# Separating Real and Virtual Objects from Their Overlapping Images 

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#### Abstract

We often see scenes where an object's virtual image is reflected on window glass and overlaps with an image of another object behind the glass. This paper proposes a method for separating real and virtual objects from the overlapping images. Our method is based on the optical property that light reflected on glass is polarized, while light transmitted through glass is less polarized. It is possible to eliminate reflected light with a polarizing filter. The polarization direction, however, changes even for planar glass and is not easily determined without information about the position and orientation of the glass and objects relative to the camera. Our method uses a series of images obtained by rotating a polarizing filter. Real objects are separated by selecting the minimum image intensity among a series of images for each pixel. The virtual image of objects is obtained by subtracting the image of the real objects from the image having the maximum image intensity among a series of images for each pixel. We present experiments with actual scenes to demonstrate the effectiveness of the proposed method.


## 1 Introduction

In real scenes indoors and outdoors there are many transparent objects with glossy surfaces, such as glass and water. We often see the reflected (virtual) image of an object on these surfaces located on the same side as we are, relative to the surface. The reflected image overlaps with other objects behind the transparent object. For example, we cannot see a person in a car clearly, because the surrounding scene is reflected on the window glass. Thus, reflected images hinder our perception of what is behind a transparent object.

It is important to separate the real from the reflected components in overlapping images. Eliminating reflected images improves the quality of TV camera images. Detecting reflected images helps a mobile robot perceive a transparent object obstructing its advance; a mobile robot can also use reflected images to detect the existence of objects outside the camera's field of view.

This paper proposes a method for separating real and virtual objects from their overlapping images. Our method is based on the polarization of light reflected on a specular surface, using a series of images captured by rotating a polarizing filter; the
polarization direction is not required in advance. Using simple comparison and subtraction, we easily separated real objects from overlapping reflected images.

There are a few related studies [1]-[4]. Wolf [1][2] used polarization to separate reflection components, and developed a polarization camera. Yamada et al.[2] developed a vision system for removing the specular reflection component so that mobile robots could correctly locate a white guide line. Polarization was used to detect wet roads [4]. In these studies the polarization direction was known in advance, and only the reflection on the surface of an opaque object, such as a floor or a road, was introduced. In contrast, this paper deals with reflections on the surface of a transparent object behind which other objects exist. We do not require the polarization direction in advance.

In section 2 we describe optics, especially polarization. In section 3 we present a model of the image formation in terms of polarization and propose a method based on the model for separating overlapping objects without prior information about the polarization direction. In section 4 we show the experiment results in real scenes and demonstrate the method's effectiveness. Finally, we offer our conclusions.

## 2 Optics on Polarization

### 2.1 Polarization of Reflected and Transmitted Light

Consider the light reflection between two materials, A and B , whose refraction coefficients are $\mathrm{n}_{\mathrm{A}}$ and $\mathrm{n}_{\mathrm{B}}$ (Fig. 1). In the figure, $\theta_{\mathrm{A}}$ is the angle of incidence and $\theta_{B}$ the angle of refraction. There is the following relation between $\theta_{A}$ and $\theta_{B}$ (Snell law):

$$
\begin{equation*}
\mathrm{n}_{\mathrm{A}} \sin \theta_{\mathrm{A}}=\mathrm{n}_{\mathrm{B}} \sin \theta_{\mathrm{B}} \tag{1}
\end{equation*}
$$



Fig. 1 Reflection and transmission
If the boundary of the two materials is completely specular, the angle of reflection is equal to the angle of incidence. As in optics, the Fresnel reflection coefficients are represented as

$$
\begin{align*}
& R_{\mathrm{s}}=\frac{\sin ^{2}\left(\theta_{\mathrm{A}}-\theta_{\mathrm{B}}\right)}{\sin ^{2}\left(\theta_{\mathrm{A}}+\theta_{\mathrm{B}}\right)}, \\
& \mathrm{R}_{\mathrm{p}}=\frac{\tan ^{2}\left(\theta_{\mathrm{A}}-\theta_{\mathrm{B}}\right)}{\tan ^{2}\left(\theta_{\mathrm{A}}+\theta_{\mathrm{B}}\right)}, \tag{2}
\end{align*}
$$

where subscripts $s$ and $p$ indicate parallel and orthogonal components to the incident plane consisting of the incident light vector $\mathbf{S}$ and surface normal $\mathbf{n}$.


