# Fuzzy Segmentation and Structural Knowledge for Satellite Image Analysis

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**Abstract.** We present a segmentation method using fuzzy sets theory applied to remote sensing image interpretation. We have developed a fuzzy segmentation system in order to take into account complex spatial knowledge involving topologic attributes and also relative position of searched areas in membership degrees images. A membership degrees image represents the membership degrees of each pixel to a given class and is supposed obtained by a previous classification (involving simple contextual knowledge). To improve this previous classification, we introduce structural rules which allow us to manage with region characteristics. These structural characteristics are obtained by using a fuzzy segmentation technique.

## 1 Introduction

We present a knowledge-based approach for satellite image interpretation taking into account structural knowledge about searched classes In a first step, we use a locally existing knowledge-based system [1] which gives us membership degrees to each searched class for each pixel. Non-structural expert knowledge describing the favourable context for each class in terms of out-image data (elevation, roads, rivers, types of soils, ...) is at that step already taken into account. This type of knowledge is based on pixel information independently of the neighbourhood.

So we have n images (if we look for n classes) representing the membership function to each class i: $\{\mu_i(x,y)\}$). We have now to introduce expert structural knowledge and out-image data to update the $\mu_i$ functions in order to give the final classification. As we have to manage with regions and relations involving different regions or objects and as we have only membership functions $\mu_i$, we produce a fuzzy segmentation of each image $\{\mu_i(x,y)\}$. We have now to produce n sets of fuzzy regions (with their fuzzy geometric attributes) and to compute fuzzy relations involving regions of different classes or objects defined on out-image data.

For each image $\{\mu_i(x,y)\}$ the fuzzy regions are defined by their level-cuts. Note that if a level-cut gives more than one connected crisp region, we define new fuzzy regions. Geometric attributes are computed for fuzzy regions (surface, perimeter, degree of compactness, shape ...). We define relations (inclusion degree, distance, ...) between fuzzy regions of different classes or out-image items (roads, rivers, soils, ...). So our fuzzy segmentation splits the n images $\{\mu_i(x,y)\}$ into hierarchical structures of fuzzy regions with their geometric attributes.

The structural knowledge concerning the searched classes is now introduced in order to update the n membership functions $\mu_i$ and finally an improved classification is obtained.

# 2 Fuzzy Segmentation

## 2.1 Convex Combination of Sets

Using fuzzy sets theory [7] we can define a fuzzy set by a random sets representation (*also called convex combination of sets*) [2]. This combination is composed of n included crisp sets $A_i$ $(A_1 \supset A_2 \supset A_3 \supset \ldots)$ with $m(A_i)$ corresponding positive weights [5]:

$$\sum_{i=1}^{n} m(A_i) = 1 \qquad (1)$$

Assume n values $\alpha_i \in [0,1]$ and $\alpha_1 < \alpha_2 < \ldots < \alpha_n$. We can compute $A = \{CUT_{\alpha_1}, CUT_{\alpha_2}, \cdots, CUT_{\alpha_n}\}$ with $CUT_{\alpha_i} = A_i$ is the crisp set obtained with the level-cut $\alpha_i$. A level-cut is defined as the set of pixels which $\mu(x,y)$ is greater than $\alpha_i$.

The weight assigned to the set $A_i$ is computed as follows:

$$m(A_i) = \alpha_i - \alpha_{i-1} \text{ and } \alpha_0 = 0 \qquad (2)$$

The membership function of a fuzzy set is obtained from its "convex combination of sets" representation. This property allows us to manage fuzzy regions only by using their level-cuts *(which are crisp regions)*.

## 2.2 Application on a Membership Degrees Image

**Fuzzy Sets Supports.** We consider the Euclidean plane of the image as the fuzzy sets referential. Fuzzy sets supports are computed by using a threshold $\alpha_1$, obtained from the membership degrees image histogram, in order to process only the significant pixels *(fig. 1&2):*

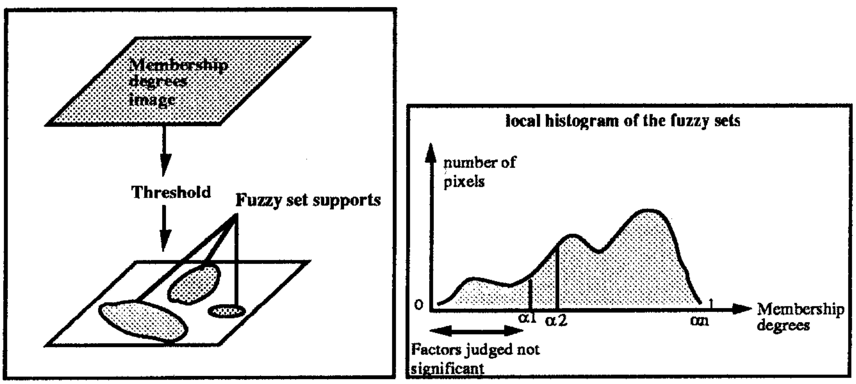



**Fig. 1. and 2.**

**Fuzzy Region.** An ordinary region is a set of connected pixels *(with a non-zero membership degrees value)* obtained from a level-cut on the $A_1$. fuzzy region support. All the crisp sets *(included in the support)* with corresponding weights define a fuzzy region *(fig. 3)*

