy's algorithm and ours is that we do not need to check that the leaves of our tree correspond to valid sets of defaults generating extensions.

We have described in [20] how the notion of prioritized conflict resolution underlies some other famous nonmonotonic logic, like McDermott and Doyle's one [18]. In [19], we have also shown how elimination functions are interesting from the knowledge representation point of view.

The elements presented above are part of a system for prioritized default reasoning that we have implemented in Caml Light, a language of the ML family.

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