Fig. 2 Coefficients of reflection and transmission vs. angle of incidence ( $\mathrm{n}_{\mathrm{A}}=1.0, \mathrm{n}_{\mathrm{B}}=1.5$ )

Figure 2 shows the changes of reflection coefficients RS and Rp with the angle of incidence. On the graph we see that $R_{P}$ is smaller than $R_{S}$ for an angle of incidence ranging from 30 to 80 degrees. Reflected light is thus polarized and has a smaller component parallel to the incident plane. Therefore, it is possible to remove light reflected on a specular surface. In contrast, the transmission coefficients in Fig. 1 are given as:

$$
\begin{align*}
& \mathrm{T}_{\mathrm{s}}=\frac{\mathrm{n}_{\mathrm{B}} \cos \theta_{\mathrm{B}}}{\mathrm{n}_{\mathrm{A}} \cos \theta_{\mathrm{A}}}\left\{\frac{2 \cos \theta_{\mathrm{A}} \sin \theta_{\mathrm{B}}}{\sin \left(\theta_{\mathrm{A}}+\theta_{\mathrm{B}}\right)}\right\}, \\
& \mathrm{T}_{\mathrm{p}}=\frac{\mathrm{n}_{\mathrm{B}} \cos \theta_{\mathrm{B}}}{\mathrm{n}_{\mathrm{A}} \cos \theta_{\mathrm{A}}}\left\{\frac{2 \cos \theta_{\mathrm{A}} \sin \theta_{\mathrm{B}}}{\sin \left(\theta_{\mathrm{A}}+\theta_{\mathrm{B}}\right) \cos \left(\theta_{\mathrm{A}}-\theta_{\mathrm{B}}\right)}\right\} \tag{3}
\end{align*}
$$

Now consider light transmitted from material A through thin material B to material A. The transmission coefficient from material B to A is easily obtained using Eq. (3) as follows:

$$
\begin{align*}
& \mathrm{T}_{\mathrm{s}}^{\prime}=\frac{\mathrm{n}_{\mathrm{A}} \cos \theta_{\mathrm{A}}}{\mathrm{n}_{\mathrm{B}} \cos \theta_{\mathrm{B}}}\left\{\frac{2 \cos \theta_{\mathrm{B}} \sin \theta_{\mathrm{A}}}{\sin \left(\theta_{\mathrm{A}}+\theta_{\mathrm{B}}\right)}\right\}^{2}, \\
& \mathrm{~T}_{\mathrm{p}}^{\prime}=\frac{\mathrm{n}_{\mathrm{A}} \cos \theta_{\mathrm{A}}}{\mathrm{n}_{\mathrm{B}} \cos \theta_{\mathrm{B}}}\left\{\frac{2 \cos \theta_{\mathrm{B}} \sin \theta_{\mathrm{A}}}{\sin \left(\theta_{\mathrm{A}}+\theta_{\mathrm{B}}\right) \cos \left(\theta_{\mathrm{A}}-\theta_{\mathrm{B}}\right)}\right\}^{2} \tag{4}
\end{align*}
$$

Then the total transmission coefficient is obtained as:

$$
\begin{align*}
& \mathrm{T}_{\mathrm{s}}^{\prime \prime}=\mathrm{T}_{\mathrm{s}} \mathrm{~T}_{\mathrm{s}}^{\prime}, \\
& \mathrm{T}_{\mathrm{p}}^{\prime \prime}=\mathrm{T}_{\mathrm{p}} \mathrm{~T}_{\mathrm{p}}^{\prime} \tag{5}
\end{align*}
$$

Figure 2 shows the changes of coefficients $\mathrm{T}^{\prime \prime}$ s and $\mathrm{T}^{\prime \prime} \mathrm{p}$. Transmitted light is less polarized than reflected light for an angle of incidence smaller than 80 degrees.

### 2.2 Direction of Polarization

This subsection describes how the polarization angle of reflected light changes depending on the position of a point on a glass surface.

