Low-level Image Processing

In-Place Covariance Operators for Computer Vision

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Abstract. Perhaps one of the most common low-level operations in computer vision is feature extraction. Indeed, there is already a large number of specific feature extraction techniques available involving transforms, convolutions, filtering or relaxation-type operators. Albeit, in this paper we explore a different approach to these more classical methods based on non-parametric in-place covariance operators and a geometric model of image data. We explore these operators as they apply to both range and intensity data and show how many of the classical features can be redefined and extracted using this approach and in more robust ways. In particular, we explore how, for range data, surface types, jumps and creases have a natural expression using first and second-order covariance operators and how these measures relate to the well-known Weingarten map. For intensity data, we show how edges, lines corners and textures also can be extracted by observing the eigenstructures of similar first- and second-order covariance operators. Finally, robustness, limitations and the non-parametric nature of this operator are also discussed and example range and intensity image results are shown.

1 Introduction

We have explored the use of covariance operators for the computation of local shape measures relevant to range and intensity (surface) image data. The covariance approach dispenses with surface parameterization and provides invariant descriptions of shape via the eigenvalues and eigenvectors of covariance matrices of different orders. The aim of this paper is to summarize the properties and results of the covariance approach as they can are applied to feature extraction in both types of surface image data.

Following Liang and Todhunter [1] we define the *local* (first–order) surface covariance matrix (C_I) as:

$$C_I = \frac{1}{n}\sum_{i=1}^{n}(\widetilde{x}_i - \widetilde{x}_m)\cdot(\widetilde{x}_i - \widetilde{x}_m)^T \ , \tag{1}$$

where $\widetilde{x}_i = (x_i, y_i, z_i)$ correspond to the image projection plane (x, y) and (z_i) corresponds to depth or intensity values at position i; $\widetilde{x}_m = 1/n\sum_{i=1}^{n}\widetilde{x}_i$ to the mean position vector; n to the total number of pixels in the neighborhood of $\widetilde{x}_i$ used to compute Eqn.(1).

The eigenvalues and eigenvectors of C_I determine three orthogonal vectors two of which define a plane whose orientation is such that it minimizes, in the least square sense, the (squared distance) orthogonal projections of all the points (n) onto the plane. Liang and Todhunter[1] originally proposed this plane as a reasonable approximation to the surface tangent plane and, so long as the two eigenvectors are chosen to preserve the "sidedness" of the surface points, it can be viewed as an analogue to this. That is, the eigenvectors corresponding to the two largest eigenvalues (λ_1 and λ_2) form this "tangent plane" if "sidedness" is preserved, otherwise the plane must be formed by the eigenvectors corresponding to eigenvalues λ_2 and λ_3. This "sidedness"-preserving or *topology preserving principle*, can be seen as a replacement of surface parameterization. It is important to note that the eigenvalues of C_I are already invariant to rigid motions and they also define the aspect rations of oriented spheres which determine the strengths of directional information in the neighborhood of a pixel. As we will see, from a geometric perspective such eigenvalues and vectors define surface types while, from an intensity image perspective, they can be used to infer edge, corners and other local shape structures in an image. First, the geometric interpretation.

2 Local geometries.

From a Differential Geometry perspective, C_I defines local tangent planes and surface normals (defining the first fundamental form) in the least-squares sense. From this an analogous definition of the second fundamental form follows. We define, about a point $\tilde{x}_0$ on a surface, a two-dimensional covariance matrix in the following manner:

$$C_{II} = \frac{1}{n}\sum_{i=1}^{n}(\tilde{y}_i - \tilde{y}_m)\cdot(\tilde{y}_i - \tilde{y}_m)^T, \tag{2}$$

with the two dimensional vectors $\mathbf{y}_i$ defined by the projection:

$$\tilde{y}_i = s_i \cdot (\tilde{x}_i - \tilde{x}_0)_{proj}, \tag{3}$$

where $\mathbf{x}_i$ is in a small neighborhood of $\tilde{x}_0$. We project the difference vector which points from $\tilde{x}_0$ to $\tilde{x}_i$ onto the "tangent plane" as determined by Eqn.(1) and weight the resulting two-dimensional vector by distance s_i which measures the orthogonal distance from the "tangent plane" to point $\tilde{x}_i$. Given both tangent vectors $\tilde{t}_1$ and $\mathbf{t}_2$ and the normal vector $\tilde{n}$ at point $\tilde{x}_0$ derived from Eqn.(2) we may write y_i explicitly as:

$$\tilde{y}_i = \underbrace{\left[(\tilde{x}_i - \tilde{x}_0)^T \cdot \tilde{n}\right]}_{s_i} \cdot \begin{pmatrix} (\tilde{x}_i - \tilde{x}_0)^T \cdot \tilde{t}_1 \\ (\tilde{x}_i - \tilde{x}_0)^T \cdot \tilde{t}_2 \end{pmatrix} \tag{4}$$

We can then define the quadratic form

$$II_C = \widetilde{v}^T \cdot C_{II} \cdot \widetilde{v} \tag{5}$$

as a "second fundamental form" based on covariance methods with the defined covariance matrix according to Eqn. (2) and a chosen unit vector v in the tangent plane. Analogous to classical computations of surface geometry we define the **principal directions** as those directions which are given by the eigenvectors of this covariance matrix. The eigenstructures of C_{II} capture how the surface points, in the neighbourhood of a pixel, depart from the estimation of the tangent plane.