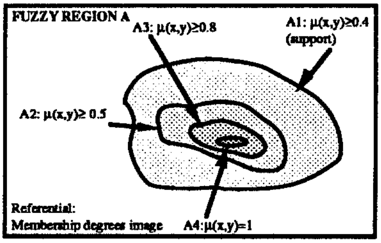



**Fig. 3.** Fuzzy Region

The level-cuts values can be computed by a constant step or by a thinness threshold as follows *(defining the thinness of each level-cut)*:

$$\alpha_{i+1} = \underset{\alpha>\alpha_i \text{ and } \alpha\leq 1}{Min}\left(\int_{\alpha_i}^{\alpha} H(x)dx \geq Thinness\right) \quad (3)$$

With : $\alpha_i$: level-cut i; $\alpha_{i+1}$: next level-cut; H(): local histogram function.

So an image is described as a set of K fuzzy regions and a fuzzy region is defined as a concentric set of crisp regions.

**Fuzzy Characteristics.** A crisp region $A_i$ is a set of connected pixels. Then we can compute topologic attributes for this region such as: Perimeter, Compactness degree, Moments of order one and two, Surface, ..:

The value of one particular topologic characteristic for each crisp region is computed separately by using a measurable function F. The final measurable function $\widetilde{F}$ of the fuzzy region, characterizing the topologic attribute, is obtained by the following formula [3]:

$$\tilde{F}(A) = \sum_{i=1}^{n} m(A_i)\,F(A_i) \qquad (4)$$

We obtain a general aspect of the fuzzy set. Therefore, the image is described as a hierarchical structure of fuzzy and ordinary regions with their geometric attributes.

**Fuzzy Relations Between Fuzzy Regions.** Moreover we can also determinate fuzzy relations between fuzzy regions such as: distance, at which degree two regions are oriented in the same direction, at which degree one region is included in another one, ...

The realisation degree of a relation between two crisp regions is computed by using a measurable function F. We can also define a measurable function $\widetilde{F}$ for the relation between two fuzzy regions [3] A and B:

- A which is composed by the crisp regions set $\{A_1, A_2, \cdots, A_n\}$
- B which is composed by the crisp regions set $\{B_1, B_2, \cdots, B_p\}$
- and a function $F(A_i, B_j)$ which links the crisp regions $A_i$ and $B_j$.

The relation between the two fuzzy regions A and B is computed as follows [3]:

$$\tilde{F}(A,B) = \sum_{i=1}^{n}\sum_{j=1}^{p} m(A_i)\,m(B_j)\,F(A_i, B_j) \qquad (5)$$

# 3 Structural Knowledge Application

## 3.1 Local Rules (Pixels)

Local rules characterize the expert's local knowledge describing the favourable context for each class in terms of out-image data (elevation, roads, rivers, types of soils,...). This type of knowledge [1] is based on pixel information independently of the neighbourhood. For example: "class X: frequently elevation >1000m and on north slope". So we obtain n images (if we look for n classes) representing the membership function to each class i: $\{\mu_i(x,y)\}$ (the membership degrees images).
Let's suppose A is a fuzzy region extracted in membership degrees image for one particular class, we define: $\Pi(A)=\{\alpha_1,\alpha_2,\cdots,\alpha_n\}$ (with $\alpha_0 < \alpha_1 < \cdots < \alpha_n$ and $\alpha_0 = 0$). A level-cut $\alpha_i$ defines a crisp region and the associated weight is computed (section 2.1) to obtain a convex combination of sets.

## 3.2 Structural Rules

These rules introduce expert structural knowledge and out-image data to update the $\mu_i$ functions in order to give the final classification.

For example: "the class X appears *principally* as <u>elongated</u> shapes <u>along</u> <u>big</u> rivers".
We have in this rule three structural information:

- <u>elongated</u> (characteristic of a region) can be computed from the fuzzy compactness topologic information,
- <u>along</u> (relation between two regions) can be computed from the distance variations between regions' skeletons (or supports),
- <u>big</u> (characteristic of a region) can be computed from the fuzzy surface topologic information.

*Principally* can be considered as the frequency degree for the class to be in this context.
As we have to manage with regions and relations involving different regions or objects and as we have only membership functions $\mu_i(x,y)$, we produce a fuzzy segmentation of each membership degrees image $\{\mu_i(x,y)\}$.
In fact, in our application, after applying the fuzzy segmentation method, we determine topologic characteristics (such as compactness, perimeter,...) and relations (such as distance, adjacency degree...) for two fuzzy regions. Then we introduce the structural information given by the rule concerning the class i in order to update the membership degrees image $\{\mu_i(x,y)\}$. In fact each $\{\mu_i(x,y)\}$ will be modified relatively to the structural knowledge concerning class i.