Suppose that a camera locates in the $z-x$ plane and its optical axis directs toward the coordinate origin O . The point $\mathrm{C}^{\prime}$ is the projection of the center of camera lens C (See Fig.3(a)). The incident plane at point P in Fig.3(a) is obtained as

$$
\begin{equation*}
n_{\text {if }}=n \times p, \tag{6}
\end{equation*}
$$

where $\boldsymbol{n}$ is the normal vector of glass, $\boldsymbol{p}$ the vector from the point C to the point P , and the symbol ' $x$ ' denotes cross product of vectors. Because $\boldsymbol{p}=\boldsymbol{p}$ ' $+\mathbf{d}$ and $\boldsymbol{n} \times \boldsymbol{p}$ ' $=$ $0, \mathrm{Eq} .(6)$ is rewritten as :

$$
\begin{equation*}
n_{\mathrm{if}}=n \times\left(p^{\prime}+d\right)=n \times d \tag{7}
\end{equation*}
$$

Eq.(7) says that the direction of incident plane, orthogonal to the direction of polarization, changes depending on the vector $\boldsymbol{d}$, the position of a point on the surface. Figure 3(b) shows schematically the change of polarization direction.


Fig. 3 The normal of incident plane (polarization direction) and its change on a surface

## 3 Separation Method

In the previous section, we demonstrated that light reflected on a specular surface is polarized, while transmitted light is less polarized. Based on this physical property, we made a model of image formation under polarization and developed a method based on the model for separating overlapping objects.

### 3.1 Model of Image Formation

In the situation shown in Fig. 4, the camera captured the image of object A transmitted through the glass, and the reflected image of the object, giving us the following:

$$
\begin{equation*}
\mathrm{I}(\mathrm{x}, \mathrm{y})=\mathrm{I}_{\mathrm{A}}(\mathrm{x}, \mathrm{y})+\mathrm{I}_{\mathrm{B}}(\mathrm{x}, \mathrm{y}) \tag{8}
\end{equation*}
$$

where $I(x, y)$ is the camera image at each pixel $(x, y), I_{A}(x, y)$ the transmitted image, and $\mathrm{I}_{\mathrm{B}}(\mathrm{x}, \mathrm{y})$ the reflected image.


Fig. 4 Situation causing overlap of reflected and transmitted images
General light sources, such as the sun and incandescent light, emit unpolarized light. In this paper, we assume that the specular component is small enough to be neglected in the reflection of any object other than glass. Light reflected on the surfaces of objects $A$ and $B$ consists only of a diffuse component and is not polarized. The light from object $B$ reflected on the surface of the glass is polarized, because glass is specular. Therefore we can use a polarizing filter in front of the camera to remove image $I_{B}$ from the camera image and to separate only image $I_{A}$.

We rewrite Eq. (8) as

$$
\begin{equation*}
\mathrm{I}(\mathrm{x}, \mathrm{y} ; \theta)=\mathrm{I}_{\mathrm{A}}(\mathrm{x}, \mathrm{y} ; \theta)+\mathrm{I}_{\mathrm{B}}(\mathrm{x}, \mathrm{y} ; \theta) \tag{9}
\end{equation*}
$$

where $\theta$ is the angle of a polarizing filter. As mentioned above, image $I_{A}$ is less polarized and does not change with the angle of a polarizing filter $\theta$;
$I_{A}(x, y ; \theta) \cong I_{A}(x, y) \quad$ for all $\theta$.
We then have:

$$
\begin{equation*}
\mathrm{I}(\mathrm{x}, \mathrm{y} ; \theta)=\mathrm{I}_{\mathrm{A}}(\mathrm{x}, \mathrm{y})+\mathrm{I}_{\mathrm{B}}(\mathrm{x}, \mathrm{y} ; \theta) \tag{10}
\end{equation*}
$$

### 3.2 Method for Separating Overlapping Images

As mentioned in 2.2, the polarizing direction (angle) depends on the relative positions of the object, glass, and camera. It changes depending on the position of the glass, especially curved glass. Thus, measuring the relative positions and the 3D
shape of the glass is complicated and unrealistic. By rotating a polarizing filter, we captured a series of images at each polarizing filter angle $\theta$; for example, every 10 degrees.