Further to this, we have an analogous operator to the Gauss map of classical Differential Geometry[2]. That is, we can determine how the estimated surface normals, in the neighborhood of a pixel, project onto the estimated tangent plane by the two-dimensional covariance matrix:

$$C_P = \frac{1}{n}\sum_{i=1}^{n}(\widetilde{v}_i - \widetilde{v}_m)\cdot(\widetilde{v}_i - \widetilde{v}_m)^T, \tag{6}$$

with the two dimensional vectors $\mathbf{v}_i$ being defined by the projection:

$$\widetilde{v}_i = (\widetilde{n}_i)_{proj} = \begin{pmatrix} \widetilde{n}_i^T \cdot \widetilde{t}_1 \\ \widetilde{n}_i^T \cdot \widetilde{t}_2 \end{pmatrix}, \tag{7}$$

where $\widetilde{t}_1, \widetilde{t}_2$ are the estimated tangent vectors obtained from Eqn.(1) and assigned to the point (x, y, z) at the center of the current window and $\widetilde{n}_i$ is the normal vector obtained by Eqn.(1) at a position (x_i, y_i, z_i) in the neighborhood of (x, y, z).

In all, then, the operators C_I, C_{II} and C_p determine local geometric characteristics of surfaces without the use of Calculus and with the constraint that such eignestructures correspond to least squares estimates of different order orientation fields. They can be used in identifying different surface types - including discontinuities - in the following ways (using appropriate thresholds, see [2]:

Jump-edge detection: The covariance matrix C_I, according to Eqn.(1), is calculated in a 5×5 pixel neighborhood at each pixel of the range image. Pixels with values of the maximal eigenvalue larger than a certain threshold are labeled as *jump*. The eigenvectors calculated in this stage are used for the next step.

Crease-edge detection: The covariance matrix C_p, according to Eqn.(6), is calculated in a 5×5 pixel neighborhood at each pixel of the range image. The maximal eigenvalue of this covariance matrix C_p is utilized as a crease-edge detector. Pixels not already labeled as *jump* and with values of the maximal eigenvalue larger than a certain threshold are labeled as *crease*.

Region segmentation: Pixels neither being labeled as *jump* nor *crease* are labeled as ***planar***, ***parabolic*** (developable) or ***curved***. In order to do so the covariance matrix C_I according to Eqn.(1) is calculated in a 7×7 pixel neighborhood at each pixel of the range image. The eigenvectors of this covariance matrix are again used as input for the next stage which calculates the covariance matrix C_p

according to Eqn.(3) from the projected normal vectors in a 7×7 pixel neighborhood where only pixels which have not been labeled so far as *jump* or *crease* are taken into account. Two thresholds for each of the two eigenvalues of this covariance matrix are applied. Pixels with smaller values of both eigenvalues than the thresholds are labeled as *planar*. Pixels with a smaller value of the smaller eigenvalue than the threshold but with values of the larger value beyond the corresponding threshold are labeled as *parabolic*. Pixels not meeting above conditions are assigned the label *curved*.

We have obtained experimental results for the proposed segmentation technique using many synthetic range images - one of which is shown in Figure 1a. The range images have neither been filtered nor preprocessed in any way. However, we have computed surface Mean and Gaussian curvatures using the bi–quadratic surface algebraic form proposed in [2]. We have also used the jump–and crease–edge detection technique presented in [3]for comparison. Together these region and boundary detection methods are termed the Smoothed Differential Geometry, or SDG method.

3 Image features

Viewing intensity images as surfaces is not that common but is natural to this covariance approach. Indeed, form this perspective, "edges", "corners" and "linear segments" all can be described by different types of surface types [5] including jumps, creases and non-planar region types. Further, we can even provide first- and second-order descriptions of "textural" features in terms of the associated eigenstructures of the covariance operators (Figure 1c). Figure 1b and c shows examples of such interpretations. Lines were determined by first-order eigenvalue ratios, corners by second-order eigenvalue strengths.

4 Discussion

The aim of this paper was to simple expose and demonstrate the covariance approach to feature extraction. It contrasts with more classical methods - and, in particular, differential and filter-based techniques, in two ways. One, there is no use of parameterization or differentiation directly. Two, there is an implicit optimization criterion: orthogonal least squares approximations of linear structures of different orders. Furthermore, we have been able to define analogous operators to the classical Weingarten map (second fundamental form) and the Gauss map on the basis of covariance matrices. The eigenvalues of the covariance matrices are invariant to rigid motion as well - since the computation operates in the tangent plane only. In addition, we have shown how the covariance method treats discontinuities in a very natural manner.

Finally, since covariance methods do not rely on a consistent local surface parameterization, the spectrum of the covariance matrix (the full set of eigenvalues) provides us with a type of smooth transformation between lines, surfaces and volumes. One non-zero eigenvalue defines merely linear components while a solution of three equal eigenvalues corresponds to data of uniform density in a volume about a point – analogous to fractile-dimensionality.

It should also be noted that covariance methods provide ideal ways of treating signals embedded in additive Gaussian or white noise.In these cases the total covariance matrix decomposes into the sum of the signal and noise covariance matrices. These covariance models also provide linear approximations to the local correlations or dependencies between pixels. Consequently, they are related to techniques for modeling the observed (local) pixel "cliques" or "support" kernels defined in Markov Random Fields and determined using Hidden Markov Models. However, space prohibits more detailed analysis of these connections.

5 References

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Figure 1 - Following Page

Top: Shows input range image; second row:jumps(white), creases(grey) for SDG(left) and covariance(right) methods; third row: region segments from SDG(left) and covariance(right) methods.

Middle: Input textures(left) and rotation invariant segmentation based on first-order covariance eigenvalues(right) and 2-class K-Means clustering.

Bottom: Input intensity image(left) lines(centre), and corners(right) using first and second-order covariance eigenvalues respectively.