## 3.3 Application of the Rules

Assume P is a property (associated to a production rule concerning class C) and $f_p(\ )$ the real function which associates at each crisp region its realization degree.
$\tilde{f}_p(A)$ will be the realization degree for the fuzzy region A.
For example:
Rule: If <Class C> Then elongated shape
(Property P: "elongated region").

For all crisp region $A_i$, $f_p(A_i)$ is the elongated degree of $A_i$. This degree, which must be a value in the ]0,1] set, may be computed from the following ratio: perimeter / surface.

Let's suppose A is a fuzzy region (included in a membership degrees image and characterizing a class C) with $A=\{A_1,A_2,\cdots,A_n\}$ the level-cuts set on A. So, we define the weight function $m_p(\ )$ as follows:

$\forall A_i \subseteq A$

$$m_p(A_i) = f_p(A_i) \qquad (6)$$

In our method, all the new weights are normalized ([0,1] set). So the combination $m(A_i)$ by $m_p(A_i)$:

- increases the $A_i$ regions' weights which best verify the property P,
- reduces the $A_i$ regions' weights which worst verify the property P.

We modify the weight attached to each crisp region:

$\forall A_i \subseteq A$

$$m'(A_i) = \frac{m(A_i)\cdot m_p(A_i)}{\sum_{A_j\in A} m(A_j)\cdot m_p(A_j)} = \frac{m(A_i)\cdot f_p(A_i)}{\sum_{A_j\in A} m(A_j)\cdot f_p(A_j)} = \frac{m(A_i)\cdot f_p(A_i)}{\tilde{f}_p(A)} \qquad (7)$$

So, the weight assigned to a crisp region $A_i$ which verify property P will be increased.

In the formula (7), the expert knowledge is considered as sure. But, generally, we have both an expert rule and a frequency degree $\beta$ for the rule $(-1\le\beta\le1$; from -1 for "never" to 1 for "always"). Then we define a new weight function to manage with this frequency degree $\beta$:

$\forall A_i \subseteq A$

$$m_p(A_i) = f_p(A_i)^\beta \qquad (8)$$

We obtain a new fuzzy region:

$\forall A_i \subseteq A$

$$m'(A_i) = \frac{m(A_i)\cdot m_p(A_i)}{\sum_{A_j\in A} m(A_j)\cdot m_p(A_j)} = \frac{m(A_i)\cdot f_p(A_i)^\beta}{\sum_{A_j\in A} m(A_j)\cdot f_p(A_j)^\beta} \qquad (9)$$

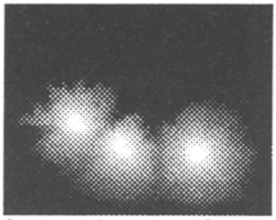
At this step, we use a MYCIN-based combination function [6] $F_{mycin}$ (see annex) to obtain the final fuzzy region:

$$\forall A_i \subseteq A,\ m''(A_i) = \frac{1+F_{mycin}\left(2m'(A_i)-1, \beta\cdot\tilde{f}_p(A)\right)}{2} \qquad (10)$$

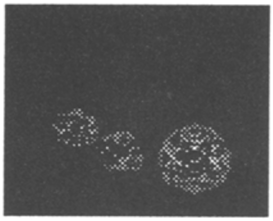
The final weights $m''(A_i)$ associated to all crisp regions $A_i$ are defined with the MYCIN-based combination [6] and from the two following values:

- $m'(A_i)$: weight for region $A_i \in [0,1]$
- $\beta\cdot\tilde{f}_p(A)$: - $\beta$ frequency degree of the rule.
  - $\tilde{f}_p(A)$ realisation degree of property P for the fuzzy region A.

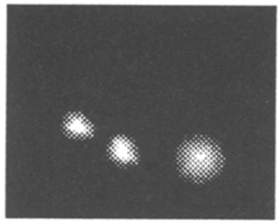
## 4 An example of Fuzzy segmentation and structural knowledge application



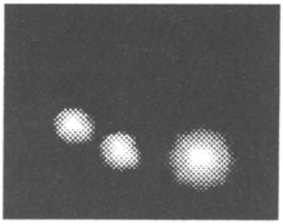
Membership degrees Image



Segmented Image



Fuzzy regions display



If <Class C> Then compacted shape - A -
( + property "compacted region").

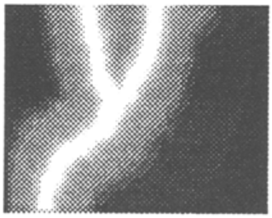
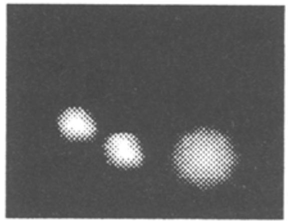


Image: "Distance to roads"



New membership Image
If <Class > then roads
(combined with - A -)

## 5 Conclusion

We have proposed a new method to introduce structural knowledge for image interpretation based on fuzzy logic and fuzzy segmentation. Our method splits the membership degrees images in a set of atomic fuzzy regions. In a second step, we use structural rules to update the membership degrees images and then to give final classification. We have applied this method on geocoded images and achieved interesting results. The application to satellite image classification is going on.

## References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