The real image of object A can be obtained by selecting a minimum image intensity of $I(x, y ; \theta)$ for each pixel:

$$
\begin{equation*}
\hat{\mathrm{I}}_{\mathrm{A}}(\mathrm{x}, \mathrm{y})=\min _{\theta \in \Theta} \mathrm{I}(\mathrm{x}, \mathrm{y} ; \theta) \tag{11}
\end{equation*}
$$

where $\Theta$ is the set of the filter angle. Next, we can easily obtain the maximum intensity image:

$$
\mathrm{I}_{\mathrm{MAX}}=\max _{\theta \in \Theta} \mathrm{I}(\mathrm{x}, \mathrm{y} ; \theta)
$$

This is ideally represented as

$$
\begin{equation*}
I_{M A X}=I_{A}(x, y)+I_{B}(x, y) \tag{12}
\end{equation*}
$$

By subtracting Eq. (11) from Eq. (12), we obtain the virtual image of the object B :

$$
\begin{equation*}
\hat{\mathrm{I}}_{\mathrm{B}}(\mathrm{x}, \mathrm{y})=\mathrm{I}_{\mathrm{MAX}}(\mathrm{x}, \mathrm{y})-\hat{\mathrm{I}}_{\mathrm{A}}(\mathrm{x}, \mathrm{y}) \tag{13}
\end{equation*}
$$

## 4 Experiments

We conducted experiments both in a dark room and outdoors. In a dark laboratory room, we set up a sheet of glass with two balls in front of it and a ball behind it and illuminated them with a fluorescent lamp located in front of the glass. The angle between the camera's optical axis and the normal surface of the glass was 56 degrees. By changing the angle of the polarizing filter attached to the camera, we captured a series of images ( 18 images, one every 10 degrees) at a fixed iris. The images are shown in Fig. 5.

Using Eqs. (11) and (13), we obtained the real image of the object behind the glass as shown in Fig. 6(a) and the virtual image of the two objects in front of the glass (Fig. 6(b) ) which were not seen directly in the camera's field of view. In Fig. $6(b)$, the right-side contour of the ball behind the glass was detected because of the slight sway of the ball, which was hanging from a string.

We conducted another experiment outdoors, at the entrance to our faculty building. The entrance is partitioned by wide glass, hindering us from seeing inside the entrance. We set a camera outside the entrance, with the optical axis creating an angle of 54 degrees. As with the previous experiment, we captured a series of images ( 18 images, one every 10 degrees) at a fixed iris. The images are shown in Fig. 7.

In Fig. 7, we know that the bicycles and trees are reflected on the glass and they corrupt the view inside the entrance. We separated the inside entrance scene and the outside scene (reflected on the glass) using Eqs. (11) and (13). The results are shown in Fig. 8. Figure 8(a) shows real objects and we can see inside the entrance clearly: three boards and light switches on the tile wall. In contrast, Figure 8(b) shows the reflected images of a bicycle, a motorcycle, trees and a wall with the same tile pattern as inside the entrance. But we can see the black-and-white reversed image of the boards on the wall. This results from over-subtracting the board image from the maximum image IMAX. Although we neglected the polarization in Eq. (13), transmitted light is somewhat polarized.

The experiment results demonstrate the effectiveness of the proposed method.


Fig. 5 Images captured by rotating polarizing filter (every 10 degrees; left to right)

(a) real object

(b) virtual objects (reflected image)

Fig. 6 Separated images


Fig. 7 Images captured by rotating polarizing filter (every 10 degrees; left to right)

(a) real objects

(b) virtual objects (reflected image)

Fig. 8 Separated images

## 5 Conclusion

We have presented a method for separating the real image transmitted through glass from the virtual image reflected on it. Our method uses the optical property of polarization and does not require any information about the object's position, orientation, or shape. Furthermore, the method requires only simple calculation, comparison, and subtraction for each pixel. It is possible, therefore, to design hardware to perform the separation in real time. Refinement of the separation model (including improving the separated image quality) and the design of a real-time separation hardware are future projects.

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